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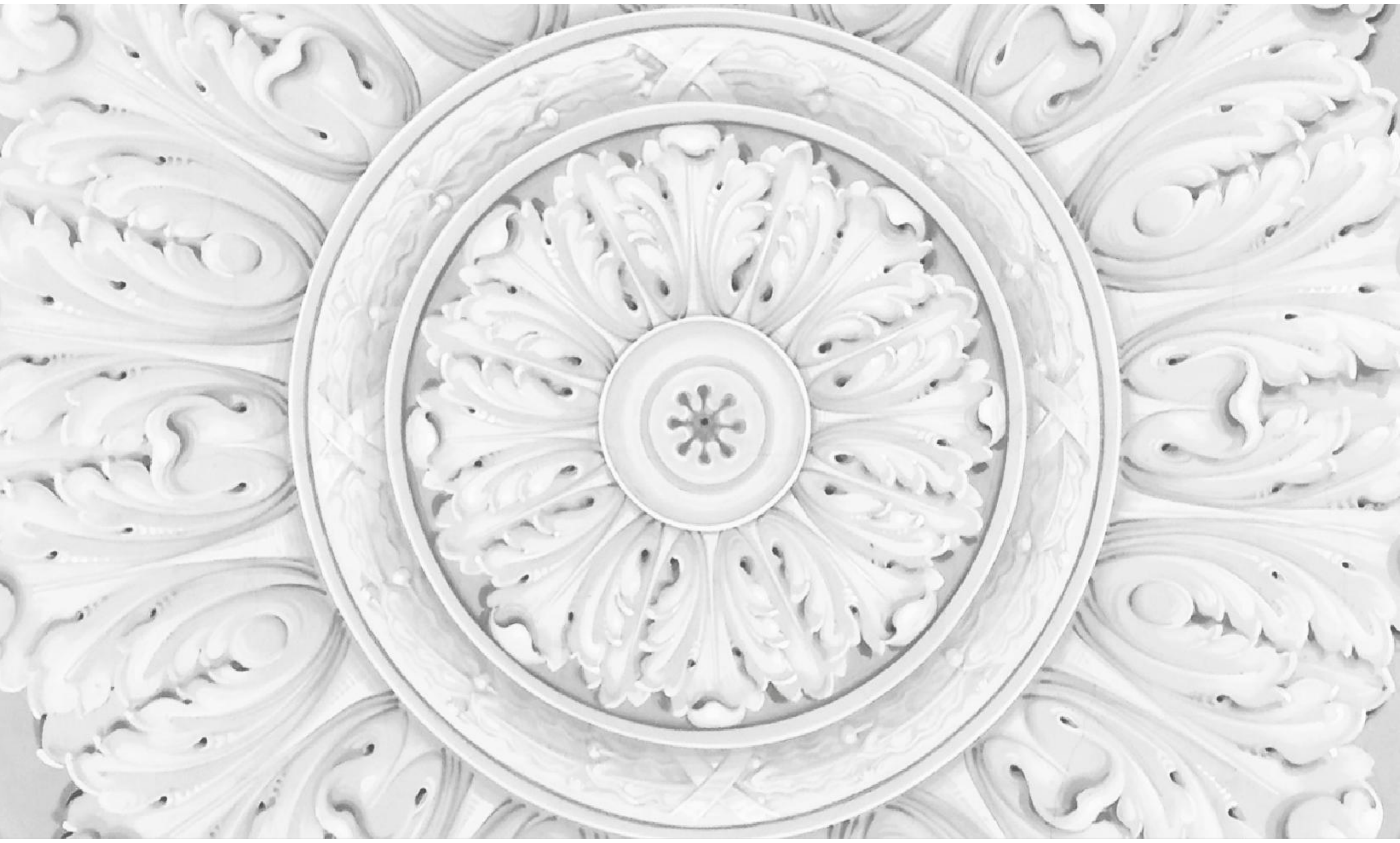

ECONOMIC RESEARCH WORKING PAPERS

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The equilibrium interest rate: a measurement for Russia

No. 13 / July 2016

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The authors would like to thank the Bank of Russia research workshop participants for their comments and also Sergey Egiev for his involvement in the discussions of the equilibrium interest rate concept and his helpful suggestions. Any errors or discrepancies belong to the authors.

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Summary

The aim of this paper is to measure the equilibrium interest rate for Russia both in the short and long run, based on three definitions of the equilibrium interest rate. A general equilibrium model for the Russian economy is being built and gauged. In this real business cycle of a commodity-centred economy with investment, we find that short-run estimates come with very extended confidence intervals (approx. +/-10 pp). In the long run, equilibrium interest rates in the model are set by one of its equilibrium conditions, which in practice will often be applied discretely to find the equilibrium interest rate. This condition-based estimate comes very sensitive to unknown parameters, and is also very uncertain. Additionally, we use the general equilibrium model to study how the equilibrium interest rate reacts to changing oil prices, a rising global rate and growth in consumers' propensity to save. These calculations complement with panel data-based calculations (those for the long-run equilibrium) and computation built on semi-structural methods (for the current equilibrium). Such estimates are also characterised by a high, for practical purposes, degree of uncertainty for the long-run equilibrium, with its point estimates equalling to 1.0% и 3.0%. The point estimate of the *current equilibrium* short interest rate based on semi-structural methods comes at the level of around 0.5%, and the one derived from the interest rate parity is 2.7%. The uncertainty in the measures of equilibrium interest rates calls for a central bank to apply *robust* monetary policy rules.

Key words: equilibrium (natural) interest rate, real business cycle with investment model, potential GDP growth, uncovered interest rate parity, minor open economy of a commodity exporting country.
JEL coded: E32, E43, E52.

CONTENTS

INTRODUCTION

Starting from Woodford's research (1999), the equilibrium interest rate[1] has been an integral component of all macroeconomic models used for decision-making in a monetary course. For all its theoretical importance, as central banks pursue their policies they will often refrain from publicly referring to this concept[2]. This prevents an external observer from understanding the importance central banks attach to the actual to equilibrium rate comparison in the course of their rate decision-making. The key difficulty for central banks stems from the high uncertainty in measuring the equilibrium interest rate[3] (extended confidence intervals of estimates), which holds true for other unobserved indicators, e.g., potential GDP. It also concerns difficulties related to the identification of key drivers weighing on the equilibrium real rate, in particular, those connected with the business to financial cycle correlation (see the criticisms of the equilibrium rate concept in the research by Borio (2015)).

In this paper, we seek to measure the equilibrium interest rate in Russia. In doing so, three definitions of the equilibrium interest rate will be used. Each of the definitions comes with its relevant models being built, which are used to estimate the level of the equilibrium rate in Russia.

Definition 1. The equilibrium interest rate is a rate that is settled in the economy in the long run, i.e., once all shocks have run their course.

Definition 2. The equilibrium interest rate is a rate which is settled in the economy with flexible prices, i.e. such that is aligned to output standing at its potential level at each point in time[4]. This level is not seen in a real economy because of its price rigidities.

Definition 3. The equilibrium interest rate is a rate which is derived in a semi-structural model once the filters are applied, and is specific to a certain non-cyclical rate level. This being

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[1] The equilibrium interest rate is normally understood to be the real equilibrium interest rate. This 'real' attribute means that an interest rate exists in an economy without money, just as the price of real resources to be borrowed. This is the price of resources which can be carried from the next period to the present one. In practice, the understanding of the interest rate level may help develop the understanding of the nominal rate level. The nominal variables (inflation expectations or an inflation target) are subsequently added to the real interest rate to see its level in practice:

$$i \stackrel{\text{def}}{=} r + \pi^e,$$

where i is the nominal interest rate; r is the real interest rate; π is inflation expectations (or inflation).

[2] See, e.g., the difference between FRS and ECB practices: a very wide use: Yellen, Janet (2015) Normalizing Monetary Policy: Prospects and Perspectives. Remarks at the conference on New Normal Monetary Policy, Federal Reserve Bank of San Francisco, compared to its total ignorance in ECB chairman Mario Draghi's statements: *Introductory statement to the press conference, ECB, 2016*.

The examples of early research papers in the field are Amato J. The role of the natural rate of interest in monetary policy. *BIS working paper*. 2005; those of later research are Carlstrom and Charles T. and Timothy S. Fuerst (2016) The Natural Rate of Interest in Taylor Rules. Economic Commentary. No. 2016-01, Federal Reserve Bank of Cleveland.

[3] Refer to: Taylor J., Wieland V. Finding the Equilibrium Real Interest Rate in a Fog of Policy Deviations, 2016. Let us remark here that measuring the nominal equilibrium interest may be more challenging than the real equilibrium interest rate, which is explained by the level of inflation expectations unobserved in practice, unless the assumption is that the expectations are anchored to a central bank target. In such a way, for the lack of observed inflation expectations, it is no easier task to measure even the actual real interest rate in the economy.

[4] This definition is tightly linked to the potential output concept and aligned to the so-called natural potential output. See. Vetlov et al. (2011).

the most blurred definition, it defines the equilibrium interest rate as derived from the Laubach&Williams model (2003), widely used for equilibrium interest rate filtering.

Definition One is focused on a long-term period when the economy enjoys a long-term balance[5] with inflation, which is equal to a central bank target, and GDP which is a match to its potential value. This valuation will serve as a reference point for interest rates in the economy in the long-term period. The second definition considers the equilibrium rate in the current period when the rate is affected by a variety of real shocks and is subject to change when such shocks unfold. The third definition also deals with the short-term dimension[6].

There have been recorded estimates of the equilibrium interest rate as per Definition One in the specific equilibrium. This means that only several key conditions are considered separately; these trigger the equilibrium of the interest rate. Yet, these conditions are not considered collectively – in the general equilibrium. These conditions include the following three arbitrage-free equations:

- Indifference in consumer choice between the current and future consumption (this choice impacts on the volume of savings, defining the correlation between the rate and consumption / potential GDP growth rates)
- No arbitrage between investment into financial assets (at a certain interest rate) and into physical capital;
- No arbitrage between investment into domestic and foreign financial assets – the uncovered parity of interest rates.

The equations describing these conditions are subsequently valuated econometrically (or their structural parameters are merely calibrated – selected based on other assumptions). The derived equilibrium interest rates will not necessarily be identical because the parameters or data may have been valuated imprecisely. It is possible to ensure that they are coherent by, for instance, building the general equilibrium model to understand the ultimate rate convergence mechanism for separate conditions: which of the equation is, in this context, the main one[7]. Importantly, the valuations viewed separately will not match the above equilibrium interest rate definitions – which is to be borne in mind when dealing with such deviations of the equilibrium interest rate. As our report shows, it is the first condition taken separately – the link between the interest rate and the potential GDP growth rate – which meets one of the equilibrium interest rate definitions (namely, Definition One)[8]. Therefore, for the quantification of the long-term equilibrium rate, we give priority to this ratio and, in addition to its calibration, we estimate a panel regression

[5] Here and elsewhere the long-term equilibrium is understood to be a deterministic equilibrium state for the DSGE-model. Only the linear model is meanwhile considered, with higher-order effects neglected.
[6] The way the definitions are different is described in, e.g., in Amato J. (2005) and even some FRS public statements (see Yellen (2015) – Footnote 4).
[7] We intend to show further that in standard equilibrium models for a minor open (commodity-focused) economy a long-term equilibrium interest rate will be determined based on the first condition which links the interest rate level to potential GDP growth rates (more precisely, consumption growth rates).
[8] The rates from the other two conditions should converge to the first condition-determined rate.

of the link between interest rates and per capita GDP growth rates – see, e.g., He et al. (2015) or Hamilton et al. (2015). We use the other remaining conditions separately only for reference purposes, with the proviso that in practice the valuations for the link between the rate and GDP growth can be less certain than derivations from other conditions, in particular, those from the parity of interest rates.

To evaluate the interest rate *in the short run*, all the three conditions will normally be considered all together – in the general equilibrium as per RBC (Real Business Cycles) models – see Mendoza (1991) and Guo&Janko (2009) because in the short run transitional dynamics are essential, which appears impossible to be determined otherwise.

To evaluate the interest rate *in the short run,* we constructed a general equilibrium model for Russia without price rigidities (flexible prices), or an RBC model. In this model, the economy has the properties of the Russian economy: it is a minor open and commodity exporting economy. For our purposes, we viewed the three-sector structure of the economy in the general equilibrium (for mining sectors, domestically producing sectors and externally producing sectors), factoring in capital accumulation (whereby businesses will optimally determine capital stock and invest).

We made an estimate of the model based on Russian statistical data[9]. In this model, we were interested not only in a recovering interest rate level in the short-term equilibrium, but also in the transitional dynamics of the interest rate in the course of its progression from one long-term balance to another. The equilibrium *long-term* level itself was measured based on other models (see above). We looked into the equilibrium rate's reaction to three types of change:

1) a growing future discount ratio (stronger thrift) – to which we attribute a larger share of elderly people and a dropping share of young people in Russia's demographic structure
2) a 1 pp growth of the global exchange rate
3) a 10% drop in oil prices.

To measure the interest rate as per *Definition 3* we replicated one of the filters, most cited on the subject – Lubach&Williams (2003, 2015); the relevant estimations follow. These estimates are reflective of the estimate for a short-term equilibrium interest rate.

Below are our key findings.

1. In the long term, the general equilibrium model for a minor open commodity-focused economy with investment has the equilibrium interest rate determined by Euler's formula, i.e. by the connection between the rate and potential GDP growth rates. In such a way, of the three arbitrage-free conditions, this condition proves dominant in determining a long-term rate. This is a standard result in such models.

[9] A model *with* price rigidities is first evaluated; whereafter, the model parameters are used for a price rigidity-free model.

2. Econometric estimates of the equilibrium rate *in the long run* based on the panel data model (which links the short interest rate of the money market with per capita GDP growth rates and other variables) have very extended confidence intervals. The current point estimate of a short equilibrium rate is equal to 1.0% with a 95% confidence interval equalling to [-10%; +12%], based on data for 30 advanced and emerging economies between 1970 and 2014. The relationship with potential GDP growth rates is meaningful. A 1 pp potential GDP is accompanied with a 1.1 pp growth in the real interest rate; a 1% growth in the saving ratio for one per cent of GDP is connected with a 0.3 pp rate reduction; increased openness of the economy (and, in particular, its capital account) also puts downward pressure on the equilibrium rate (with a non-robust effect, however, which depends on the specification). In developing countries, the rate is more sensitive to the saving ratio, compared to advanced economies, and is in negligible correlation with per capita GDP growth. The latter observation may reflect, in our opinion, beyond possible structural shifts in the estimates, the imperfection of financial markets disallowing a flattening-out of consumption.

3. Point estimates for the equilibrium interest rate in the long run derived from the three arbitrage-free conditions (generated in a specific equilibrium) are shown in Table 1.

Table 1. Equilibrium interest rate calculations in specific equilibrium models

	Choice: consumption vers. saving	Choice: physical capital vers. financial assets	Uncovered interest rate parity
Rate level (range) in the long-run equilibrium	Rate average[10] of deposits and household loans 4.4%–4.7% with no additional assumptions on parameters 0.3%–83.6%	Rate average of deposits and loans to businesses 5.5%–12.5%	Short-term rate in the money market 2.5%–3.5%

Long term, the high level of the interest rate driven by the three conditions varies: the interest rate parity-determined estimations come lower than the rates based on conditions reliant on domestic economic factors. This is partially down to the fact that, in contrast to the interest rate parity calculated for short rates in the money market, the rates derived from the other theoretical conditions are average ruble rates for deposit and loan operations being made by the relevant agents, households and businesses.

[10] Guided by model assumptions, we are unable to compare this rate to either a deposit or a loan rate; we therefore interpret same as a deposit and loan rate average, which might be somewhat unfair from the model perspective.

In addition, these rates price in both risk and term premium, which is why they should exceed those in the money market[11] - see Wieland (2014). In this paper, we show that the rate from the 'consumption vers. savings' choice is the most approximate match to the long-term equilibrium interest rate definition; this estimation however proves very sensitive to the discount ratio, which is estimated with broad confidence intervals.

Should our assumption as regards the latter be wrong, the range of [4.4–4.7%%] turns into [0.3–83.6%%]. Following the available financial market data – which helps improve the quality of estimation – to test the range of possible values we use rates in the *long-term* equilibrium[12] . The interest rate parity-defined equilibrium interest rate in the long term turns out to be around 2.5–3.5%, which fits the confidence interval in the panel data-based estimation (see Para. 2 above), as well as the range of estimations derived from the 'consumption vers. savings' choice. Ultimately, with the panel data-based point estimation factored in, we come up with possible values of the short rate in the future {1.0%; 3.0%}.

4. The estimates for the equilibrium interest rate for Russia in the short term as derived from the general equilibrium model also come with very extended confidence intervals ((over +/-10 pp for the real equilibrium short interest rate in the money market). A precise estimate for the equilibrium interest rate in the money market is very volatile, has sharp fluctuations and is in negative and strong correlation with oil price movements (a real shock for the economy). In such a manner, the current equilibrium interest rate in the general equilibrium will be estimated to total 14% in 2015 Q3, while before the oil price drop in 2014 Q2 it equalled to 4.5%, whereafter in 2015 Q1 it picked up to the peak point of 23%. Our suggestion is not to be guided by its actual levels owing to the high uncertainty of the estimates for the level.

5. Deviations from the long-run equilibrium connected with a changing discount ratio, a falling oil price oil or a growing global interest rate are known to have the following influence on the equilibrium rate in the short-term and long-term periods in the general equilibrium:

- A 0.01 pp growth in the discount ratio (increasingly thrifty consumers) results in a reduced interest rate, a growing stock of capital, expansion in consumption and output in the long-term equilibrium. Short term, the interest rate proves lower than the equilibrium value. This is triggered by agents who, cautious of their future

[11] By how much higher would depend on a balanced excess of the real average deposit and loan rate of banks over the short rate in the money market. Estimates suggests that between 2013-2014 the average spread for deposits / loans to individuals for one year or less was equal to around 7.5 pp, for loans/deposits to/of non-financial organisations for for one year or less the spread was 1.5 pp; yet, this spread is expressed in nominal values and takes no account of, e.g. inflation volatility risk that is difficult to gauge. This is the reason estimates for the real rate may be undervalued.

[12] The estimate from the selection 'physical capital vers financial assets' is also very sensitive to parameters in calibration.

consumption, are energetically cutting back on their debt and make external debt repayments, which finds it way in a dropping risk premium in the interest rate parity

- A 1 pp growth in the US interest rate has no implications for the interest rate in the long run because of repayments of external debt, more expensive now, and the compensatory reduction of risk premia. Having said this, in the short-run equilibrium, the interest rate is quick to drop by 1.4 pp and as quickly (with most of the adjustment taking two years) comes back to its level. The forex exchange rate in the long-run equilibrium strengthens 2%, thanks to reduced external debt repayments. Dynamic debt repayments in the long run, together with reduced risk premia, while temporarily reducing the interest rate, provide incentives for capital investment to expand (a 3% YoY peak growth vers. the equilibrium growth, albeit rapidly decelerating in the aftermath of the peak) and growth in consumption (propelled by rate hike expectations). Eventually, output in the first five years following the shock expands 0.5%

- A 10% drop in the price of oil leaves the long-term interest rate unchanged as the interest rate is determined by long-term growth parameters, and growth is unchanged. The short-term rate meanwhile hovers over the long-term level, gaining 3 pp, it being the way the rate reacts to a growing risk premium because of lower oil prices and to the shortage of resources caused by household consumption slowly adjusting to the new low standards in crude prices. As the economy is undergoing a transition to the long-term equilibrium, consumption drops 1.5% YoY for the peak period, falling almost 4% in the long run, and so does investment. We observe a weakening in the real exchange rate – in the long-run equilibrium it drops 5%. The output of investment goods in the short-term falls almost 4%, while the output of consumer goods, which compete with imports, rises 1%. In the long term, output drops 1 pp. At a point in time, the rate of deceleration reaches 0.5% YoY, albeit slowing down rapidly. Imports of consumer products in the long-run decrease 8.5%, with non-commodity exports rising 3.5%.

6. The valuation of the equilibrium interest rate for loans to businesses for a term of shorter or equal to one year (the last point – 2015 Q1) in semi-structural models proved to total around 3% (which is aligned to the short real rate in the money market of around 0.5% at the average, since c 2013, spread of nominal rates for loans for a term of one year or less to non-financial organisations to the nominal MIACR rate in the money market, which is

2.5 pp[13]). The actual 2007 and 2011 rates came down to levels lower than the equilibrium ones, thereby probably triggering acceleration of inflation in these periods. This, for all the growth in inflation expectations, was not the case in 2014, subsequently helping slow down inflation (consequentially, base effect-free) in 2015.

7. *The short-term estimate of the short equilibrium actual interest rate* derived from the interest parity appeared equal to 2.7%. This method-based assessment is generally no match to any of the equilibrium rate definitions, but in practice may serve as a reference point for the equilibrium rate (in particular, in quarterly forecast models used in a number of central banks). Its advantages include the simplicity of calculation and interpretation.

The paper is structured as follows. In **Chapter 1**, we provide the overview of literature, together with the description of theoretical approaches to the quantification of the equilibrium interest rate and the results of their empirical testing. In **Chapter 2**, we look into three arbitration-free conditions as components of the general balance equilibrium model, key to the quantification of the interest rate. **Chapter 3** presents an assessment of the general equilibrium model, as well as estimates of the equilibrium rate as we analyse the rate's reaction to change in individual variables or parameters (oil prices, the global interest rate, change in the subjective discount factor). **In Chapter 4**, we provide estimates for the equilibrium rate in semi-structural models, i.e. those based on Definition Three, and an assessment of the equilibrium rate on the panel data-based model. **The Conclusions** present key findings of this research.

[13] This spread, as long as it leaves out inflation volatility premium, may in real terms be even lower, while the real equilibrium rate may be higher.

1. REVIEW OF LITERATURE

Various approaches to measuring the equilibrium rate as exemplified by the US economy are provided in the papers of Woodford (2000), Amato J. (2005), Laubach&Williams (2015), Pescatori&Turunen (2015); empirical estimates for the equilibrium interest rate (for a large group of countries) are given in the works of Hamilton et al. (2015). A clear account of definitions and calculations for the actual interest rate is presented in Lundvall, Westermark (2011).

The equilibrium interest rate, as well as the rate corresponding to the long-term equilibrium of the economy (and target inflation), i.e. to Definition One, is investigated in the works of Fuentes&Gredig (2007), He et al. (2015).

The models for estimating the interest rate in the short-run equilibrium include those of a real business cycle (or neo-Keynesian models with nominal rigidities disabled) – see Giammarioli N. and Valla N. (2004), Barsky et al. (2014), Curdia et al. (2015).

Estimates as per Definition Three which are, as a rule, based on the semi-structural approach are found in the works of Laubach, Williams (2003, 2015).

Empirical estimates for equilibrium interest rates are fairly common in the literature: Kirker (2011), IMF World Economic Outlook (2014), He et al. (2015).

The general equilibrium models set key patterns (equations, restrictions) which predetermine the dynamics of interest rates (including the equilibrium one). In particular, the general equilibrium models are used to conclude that the equilibrium interest rate meets three restrictions (hereinafter, arbitrage-free conditions):

1. Consumer choice equilibrium (Euler's formula)

The adjacent real equilibrium rate concept is based on C-CAPM (see Giammarioli N. and Valla N. (2004)). Empirical estimation is contained in: He et al. 2015, Fuentes&Gredig (2007):

$$\mathrm{r} = \sigma g - ln\beta, \quad (1)$$

where r is the real interest rate; g is per capita GDP growth rate (income or consumption); σ is the value inverse to the elasticity of intertemporal substitution; β is the subjective rate of intertemporal preferences (discount ratio).

These key predictions under this model are definitely supported by practice – see Hamilton et al. (2015), Kocherlakota (1996) and Mehra and Prescott (2003).

2. The uncovered interest rate parity as applicable to the Chilean economy – see in Fuentes&Gredig (2007), for Brazil – Perrelli, Roache (2014):

$$r = r^f + \Delta E[p_{t+1}] + \ risk\_premia, \quad (2)$$

where r^f is the forex interest rate used for arbitrage operations; $\Delta E[p_{t+1}]$ is the expected real forex exchange weakening *p*; $risk\_premia$ is the premium for the risk of investment into national assets.

3. The uncommon (common only for models with investment) condition is determined by the requirement to level off investment into financial assets and physical capital. This condition is described by the equation – see Hall and Jorgenson (1967) or Creedy&Gemmel (2015)[14]:

$$r + \delta - \Delta P_{capital} - \Delta P_{GDP} = MPK, \qquad (3)$$

where MPK (marginal product of capital) is the marginal product of physical capital (in terms of real GDP); $r + \delta - (\Delta P_{capital} - \Delta P_{GDP})$ is the user cost of capital) in real terms; $\Delta P_{capital} - \Delta P_{GDP}$ is change from the relative price of physical capital in terms of consumer products or GDP deflator; δ is the rate of capital depreciation for the period.

The literature also contains descriptive models of the equilibrium between the supply (savings) of loanable funds and demand for them (investment), which use demand and supply factors to investigate the price of loanable funds. These models rely on individual results of a number of theoretical models and estimate the direction of its change, rather than its actual level. Most often than not, in these non-model descriptions (see Blanchard et al. (2014), IMF WEO (2014)) the interest rate depends on factors determining national savings, investment and investors' risk sentiment (the relative demand for risk-free and risky assets).

Here we come across the following saving factors:

1. The population's age structure (shifts in intertemporal preferences). The age structure has a role in both the economy-average saving ratio and potential output growth rates (when younger workers can perform better because of technological advance). A growing saving ratio (average age dominance) puts downward pressure on the equilibrium interest rate – see Ikeda&Saito (2012).
2. Higher potential GDP growth rates lead to growth in the interest rate, other things being equal. The underlying factor in this impact is consumption smoothing (as individuals desire to have a steady consumption plan for the rest of their life, resulting in partial preservation of the additional current income), with expectations that the income will in the

[14] Formula 3 is sometimes adjusted for tax. In particular, the marginal corporate tax is deducted from the marginal product. As this adjustment is being made, however, it is important that return on financial assets be also adjusted for tax, that is, we deal with the need to introduce the difference between the interest rate *before* and *after* tax. Below we show that with tax amounts factored in, given the different rates for interest rate taxes and corporate taxes in Russia, the equilibrium interest rate will have no implications, conditional to the equality of physical and financial asset return: which is why we leave tax out to simplify the formula.

future grow quicker than before. This link between growth and the rate is in theory set by Euler's formula.

3. Growing debt load with subsequent deleveraging – see Eggertsson, Mehrotra (2014). Accumulation of debt is accompanied with high demand for savings, which at the time of deleveraging is replaced by rising savings enabling the repayment of debt. The associated contraction in demand needs to be set off by reducing interest rates, which stimulates consumption and investment (decreased savings) at the expense of the resources released in the economy.

4. Fiscal policy. Budget deficits are considered to trigger a rise in the interest rate as the economy's private sector obtains for investment a smaller part of the national savings. This logic is overly simplified and fails to take heed of, in particular, the fact that a softer budget policy impacts on income. The resulting total savings in the economy may grow up – instead of contraction. Neither recognised is the open type of modern economies or their close connection with the international capital market. The channel really important for the budget policy impacting on interest rates is the way investors evaluate sovereign risks – which is directly included into the interest rate parity-based definition of the interest rate. Accumulation of public debt (especially that of external one) is in positive correlation with the assessment of sovereign risk – see B1 (2011).

5. Restricted capital flows (closed financial account). Restricted national access of the global pool of savings leads to growth of equilibrium interest rates in the economy, other things being equal.

Investment factors are user costs of capital, which are different from interest rates: the speed of capital depreciation, the speed of change in the price of investment products and, on some occasions, taxes. The relative cheapening of investment products, e.g., because of technical advance or shifts in the structure of the economy in the service sector, will impact on investment (possibly, temporarily) that also has implications for the equilibrium interest rate.

Risk premium factors:

1. Fiscal policy: core sovereign risks are usually related to fiscal policy.
2. The Inflation volatility premium.
3. The uncertainty in the economic policy or oil prices.
4. The extent of risk aversion (equity premium).

The estimates for the global equilibrium interest rates are reflective of a number of global trends widely discussed – see IMF WEO (2014), Lubik&Matthes (2015):

- Savings glut and the trend towards a decline in equilibrium interest rate rates since 2000s

- A declining relative price of investment products
- Potential GDP growth slowdown measured as the average growth over an extended time period, and the directly proportional link between the rates and potential GDP.

In the chapter that follows, we will elaborate on real interest rate modelling based on the above-mentioned three key equations.

2. THE EQUILIBRIUM INTEREST RATE IN THE SHORT AND LONG RUN: SPECIFIC EQUILIBRIUMS

In this chapter, we will elaborate on the three arbitrage-free conditions which determine the equilibrium interest rate in structural models (in the long run and, less often, in the short run). The resulting interest rate is an equilibrium one only in the context of it being a match to a specific condition, and it therefore does not have to be aligned with the above definitions of the equilibrium interest rate (save for, as we intend to show in the next chapter, the link between the growth in the interest rate and potential GDP growth). However, the need to look into specific equilibrium stems from the tribute to tradition in the applied research of equilibrium rates, for one, and, second, this review enables to get a feel for their level (back of envelope calculations) and determine how reliable such estimates are to a change in parameters.

2.1. The equilibrium interest rate and potential GDP growth

The key condition to measure the equilibrium interest rate is the optimal choice made by households may be between current and future consumption (savings), or Euler's formula[15]:

$$E[\frac{c_{t+1}}{c_t}]^{-\sigma} * \beta * \ (1 + r_t) \ = 1, \qquad (4)$$

where c_t is the per capita consumption in the economy in the current period; c_{t+1} is the per capita consumption in the economy in the subsequent period; E [*] is the expectation operator; $0 < \beta < 1$ is the rate of intertemporal preferences (the future discount ration); $\frac{1}{\sigma}$ is the elasticity of intertemporal substitution.

This equation sets the supply of savings (financial assets) in the way that the higher the rate is, the lower the current consumption is (and the higher growth for the future consumption) and the higher the resources are for investment, other things being equal. The consumption growth and interest rate link may be inverse: the higher consumption growth means that tomorrow's consumption (income) will be higher than the current one. The reason to smooth consumption (individuals' desire to have a steady consumption plan for life) leads to demand for consumption growing today when the income is still insufficiently high. It triggers growth in the actual interest rate through stronger demand for loanable funds for consumption smoothing. As a result, there is a link emerging between long-term income growth (more precisely, consumption growth) and the actual interest rate for deposit and loan operations of households – Equation 1:

[15] The equation is derived for the function of utility of the form $C \ = \frac{c^{1-\sigma}-1}{1-\sigma}$. The setting of the problem and its solution are found in: Barro, Sala-i-Martin, Economic growth, MIT Press, 1999.

$$r = \sigma g - ln\beta,$$

where g is the per capita economic growth rate; σ is the value inverse to the elasticity of intertemporal substitution.

Both the literature and practice show the very common use of exactly this ratio for measuring the equilibrium interest rate level. Importantly, limitations of this approach need to be remembered, as detailed below:

1) Equation One is one of the conditions which set the equilibrium interest rate in the general equilibrium, yet not the only one. It really is sufficient for the long run (the rate is determined by potential GDP growth dimensions – the rate of change in the population and technological advance); however, to understand the short-term dynamics of the equilibrium interest rate in the aftermath of some shocks, we should examine the interest rate in the general equilibrium. Indeed, in the short run, a shock in risk premium, along with capital accumulation constraints, may lead to lower growth along the new equilibrium path. However, in the long-run the equilibrium interest rate will may be set by Equation One (1).

2) The formula assumes that individuals smooth consumption, in particular, that financial markets make it possible for consumption to be deferred until the future or to borrow, i. e., there are no credit restrictions. In the case of credit restrictions, the interest rate is higher – see the Krugman-Mehrotra model (2014);

3) The formula neglects fiscal policy effects.

We calibrated key elasticities in Formula (1) enabling to estimate the equilibrium rate for Russia in the long run.

The equation's calibrated parameters are shown in Table 2.

Table 2. Parameters for estimating the equilibrium rate for Russia from Euler's formula

Parameter	Value	Source	Note
$\frac{1}{\sigma}$	4,167 1	Khvostova et al. (2014) – for Russia Standard calibration - see Christoffel et al. (2008)	Elasticity of Intertemporal substitute
β quarterly	0,85–1,098 0,99	95% – confidence interval Khvostova et al. (2014) – for Russia Standard calibration – see Yogo (2005)	95% – confidence interval for the subjective discount factor

Our estimates for potential Russian GDP in the per capita growth model are shown in Appendix 1.

With the confidence interval for β used for Russian micro data, according to Khvostova et al. (2014), factored in, as well as the content restriction for $\beta \leq 1$ for these estimates where per capita annual GDP growth is 1.2%-2.6%, we come up with the range for the real rate to be at the level [0.3%; 83.6%] per annum. The equilibrium rate estimate proves extremely sensitive to the subjective discount factor valuation. With the discount ratio applied (which is most common in the literature, see Yogo (2005), at 0.99 a quarter, which enters the confidence interval for Russian data-based quarterly estimates, we will come up with a range for the interest rate (subject to economic expansion rates) of a [4.4%; 4.7%] p.a. type[16].

Overall, the uncertainty in the estimates of the subjective discount factor takes much from the practical value of the method[17].

2.2. A Relationship with the global interest rate

Equilibrium in the demand for domestic and foreign financial assets, or interest rate parity, is determined by a relationship between the domestic asset market with that of the global market. Deviation in the global interest rate from the domestic interest rate will imply, all other conditions being equal, movement (with a lag) of global resources into the country or that of domestic ones – from the country.

The ruble equilibrium interest rate will then must be a composite of the real US dollar equilibrium rate[18] with the same maturity, the equilibrium premium for sovereign risk and equilibrium change in the real ruble to dollar rate. We will consider each of these components separately in connection with the short rate of the money market in Russia[19].

A. The US dollar rate

The FRS-calculated[20], short-term (Federal funds rate) equilibrium real rate in the US is estimated in the short term at the level of about zero, in the long run (beyond 2018) its upward revision of 1.0–1.5% is expected. The annual term premium is estimated by the FRB of New

[17] Updated estimates for discount factor and intertemporal substitution elasticity, for example, from DSGE model or econometric calculations, could provide more precise valuations of the equilibrium rate; however, in case of Russia, we face the problems of structural shifts and a short sample length. In such model, estimates for an equilibrium rate will be a certain average of historical data with several reservations.

[18] Parity may also take account of a portfolio of foreign currencies, which in linearity can be reduced to parity with one currency (US dollar), thereby adding to interest rate parity the change in rates of these currencies to the US dollar. It is therefore sufficient to consider parity only with one currency.

[19] To reiterate, the rates estimated in this method in the short run have nothing common with the above three definitions, providing merely an illustration to the rear cases of them materialising in practice. These estimates, while they bear no sense in the context of short-term real equilibrium rates, are used in practice as reference points.

[20] US Fed Chairman Janet Yellen's speech at the economic club meeting in New York, 29 March 2016, where she refers to the latest equilibrium rate estimates in the USA. See also FOMC Minutes dated 6 July 2016. https://www.federalreserve.gov/monetarypolicy/files/fomcminutes20160615.pdf

York[21] to be about 50 bps – this is the amount needed for the short rates of the money market to annual rates.

B. Equilibrium risk short-term and long-term premia[22]

There exist several methods for estimating the premium for Russia's sovereign risk, an important factor in investment decision-making as regards purchase of Russian assets.

B.1. **CDS premium-based estimates**. CDS premium is reflective of the payment of insurance against default under government bonds in basis points to the face value[23].

To calculate the premium risk, we used two comparison bases: that of CDS Russia to the USA premium and the median CDS group premium for the countries with a credit rating similar to Russia (Moody's long-term foreign currency rating) to a median rating of highest credit rating countries (including the USA). In the first case, the risk premium will cover premia for risks specific to Russia.

Our calculations used five-year CDS premia which, unlike one-year ones, appear to be a more liquid tool taking account of risks over a five-year horizon, which is why they can overvalue both the risks and the equilibrium rate estimate in the short run[24].

Our econometric estimates show that the average five-year CDS premium for the countries with a credit rating comparable to Russia (Ba1 according to Moody's grading scale) totals 190 bp to a CDS premium of the highest credit ranking countries with (see Figure 1). Given that long-term credit ratings reflect long-term trends in risk assessment, the above estimate can be considered as an estimate for a long-term risk premium, i.e. +190 bp.

The Russia risk premium in relation to the USA until 2014, dropping oil prices and negative geopolitical factors, was hovering between 100–200 bp, 150 points on average (slightly below the previously calculated +190 bp). Since 2014, the average risk premium was 300 bp. This value can be considered as an estimate for the short-term risk premium.

B.2. **Estimates based on inflation-protected bonds** – see. Bomfim A.N. (2001).

Estimates on the basis of seven-year bonds providing protection against inflation enable to get a general sense for inflation expectations of financial market participants (including the inflation volatility premium) and about the real yield of bonds priced in by market participants.

[21] URL: https://www.newyorkfed.org/research/data_indicators/term_premia.html.

[22] We use the data specific to long rates to quantify the risk premium. With the risk premium different from the term premium, the estimates for risk premiums may also be applied for short rates.

[23] It should be noted that the estimate as the cost of to the face value ratio is not a strictly mathematical estimate of the risk premium. The precise estimate is complicated enough and depends of a multitude of factors.

[24] With the liquidity premia for long and short CDS being different, the ultimate difference will be minor.

Comparison of the actual interest rates of inflation-protected bonds (TIPS) in the US and Russia enables to get an idea of the real exchange rate changes expected by market participants and the sovereign risk premium (see Figure 3).

The spread, which totalled about 3 bp in the middle of 2015, decreased in October 2015 to 2.0 bp against the backdrop of a weaker ruble and the expected Fed rate hike. This was followed by the January's weakening of the ruble (to adjust expectations for a further weakening from the interest parity perspective) and expectations for a more moderate growth of the Fed rates, when the spread rose.

Growth continued into March, reflecting expectations for a more moderate normalisation of the US Fed's monetary policy and the actual strengthening of the ruble (slighter strengthening in the expectations). Since October 2015 the spread had hovered within the 1.8 bp-2.2 bp bound, which, considering the long term to maturity, represents a long-term estimate for the risk premium (taking into account the expected change in the real currency rate over a seven-year horizon). In this case therefore, we come up with financial markets' estimates for both the risk premium and change in the real currency rate over the specified time horizon.

B.3. **Estimates based on the yield spread between Russian sovereign Eurobonds to the US Treasury securities with the same term to maturity**. This spread will also cover the liquidity premium). For a liquid issue of the Russian Eurobonds with repayment for 2023, the spread to yield before repayment of the US Treasury securities with the same term has since 2013 averaged 3 bp, without the crisis period between the second half of 2014 and the first half of 2015, was 2.5 bp (see Figure 4). In the 2014 crisis, the spread grew to 6.5 bp. The spread reflects the liquidity premium over the specified horizon. The similar spread between two-year synthetic bonds is also shown in Figure 4, demonstrating similar movements in risk premium estimates (inclusive of the liquidity premium) – about 2.5-3 pp.

B. Equilibrium change in the currency exchange

The last item to enable us to come up with the equilibrium interest rate level for Russia is the adjustment for the market-expected weakening in the real currency rate. We derived this estimate from the Bloomberg forex rate forecast model. Density in the distribution of the nominal ruble to US dollar exchange rate at the end of the first quarter of 2017, as evident from the volatility of fx-options, helps measure the expected dynamics of the nominal ruble exchange rate. The adjustment of this estimate to an inflation differential (Russia vers. USA) for one year ahead allows to measure the expected dynamics of the *real* currency rate. These estimates show that as early as late 2017 Q1, the real ruble to dollar exchange rate is set to strengthen by about 0.4%[25].

We neglect this amount. In the long term, we believe that the real currency rate (subject to constant global oil prices) will be invariable.

In general, for a real *short* interest rate (a rate of the money market) in Russia from the arbitration-free standpoint, the following data are obtained (see Table 3).

Table 3. Components of the real *short* equilibrium interest rate based on the interest rate parity condition

	Real rate, %	Equilibrium real US rate, %	Risk premium estimates, pp	Expected forex rate weakening
Short term	2.5–3.0	0	2.5–3.0	0
Long term	2.5–3.5	1.0–1.5	1.5; 1.9; 2.0	0

Consequently, considering the current higher risk premia, an equilibrium real arbitrage- free interest rate is estimated at about 2.7%, this being the average of 2.5 and 3.0%. In the long term, it is set to decrease on the back of 0.7–1.2 pp reduction in the risk premium, to be set off by an increase in the US risk-free rate. Taking heed of market participants' expectations (Bloomberg data) as regards inflation reaching 6.8% in Russia by the end of 2017 Q2, it is possible to forecast that the current equilibrium nominal interest rate in rubles will hit the mark of 9.5%, while the equilibrium *short* rate in the long run is estimated to be at about 6.7-7.5% (4% of inflation + 2.7 or 3.5%). To calculate the annual equilibrium rate, 0.5 pp should be added.

The fact that direct estimates of the interest parity's components are possible, on the one hand, renders this method-based estimation of the equilibrium interest rate more reliable than others; however, a number of assumptions (as to estimates for the risk premium / equilibrium real exchange rate) make ultimate reliance on its accuracy impossible.

2.3. A Relationship with the performance of physical capital

Let us consider an arbitration-free choice (equilibrium choice) between investment into real or financial assets. The interest rate is meant to counterbalance return from financial vers. real asset investment.

[25] Calculations based on Bloomberg fx rate forecast model: weakening in the median nominal exchange rate by the end of the first quarter of 2017 analysed as per the distribution of currency rate expectation vers. the current level (31 May 2016) is equal to 3.5%. Adjusted for the inflation accumulated since June 2016 through the end of the first quarter 2017 (which indirectly, as per Bloomberg expectations, totals 5.5% less the level of inflation expected for 10 months in the USA – that of 1.7%, strengthening in the yearly real currency rate by the end of the first quarter 2017 will reach about 0.3 pp, i.e. about 0.4 pp in annual terms.

Let us imagine an individual choosing whether to acquire a real asset (plant) or invest into a financial asset (to purchase bonds or a deposit) for a term of one year. The return on investment in a real asset will consist of the cost of a product which this asset will create through an additional capital unit (marginal productivity of capital – MPK) and the resale price of the capital in one year, less depreciable cost (physical wear-related losses) and tax on capital – see, e.g. Hall&Jorgenson (1967). Investment into a deposit will bring some real interest.

The non-arbitration condition calls for compliance with the following ratio (Equation 3):

$$r + \delta - \Delta P_{capital} - \Delta P_{GDP} = MPK.$$

It is clear from this ratio that the additional product from increased stock of the capital per unit of (marginal product) has to be equal to the costs of acquisition of this additional capital (user cost of capital). Any deviation is adjusted through a capital stock. This equation sets demand for investment (demand for capital). The higher the rate is, the lower demand is for the real capital.

Higher capital productivity results in return on investment into real assets exceeding that into financial assets, the rate being equal. We see resources flow from financial into real assets, levelling off the two types of return on assets. In the general equilibrium, all other conditions being equal, interest rates will be expected to grow.

The equilibrium interest rate expression follows from this expression[26]:

$$r = MPK + \Delta P_{capital} - \Delta P_{GDP} - \delta. \quad (5)$$

For further calculations of the current equilibrium interest rate we assume that δ = 5% p.a. $(\Delta P_{capital} - \Delta P_{GDP})$; the difference in growth rates by investment and GDP deflator. The greatest difficulties are presented by estimations of marginal MRK capital performance. The simplest solution for this problem in the literature is the Kobba-Douglas production function of (see He et al. (2015)) for which the marginal capital performance is equal to:

$$MPK = \alpha * \frac{Y}{K}, \quad (6)$$

where α is the contribution of capital to GDP; $\frac{Y}{K}$ is capital productivity (GDP to the stock of capital ratio). For Russia, $\frac{Y}{K}$ averages for 2000–2014 about 0.3–0.5 according to various stock of

[26] We also estimated this ratio as adjusted for taxes. As a corporate profit tax for Russia, a KPMG database indicator was used; this one defined the rate for Russia at 20%. URL: https://home.kpmg.com/xx/en/home/services/tax/tax-tools-and-resources/tax-rates-online/corporate-tax-rates-table.html. The tax interest income tax rate was assumed to be 35%. Once included, taxes made only a negligible effect on the estimates.

capital estimations we made based on Rosstat, World KLEMS Russia и IMF data[27]. The contribution of capital to product is roughly 0.35. Starting from 2011, capital products were on average becoming more expensive at a 1% higher rate than end products.

In all, the interest rate receives the following range of values [5.5%; 12.5%] p.a., which is explained by different estimates for capital productivity in Russia.

The range of estimates $PK - (\Delta P_{capital} - \Delta P_{GDP})$ for other countries is contained in the paper of Caselli&Feyrer (2007). In Russia, the corresponding value is equal to 0.083–0.155, subject to the stock of capital, which is within this range for 53 countries [0.07; 0.23], but closer to its lower bound which corresponds to developed countries.

Overall, this method-based estimation is rather unreliable as it is highly problematic to estimate the marginal capital product in the economy and capital depreciation rates.

[27] The calculations were made based on an inventory count method, being built on Rosstat data on fixed capital investment in 2008 prices and depreciation rates, which depend on fixed capital employed (as per the Russian barometer), which, with the 2000–2014 average being assumed to be 5% p.a. World KLEMS Russia data (http://www.worldklems.net/data.htm) include the capital services indicator, which, similar to Rosstat data, can be used to estimate the stock of capital. IMF data as requested in the Russian Federation: Selected Issues, IMF Country Report № 14/176, 2014.

3. THE EQUILIBRIUM INTEREST RATE IN THE GENERAL EQUILIBRIUM

In this chapter, we are going to consolidate all the tree conditions which quantify the real interest rate, through a RBC general equilibrium model. Appendix 2 includes a detailed description of this model for the Russian economy. We consider a standard RBC model of a minor open economy (see the similar model that, however, does not consider capital – in the work of Kreptsev-Seleznev (Крепцев-Селезнев (2016)).

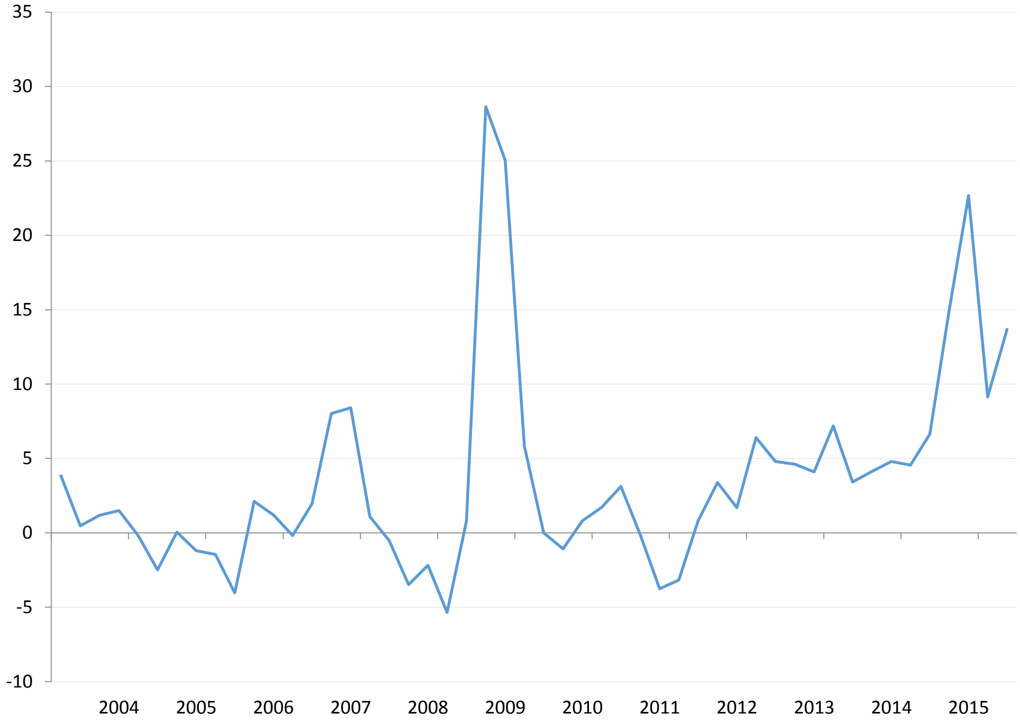
The model for Russia features the following key qualities:

- Recognition for the structure of the economy and structural shifts in the aftermath of real shocks (the oil price shock)
- Capital accumulation by firms in the environment of capital adjustment costs. More often than not, similar models, as they quantify interest rates, would leave out capital accumulation, thereby disabling one of the equilibrium rate defining conditions – see Barsky (2014)
- Due regard for household-implemented optimisation of both their consumption and the number of hours worked, as well as consumption habits
- Some parameters of the model are quantified based on statistical data from the complete model including price rigidities and a central bank. For quantification, the model uses data on real GDP (both for Russia, including expenditure data, and the global economy), inflation (in Russia and its trading partners) and the MIACR interest rate. Some of the parameters are calibrated – see Kreptsev-Seleznev / Крепцев-Селезнев (2016).

The estimate for the current equilibrium interest rate (according to Definition Two) in this model is shown in Fig. 5. This estimate is very rough and has very extended confidence intervals (they are not shown in the figure)[28].

[28] The confidence intervals are extended because estimation parameters come with uncertainty, as well as because of the uncertainty arising when the Kalman filter is used to quantify unobserved values. These estimations are also specific in the sense that we study linear approximation, leaving strongly negative interest rates totally unrestrained.

Figure 5. The current equilibrium *real* rate of the money market (MIACR) in the general equilibrium for Russia model, % p.a.



It was essential that we use this model to analyse how the equilibrium interest rate reacts to the following shocks:

1. Demographic changes related to the growing share of older people in the demographic structure, which is characterised with weaker future discounting: households tend to show more propensity to save. The challenges related to search for the alignment between the current demographic changes and their impact on the discounting ratio are behind the reason why this case is mostly interesting from the academic perspective. We consider the yearly ratio β to grow as determined by the formula similar to equation (4), by 0.01.
2. A 1 pp growth in the global interest rate
3. A 10% decline in the price of oil.

Below are the results of each of the changes disrupting the general equilibrium (see also figures in Attachment 3):

1. **Increased rate of intertemporal preferences** (a 0.01 growth in discounting ratio).

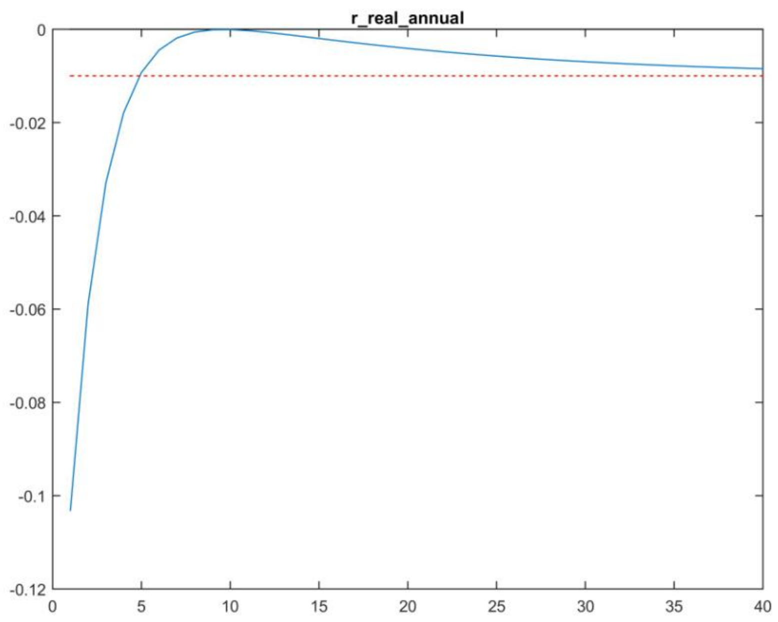
This similar structural shift is supposed to reflect change in the demographic structure of the population – a dwindling share of younger people who aggressively discount the future income (‘let us live for the moment’, or *impatience*). Growth in the value of future consumption leads to an almost 1 pp decrease in the equilibrium rate in a new equilibrium (see Figure 6), with the interest rate beating its equilibrium value, dropping by 10 pp and embarking on a subsequent monotonous growth.

A decrease in the equilibrium rate is triggered by households beginning to reduce the debt actively (to accumulate savings) to provide for a higher level of future consumption – which they value more now. The interest rate is 10 pp lower than the new long-term equilibrium. It is set to grow afterwards in one year's time, to reach a new balance. Because of the smaller long-term level of debt, future consumption is 10% higher. This level is reached in 10 years.

The real forex rate in the equilibrium is 7.5% stronger. As firms are undergoing a transition to the new equilibrium, they are accumulating capital, using the benefit of the current low rates and expecting their growth. The resulting GDP growth rate accelerates temporarily by 2 pp and thereafter gradually decreases. In response to stronger demand for capital the price of capital products grows, partially setting off the lowering effect on capital costs from lower rates. In response, firms will increase the imports of investment goods.

Lower external debt leads to decreased risk premia which, together with the strengthening of the real forex rate, ensure compliance with the non-arbitration condition while interest rates in the long term are lower.

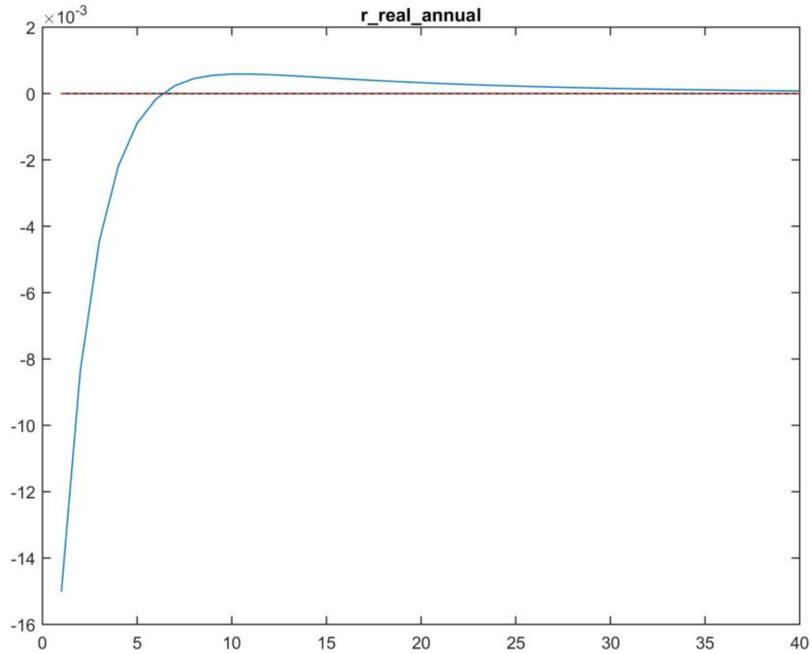
Figure 6. The equilibrium rate's reaction to a weaker future discounting in the long-term equilibrium and transition-to-the-equilibrium data, 0.1= 1 percentage point



2. A 1 pp growth in the global interest rate brings no change to a long-term real rate in the model for Russia because of the offset coming from the declining country risk premium[29]. In the short term, the equilibrium rate drops 1.4 pp (below the equilibrium level) and then in a year returns to the equilibrium level (see Figure 7).

A reduction in the short-term equilibrium rate occurs on the back of rapid external debt repayments as the external debt becomes overly expensive because of rising external rates. External debt repayments put strong downward pressure on the risk premium, leading to a recovery in the interest rate parity. The real currency rate therefore gradually strengthens, rising 2% against the old long-term equilibrium. Expectations for rates to grow, while the current rates are low, spur growth in consumption and investment. Investment rises with a peak rate of 3% to the corresponding quarter of the previous year, but slows down quickly. The temporary period of lower rates results in the economy increasing output (GDP growth with a slower speed), accelerating 0.2 pp at its peak YoY, so that in 10 years its level gains about 0.5 pp.

Figure 7. The equilibrium rate's reaction to a 1 pp growth in the global rate and transition-to-the-equilibrium data, 0.01=1 percent point



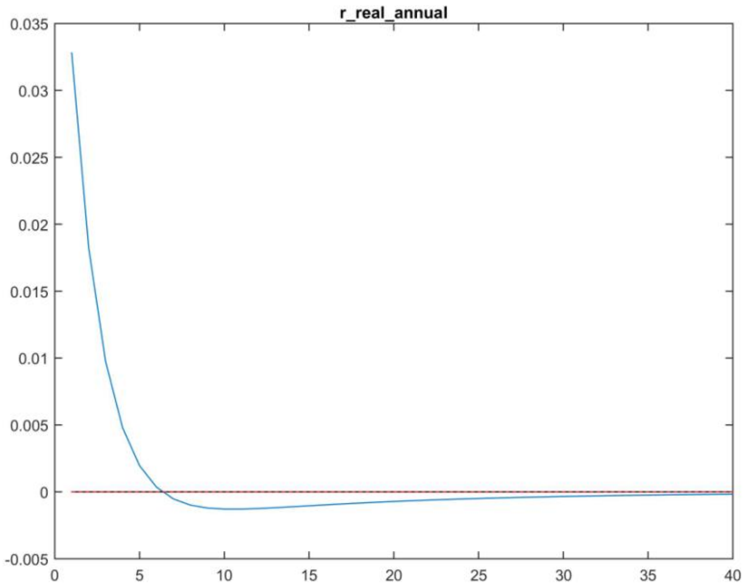
3. A 10% drop in the price of oil in the long run leaves the equilibrium interest rate unchanged, while in the short run it is set to rise 3 pp (see Figure 8).

[29] This model result is derived because the level of debt is the only country risk factor. If the sovereign premium is less sensitive to the level of debt (or absolutely insensitive), the domestic rate is set to grow; however within the standard small open economy model we consider this fact cannot be simulated for a number of technical restrictions (for more details see Schmitt-Grohe S., Uribe M. Closing Small Open Economy Models. Journal of International Economics. 61 (October 2003). P. 163–185

In the economy at the stage of transition to the long-term equilibrium, consumption falls (minus 1.5% of Y/Y for the peak values); in the long term it decreases 4%, and so does investment. The real forex rate is weakened – in the long term, it is 5% weaker.

The output of investment goods drops almost 4%, and that of consumer goods, competing with imports, grows 1%. Long term, production drops 1 pp. In the moment, the rates of output reach 0.5% YoY, but quickly slow down, remaining in negative territory (see Appendix 3). Imports of consumer goods decline 8.5 in the long term %, non-commodity exports grow 3.5%. The trading balance adjusts quickly, taking up one year from the occurrence of the shock.

Figure 8. The reaction of the equilibrium interest rate to a 10% drop in oil prices and transition-to-the-equilibrium data, pp



4. SEMI-STRUCTURAL ECONOMETRIC MODELS OF THE EQUILIBRIUM INTEREST RATE

4.1. Long-term equilibrium estimates in panel data

Our empirical estimate for the equilibrium interest rate in Russia uses the commonly used specification (see Hamilton et al. (2015), He et al. (2014)) of the dependence of the interest rate (the short rate of the money market, in our case – see data descriptions in Appendix 4) on potential GDP growth rates and other factors:

$$r_{it} = c + \beta_1 g_{it} + \beta_2 s_{it} + \beta_3 KAopennes_{it} + \varepsilon_{it}, \quad (7)$$

where r_{it} is the real interest rate in the country *i* in the period *t*; g_{it} is per capita potential GDP growth in the country *i* in the period *t*; s_{it} is saving ratio in the country *i* in the period *t*; $KAopennes_{it}$ – economic openness indicator (capital account openness) or a financial development level.

This is the specification we use to actually estimate Euler's formula (1), which connects the rate with potential output growth rates, the subjective rate of intertemporal preferences (with their impact on the rate to be reflected by the saving ratio) and economic openness indicators. As follows from the general equilibrium model, which fails to cover rate equilibrium factors, the long-run equilibrium rate can change as a result of shocks, and the above-mentioned factors are supposed to cover these changes. The choice of regressors may probably be more accurate. This paper presents only the first method results based on long-term statistics available for a large number of the countries.

Our input data were average for five consecutive years on 30 developed and developing countries from 1970 to 2014 (8 five-year periods). This type of averaging was used so the business cycle-related effects could be eliminated. Data sources are shown in the table in Appendix 4. The real interest rate is calculated as an ex-post real rate (the World Bank's alternative data came in very close). As a tightness indicator for the relationship with the global economy and financial markets, we used the capital account openness indicator Chinn-Ito (2006) [30], which the authors calculate regularly.

To eliminate the problem of endogeneity, we applied the instrumental variable method. As an income growth tool, GDP growth was used in the previous point in time (including also the attempt to use the economic (capital account) openness indicator as a GDP tool in the previous point in time), the instrument variable for a saving ratio was the share of the population aged 15–65 years and the current account level (in terms of % to GDP).

This index is stationary by construction and changes within the -1.9 +2.4 selection

The resulting estimates are shown in Appendix 4. Estimates for ratios with per capita GDP growth and a saving ratio are significant. As follows from the calculations, an 1.1 acceleration in potential per capita GDP growth is reflected in a 1 pp growth of the real rate, other conditions being equal, and a 1 pp growth in the saving ratio corresponds to a 0.3 pp reduction in the interest rate.

The Appendix 4 equation receives substitutions for data on Russia which is believed to characterise the economic performance over the next five years. Per capita GDP growth in the next five years is projected to total 1.9% (the average of 1.2 and 2.6% from Appendix 1). We quantify the saving ratio taking into account its trend towards a slow decline on the order of 28%, and we set the reading for Russia’s capital account openness at the 2013 level. This brings us to the estimate for the equilibrium short interest rate **to be 1.0% p.a**. The 95 per cent confidence interval for this assessment turns out to be very wide (between +12 and -10% p.a).

In addition to the model assessment based on complete sample covering developed and developing countries, we used the selection of only developing countries (17 countries). As a result (see Appendix 4), we saw more sensitivity of the equilibrium rate to the national saving ratio, and the reaction to GDP growth became statistically insignificant (even with 10%). There may be content-based explanations behind these differences: these can be related to imperfect markets and the difficulties in consumption smoothing.

The average rate level also grew 0.5 pp. In summation, the equilibrium rate based on data substitution for Russia dropped 0.3 pp. Our understanding is that this instability may relate to the difference in economic structures that regressors fail to capture. The general equilibrium model alone shows that different discount ratios result in different results. The more flexible panel data-based models, e.g. random slopes, might show different results.

4.2. Measuring the equilibrium rate consistent with steady inflation: the Laubach–Williams model (2003)

Most research will define the equilibrium interest rate as the level of an interest rate corresponding to inflation-neutral output data (this definition is also aligned to the natural per cent rate theory by Cnut Viksel – ("Wicksellian interest rate") and to a GDP reading at the potential level.

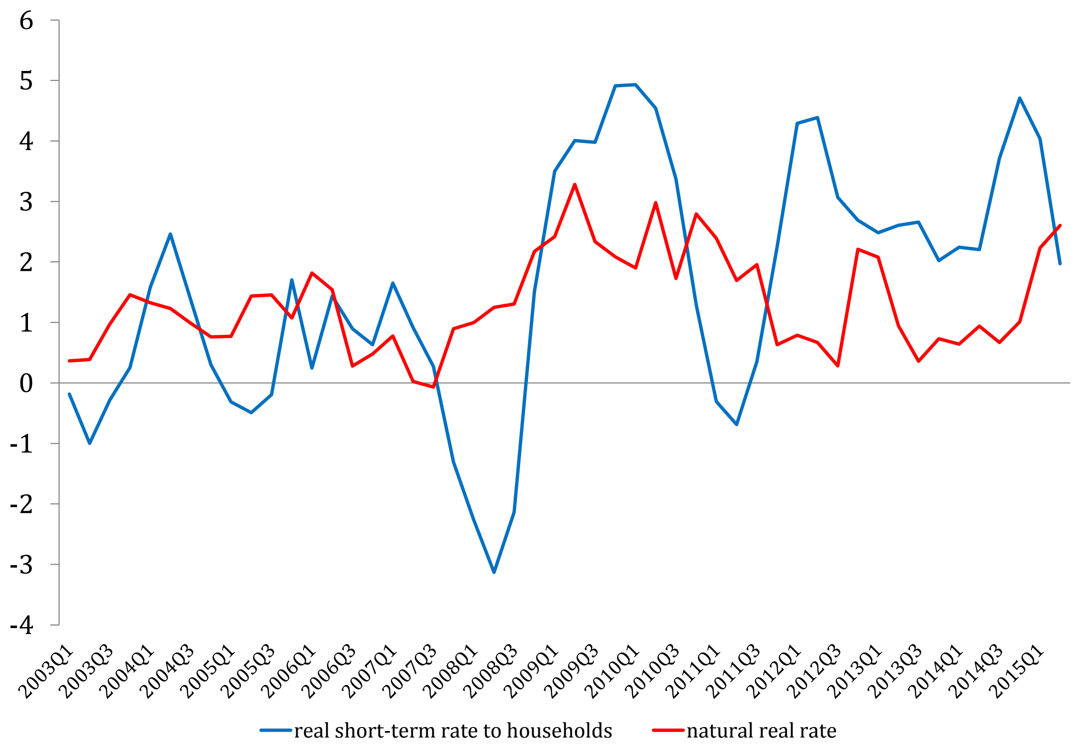
The authors (Laubach, Williams (2003) use the Kalman filter to present, for the first time, a comprehensive modelling approach to simultaneous calculation of potential GDP, its growth rate and the equilibrium actual interest rate for the US economy.

We reproduced this approach on Russian data. The ‘canonical’ model version as proposed in Laubach, Williams (2003) includes three key equations:

- Phillips's curve (function of expectations for inflation and output gap[31])
- an aggregate demand curve (function of output gap and of the real interest rate deviation from its natural level in the preceding quarter)
- the neutral interest rate equation of (dependence on potential GDP and other fundamentals connected with intertemporal preferences).

The model parameters were measured on quarterly data from the selection of the first quarter of 2003 through the first quarter of 2016 (53 observations).

Figure 9. The actual and equilibrium real interest rate for short-term loans to the economy (the Kalman filter-based estimates using the potential output model), % p.a.



The short-term rate for loans extended to non-financial organisations for a term of up to 1 year was used as the initial actual interest rate. It was quite problematic to use for calculations a short-term rate in the interbank lending market (IBl), mainly because the IBL rates in the period of high oil prices and a persistent liquidity surplus before the 2008-2009 banking system crisis remained near the lower bound of the BoR percentage corridor, and was therefore materially lower than expected inflation. The percentage corridor in itself did not play the role it is playing today: the volatility of rates was much higher, and the movements of rates were determined by BoR forex interventions.

[31] In the course of calculations based on the Laubach&Williams (2003) on Russian data, the equation of Phillips's curve also covered the readings of export and import prices.

The actual real interest rate was calculated taking into account the performance of the nominal rate and inflation expectations in each point of time. Considering that, retrospectively, economic agents' inflation expectations for objective reasons (there was no Bank of Russia inflation targeting) were dominated by the adaptive component, in expected inflation calculations, the bigger weight (75%) was assigned to the current inflation and the rather smaller weight (25%) to the future inflation (the rational expectations' component according to 'perfect foresight). It should be noted that the above procedure for calculating inflation expectations had little effect on the results of our calculations from the standpoint of the direction of deviations of the real interest rate from equilibrium level.

Importantly, these estimates are common in that they show a high degree of uncertainty: e.g., for the US, from 200 to 250 bp – see Laubach and Williams (2003).

The resulting quantifications point to the current equilibrium rate level for short-term loans to enterprises standing at 3%. With the 2013-2014 average spread to the money market rate standing at 2.5 pp, this suggests the equilibrium interest rate in the money market at the level of 0.5%. However, this is a nominal spread as long as it factors in the inflation volatility premium and therefore underestimates the real short interest rate.

Since early 2014, the equilibrium rate for loans has grown 2.5 pp.

As another interesting supervision, 2007 and 2011 episodes of accelerated inflation fell on the time when the actual rate was below the equilibrium level. On this background, the Bank of Russia's policy in the time of accelerated inflation in 2014 is profoundly different.

CONCLUSIONS

Below are our key conclusions:

1. Estimates for the real equilibrium interest rate in Russia are of minor practical value because of extended confidence intervals. The country's central bank has to act in a highly uncertain environment as regards key structural economic indicators (this is true of both key rate setting and potential GDP growth), which has strong implications for decision-making and policy communication. In particular, the need for a *robust* policy rule tops the agenda[32]. The robust rule is termed as a monetary decision-making mechanism resistant to erroneous assumptions on certain unobserved economic indicators (potential expansion, the equilibrium interest rate).

2. Short-term equilibrium rates are prone to strong and sharp change after the manner of the trade environment; however, they will remain the same in the long-term outlook. This means that the central bank-set interest rate may be affected by a reverse trend in a tight monetary stance within a short period of time. This brings about the issue of a central bank reacting optimally in this situation: should its key rate also become more *volatile* and respond to short-term shocks? This may become the subject of subsequent research.

3. A rising global interest rate in the model has no implications for the long-run interest rate as the adjustment to a higher global rate is set off by deleveraging and reduced risk premia. Where there are constraints on the risk premium's sensitivity, our model under study may have no solution.

4. The paned data-based calculations (for the long-run equilibrium) and those based on semi-structural methods (for the current equilibrium) also help generate estimates for the equilibrium interest rate. In the same manner, such estimates will be characterised by the high degree of uncertainty from the practical application prospective[33].

5. Point estimates for the short rate in the long-term equilibrium are 1.0% in the panel data-based model or 3.0% by the interest rate parity, which is least dependent on unknown parameters from three specific conditions. The point estimate of the current equilibrium

[32] Refer to: Orphanides A., Williams J. Robust monetary policy rules with unknown natural rates. Brookings Papers on Economic Activity. Vol. 2002. No. 2 (2002). P. 63–118. See also the book L. P. Hansen, T. J. Sargent, *Robustness*, Princeton University Press, 2008

[33] The issue in need separate research is the cause for the high uncertainty of estimates. According to the article M. Aguiar, G. Gopinath, Emerging Market Business Cycles: The cycle is the trend, Journal of Political Economy, 2007, vol. 115, No. 1, the often permanent shocks (trend shifts) in Russia can be behind the strong volatility and instability of estimates based on standard methods which assume the dominance of temporary shocks.

short interest rate based on semi-structural methods appears to be at the mark of 0.5%, while that derived of the interest rate parity is equal to 2.7%.

6. In recognition of market's (July 17 Bloomberg data) and the Bank of Russia's expectations for inflation for one year ahead (for June 2017) standing at 5.7% and under 5%, respectively, the actual real rates prove to be higher than equilibrium ones, even if the latter are taken in their upper bound. For the money market rates (overnight MIACR) at the level of 10%, the current real rate proves to be at the level of 4.3%– above 5.0%. The deviation of the actual current real rate in the money market from the equilibrium one reflects the rigidity degree of the Bank of Russia's monetary policy and points to a potential downside revision. A 1 pp drop in inflation expectations translates into the real rates automatically growing to 5.3%, that is, above 6.0%. In this way, the potential cut in nominal rates in the money market (from the mark of about 10%) is approximately 4 pp, long term.

7. Accelerated growth of per capita potential GDP has the capacity to raise the equilibrium rate in the long term nearly in the 1:1 ratio. In case of developing countries, this statistical link between GDP weakens (loses its significance), while the significant link with national savings remains.

This research could be followed by an effort to study what a central bank's optimal policy is in the environment of uncertainty over equilibrium rate estimates. See, e.g. Orphanides and Williams (2002), Taylor&Williams (2010).

REFERENCES

1. Aguiar, M., G. Gopinath, Emerging Market Business Cycles: The cycle is the trend, Journal of Political Economy, 2007, vol. 115, No. 1
2. Amato J. The role of the natural rate of interest in monetary policy. *BIS working paper*, 2005.
3. Baliño T., Bennett A., Borensztein E. Monetary Policy in Dollarized Economies. *Occasional Paper*, No. 171. Washington: International Monetary Fund, 1999.
4. Bi H., Sovereign default risk premia, fiscal limits and fiscal policy. *Bank of Canada WP*, 2011.
5. Blanchard O., Furceri D., Pescatori A. A Prolonged Period of Low Real Interest Rates? *Secular Stagnation: Facts, Causes and Cures*. Edited by Teulings C. and Baldwin R. VoxEu Aug. 2014.
6. Bomfim A.N. Measuring equilibrium real interest rates: What can we learn from yields on indexed bonds? Board of Governors of the Federal Reserve System, mimeo, 2001.
7. Borio C. Rethinking three intellectual pillars of monetary policy received wisdom, speech, 2015.
8. Chinn, Menzie D., Hiro Ito (2006). What Matters for Financial Development? Capital Controls, Institutions, and Interactions. *Journal of Development Economics*, vol. 81, issue 1, pp. 163–192 (October).
9. Christoffel, Kai & Coenen, Günter & Warne, Anders, 2008. The New Area-Wide Model of the euro area: a micro-founded open-economy model for forecasting and policy analysis. *Working Paper Series 0944*, European Central Bank.
10. Creedy J., Gemmell N. Taxation and the User Cost of Capital: An Introduction. *Working Paper Series 4236*, Victoria University of Wellington, Chair in Public Finance, 2015.
11. Curdia V., Ferrero A., Ng G., Tambalotti A. Has U.S. Monetary Policy Tracked the Efficient Interest Rate? *Journal of Monetary Economics*, 2015, 70.
12. Duarte F., Rosa C. The Equity Risk Premium: A Review of Models. *New York Federal Reserve Staff Report,* 2015.
13. Fuentes R., Gredig F. Estimating the Chilean Natural Rate of Interest, 2007.
14. Giammarioli N., Valla N. The Natural Real Interest Rate and Monetary Policy: a Review. *Journal of Policy Modelling*, 2004, 26.
15. Guo J., Janko S. Reexamination of Real Business Cycles in a Small Open Economy. *Southern Economic Journal, Southern Economic Association*, 2009, vol. 76 (1), pp. 165–182, July.
16. Hall R.E., Jorgenson D.W. Tax Policy and Investment Behavior. *American Economic Review*, 1967, 57 (3), pp. 391–414.

17. Hamilton J., Harris E., Hatzius J., West K.The Equilibrium Real Funds Rate: Past, Present and Future, 2015.
18. He D., Wang H., Yu X. Interest rate determination in China: past, present and future. *International Journal of central banking*, 2015.
19. Ikeda, Daisuke & Saito, Masashi. The effects of demographic changes on the real interest rate in Japan. *Japan and the World Economy*, Elsevier, 2014, vol. 32 (C), pages 37–48.
20. International Monetary Fund (2014): World Economic Outlook April.
21. Jang-Ting Guo & Zuzana Janko. 2009. Reexamination of Real Business Cycles in a Small Open Economy. *Southern Economic Journal, Southern Economic Association*, vol. 76 (1), pp. 165–182, July.
22. Khvostova I., Larin A., Novak A. Euler equation with habits and measurement errors: estimates on Russia micro data. *NRU HSE Working paper series*, 2014.
23. Laubach T. and Williams J. (2003) Measuring the natural Rate of Interest. *Review of Economics and Statistics*, 85 (4).
24. Laubach T., Williams J.C. 2015. Measuring the natural rate of interest redux. *Working Paper Series 2015–16*, Federal Reserve Bank of San Francisco.
25. Lundvall H., Westermark A. What is the natural interest rate? *Sveriges riksbank economic review*, 2011.
26. Mendoza E.G. Real Business Cycles in a Small Open Economy. *American Economic Review, American Economic Association*, 1991, vol. 81 (4), pp. 797–818, September.
27. Orphanides A., Williams J. Robust monetary policy rules with unknown natural rates. *Brookings Papers on Economic Activity*, vol. 2002, no. 2, pp. 63–118.
28. Perrelli R., Roache S. Time-varying Neutral Interest Rate – The case of Brazil. *IMF Working paper*, 2014.
29. Pescatori A., Turunen J. Lower for Longer; Neutral Rates in the United States. *IMF Working Papers 15/135*, International Monetary Fund, 2015.
30. Barsky R., Justiniano A., Melosi L. The Natural Rate of Interest and Its Usefulness for Monetary Policy. *American Economic Review*, May 2014, vol. 104, no. 5, pp. 37–43.
31. Schmitt-Grohe, S., Uribe M. Closing Small Open Economy Models. *Journal of International Economics* 61 (October 2003), pp. 163–185.
32. Taylor J., Wieland V. Finding the Equilibrium Real Interest Rate in a Fog of Policy Deviations, 2016.
33. Taylor J.B., Williams J.C. Simple and robust rules for monetary policy. *Working Paper Series 2010-10*, Federal Reserve Bank of San Francisco, 2010.
34. Vetlov I., Hledik T., Jonsson M., Kucsera H., Pisani M. Potential output in DSGE models, *Working Paper 1351*, European Central Bank, 2011.
35. Williams J. The Decline in the Natural Rate of Interest. Manuscript, 2015, March.

36. Woodford M. A neo-Wicksellian framework for the analysis of monetary policy, manuscript, Princeton University, 1999.

APPENDIX 1

Measuring the equilibrium (long-term) GDP growth

A GDP growth rate is measured according to the following formula[34]:

$$g_Y = \frac{g_A}{\alpha} + g_L,$$

where g_Y is a long-term GDP growth; g_A is a long-term growth in the total factor productivity (TFP); g_L is a long-term population growth; α is the parameter describing the share of labour costs in the production of GDP.

We model the aggregate factor productivity as approximation to the technological leader (here and elsewhere, the USA), i.e.

$$g_A = g_A^* + k * \left(\frac{Y^*}{Y} - 1\right) * 100,$$

where g_A^* is the technological leader's speed of TFP growth; Y^* is per capita GDP in the technological leader; Y is per capita GDP for Russia.

For the US, g_A^* is estimated to be 2% p.a.[35]. This reading is used as a maximum one, with the minimal estimate assumed as equal to 1%.

The contribution of approximation to the technological leader is estimated to total 0.2%–0.6%, subject to the calibration applied. The working-age population is assumed to contract at the rate of 0%–0.6% p.a[36]. The final range of estimates for GDP growth is 0.6%–2.6% p.a.

Table. Estimates based on the arithmetic of growth, %

	Upper bound	Average estimate	Lower bound
Contribution of the technological leader's TFP	*2.0*	*1.5*	*1.0*
Contribution of catching-up development	*0.6*	*0.4*	*0.2*
Contribution of labour	*0.0*	*-0.3*	*-0.6*
GDP growth	*2.6*	*1.6*	*0.6*

Mathematic description

The production function is described by the formula:

$$Y_t = A_t * L_t^{\alpha} * K_t^{1-\alpha}.$$

[34] See the mathematic description below.
[35] See., e.g. Jones C. (2015) The Facts of Economic Growth. *Preliminary and incomplete draft for Handbook of Macroeconomics, vol. 2.*
[36] ИДЭМ НИУ ВШЭ (Higher School of Economics).

Through a logarithm and then a differentiation, we deduce:

$$\frac{dY_t}{Y_t} = \frac{dA_t}{A_t} + \alpha \frac{dL_t}{L_t} + (1 - \alpha) \frac{dK_t}{K_t}$$

or

$$g_{Y,t} = g_{A,t} + \alpha\, g_{L,t} + \; 1 - \alpha \; g_{K,t}.$$

In the equilibrium $g_{K,t} = g_{Y,t}$, then $g_{Y,t} = \frac{g_{A,t}}{\alpha} + g_{L,t}$.

APPENDIX 2

Symbols for the below variables (the model data are quarterly in frequency):

c is consumption adjusted for technological advance;

r_real is the real rate;

d is external debt;

w is real wages adjusted for technological advance;

l is hours worked;

y_e is the volume of the intermediate product output adjusted for technological advance (in levels);

p_e is the relative price of intermediate products (the price of intermediate products in terms of consumer products);

q_x is non-oil exports adjusted for technological advance (in levels);

p_x_star is the relative non-oil export price in terms of foreign products;

rer is the real forex rate (increase = weakening in the real forex rate);

k is capital , adjusted for technological advance (in levels);

zk is real cost of capital;

q is *q-Tobin* (a Lagrange multiplier in the equation determining the accumulation of capital);

i is investment adjusted for technological advance (in levels);

p_i is the relative price of investment;

h is domestic ultimate products adjusted for technological advance (in levels);

p_h is the relative price of domestic ultimate products;

h_i is domestic ultimate products in the consumer basket (in levels) adjusted for technological advance

h_c is domestic ultimate products in the investment basket (in levels) adjusted for technological advance;

im is imports adjusted for technological advance (in levels);

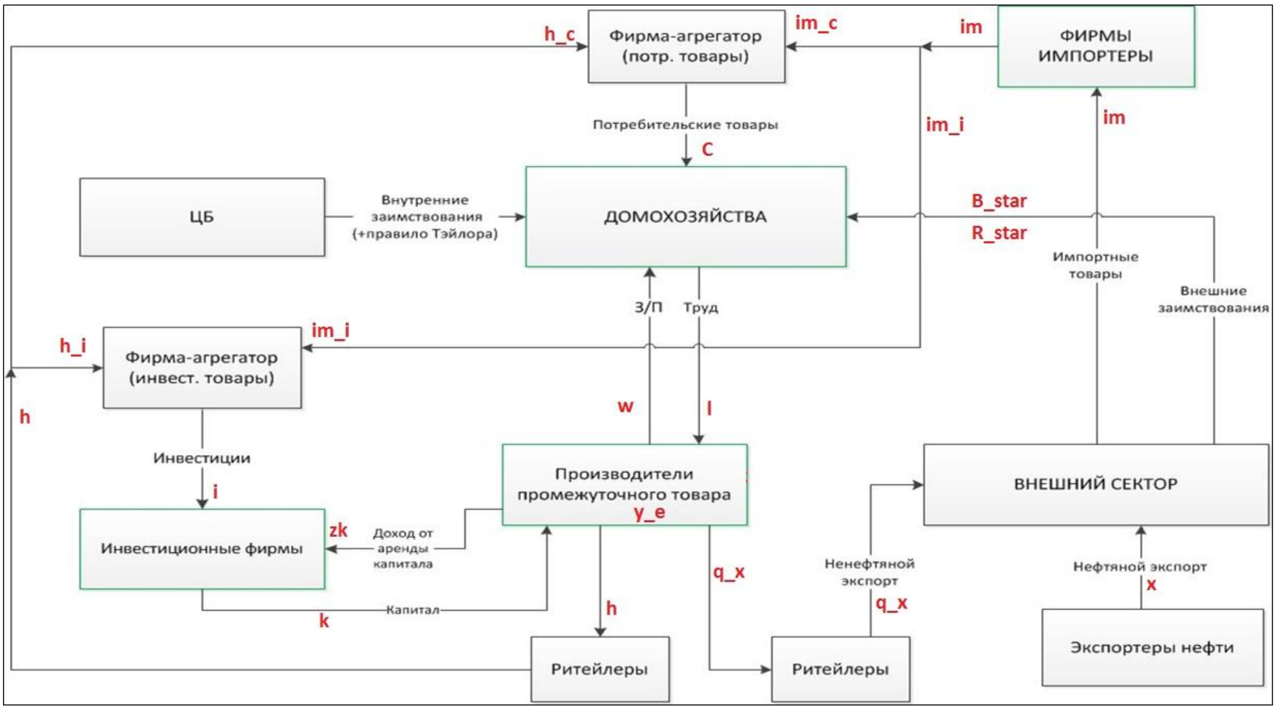
im_c is imported products in the consumer basket (in levels) adjusted for technological advance;

im_i is imported products in the investment basket (in levels) adjusted for technological advance);

p_f is the relative price of imported products;

p_oil is the relative oil price in terms of foreign products;

R_star is the foreign rate (nominal).



Households

Households will choose the level of consumption, saving, as well as the number of hours worked, while maximising utility in of the following form:

$$U_t\ j = E_t \sum_{i=0}^{\infty} \beta^i * e^{z_t^{pref}} * \frac{\left(C_{t+i}\ j - hC_{t+i-1}\right)^{1-\sigma_c} - 1}{1-\sigma_c} - A_L \frac{\left(L_{t+i}\ j\right)^{1+\phi}}{1+\phi}$$

with the budget constraint being

$$P_t^{cpi} C_t\ j + \left(R_t\right)^{-1} * B_t\ j + \left(R_t^* * rp\ b_t^*, p_t^{oil}, e^{RP}\right)^{-1} * \mathcal{E}_t * B_t^*\ j =$$
$$= W_t * L_t(j) + B_{t-1}\ j + \mathcal{E}_t * B_{t-1}^*\ j + \Pi_t(j),$$

where P_t^{cpi} is consumer price index; R_t is the domestic interest rate; R_t^* is foreign interest rate; $rp\ b_t^*, p_t^{oil}, e^{RP}$ is risk premium depending on the relative level of debt (see below), the real dollar oil price, the stochastic part of the risk premium; $\mathcal{E}_t$ is the nominal forex rate; W_t is wages; $C_t\ j$ *is* *j*-household consumption; $L_t\ j$ is *j*-household labour; $B_t\ j$ is net assets in a domestic currency; $B_t^*\ j$ is net assets in a foreign currency; $\Pi_t\ j$ is non-recurrent payments.

The variables are corrected by a quantity of $F_t = A_t^{1/(1-\alpha)}$. $f_t = F_t/F_{t-1}$.

The first order conditions are of the following forms:

$$1 = \beta * R_t * E_t \;\; \frac{e^{z_{t+1}^{pref}}}{e^{z_t^{pref}}} \frac{C_{t+1} - h * C_t \;\; ^{-\sigma_c}}{C_t - h * C_{t-1} \;\; ^{-\sigma_c}} * \frac{P_t^{cpi}}{P_{t+1}^{cpi}} \;\; , \qquad (1)$$

$$1 = \beta * R_t^* * rp \;\; b_t^*, p_t^{oil}, e^{RP} \;\; * E_t \;\; \frac{e^{z_{t+1}^{pref}}}{e^{z_t^{pref}}} \frac{C_{t+1} - h * C_t \;\; ^{-\sigma_c}}{C_t - h * C_{t-1} \;\; ^{-\sigma_c}} * \frac{P_t^{cpi}}{P_{t+1}^{cpi}} * \frac{\mathcal{E}_{t+1}}{\mathcal{E}_t} \;\; , \qquad (2)$$

$$\frac{W_t}{P_t^{cpi}} * \;\; C_t - h * C_{t-1} \;\; ^{-\sigma_c} = A_L * \;\; L_{t+i} \;\; ^{\phi}. \quad \text{(3, see the note below)}$$

Note. The labour supply equation used in the model is of a somewhat different form, with the labour market's rigidities introduced – similarly to price rigidities.

The aggregator firm will aggregate differentiated household labour into a single one and sells it to other firms.

Demand for labour:

$$L_t \;\; i \;\; = \;\; \frac{W_t \;\; i}{W_t} \;\; ^{-\frac{\varphi^W}{\varphi^W - 1}} L_t.$$

Firms

Intermediate product producer

The production function of domestic firms depend only on hired labour:

$$Y_t^E \;\; j \;\; = A_t * K_t \;\; j \;\; ^{\alpha} * \;\; L_t \;\; j \;\; ^{1-\alpha} \qquad (4)$$

where $Y_t^E \;\; j$ is the production of *j*-firm producing intermediate products; $L_t \;\; k$ is labour hired by the firm; A_t is total factor productivity; $K_t \;\; k$ is capital used by the firm; P_t^E is the price being set.

Producers will maximize profits:

$$E_t \sum_{i=0}^{\infty} \lambda_{t,t+i} \;\; \frac{P_{t+i}^E \;\; k}{P_{t+i}^{cpi}} * Y_{t+i}^E \;\; j \;\; - \frac{W_{t+i}}{P_{t+i}^{cpi}} * L_{t+i} \;\; j \;\; - \frac{ZK_{t+i}}{P_{t+i}^{cpi}} * K_{t+i} \;\; j \quad .$$

The demand for an individual firm's products (again, from a certain wrapper's side)

$$Y_{t+i}^E \;\; j \;\; = \;\; \frac{P_{t+i}^E \;\; k}{P_{t+i}^E} \;\; ^{-\frac{1}{\nu_E}} Y_{t+i}^E,$$

where $P_t \;\; k$ is the price set by *k*-firm; Y_t is the volume of domestic products produced; $\lambda_{t,t+i}$ is discount-factor applied by the firm; $\pi_t = \frac{P_t^{cpi}}{P_{t-1}^{cpi}}$.

The first order condition:

$$\left(1-\frac{1}{\nu_E}\right)+\frac{1}{\nu_E}\frac{W_t*L_t}{Y_t^E*(1-\alpha)*P_t^{cpi}}*\frac{1}{p_t^E}=0 \qquad (5)$$

where $p_t^E=\frac{P_t^E}{P_t^{cpi}}$.

$$\frac{W_t*L_t}{ZK_t*K_t}=\frac{1-\alpha^H}{\alpha^H} \qquad (6)$$

Retail firms

Domestic products

$$\max\ E_t\left[\sum_{i=0}^{\infty}\lambda_{t,t+i}\left(\frac{P_{t+i}^H(j)*H_{t+i}(j)}{P_{t+i}^{cpi}}-\frac{P_{t+i}^E*H_{t+i}(j)}{P_{t+i}^{cpi}}-\frac{k_H}{2}*\left(\frac{P_{t+i}^H(k)}{P_{t+i-1}^H(k)}-\left(\pi_{t+i-1}^H\right)^{\iota_H}*\left(\pi_*\right)^{1-\iota_H}\right)^2*H_{t+i}*\frac{P_{t+i}^H}{P_{t+i}^{cpi}}\right)\right],$$

$$H_{t+i}(j)=\left(\frac{P_{t+i}^H(k)}{P_{t+i}^H}\right)^{-\frac{1}{\nu_H}}H_{t+i}.$$

The first order conditions:

$$\left(1-\frac{1}{\nu_H}\right)+\frac{1}{\nu_H}\frac{p_t^E}{p_t^H}-k_H*\left(\pi_t^H-\left(\pi_{t-1}^H\right)^{\iota_H}*\left(\pi_*\right)^{1-\iota_H}\right)*\pi_t^H+$$

$$+k_H*\beta*E_t\left[\frac{e^{z_{t+1}^{pref}}}{e^{z_t^{pref}}}\frac{\left(C_{t+1}-h*C_t\right)^{-\sigma_C}}{\left(C_t-h*C_{t-1}\right)^{-\sigma_C}}*\left(\pi_{t+1}^H-\left(\pi_t^H\right)^{\iota_H}*\left(\pi_*\right)^{1-\iota_H}\right)*\left(\frac{\pi_{t+1}^H}{\pi_{t+1}}\right)^2*\pi_{t+1}*\frac{H_{t+1}}{H_t}\right]=0$$

(7)

Exports (local currency price setting)

$$\max\ E_t\left[\sum_{i=0}^{\infty}\lambda_{t,t+i}\left(\frac{P_{t+i}^{*X}(j)*\varepsilon_{t+i}*Q_{t+i}^X(j)}{P_{t+i}^{cpi}}-\frac{P_{t+i}^E*Q_{t+i}^X(j)}{P_{t+i}^{cpi}}-\frac{k_X}{2}*\left(\frac{P_{t+i}^{*X}(k)}{P_{t+i-1}^{*X}(k)}-\left(\pi_{t+i-1}^{*cpi}\right)^{\iota_X}*\left(\pi_*\right)^{1-\iota_X}\right)^2*Q_{t+i}^X*\frac{P_{t+i}^{*X}(j)*\varepsilon_{t+i}}{P_{t+i}^{cpi}}\right)\right]$$

$$Q_{t+i}^X(j)=\left(\frac{P_{t+i}^{*X}(k)}{P_{t+i}^{*X}}\right)^{-\frac{1}{\nu_X}}Y_{t+i}^X.$$

The first order conditions:

$$\left(1-\frac{1}{\nu_X}\right)+\frac{1}{\nu_X}\frac{1}{rer_t}\frac{p_t^E}{p_t^{*X}}-k_X*\left(\pi_t^{*X}-\left(\pi_{t-1}^{*X}\right)^{\iota_X}*\left(\pi_*^{*cpi}\right)^{1-\iota_X}\right)*\pi_t^{*X}+$$

$$+k_H*\beta*E_t\left[\frac{e^{z_{t+1}^{pref}}}{e^{z_t^{pref}}}\frac{\left(C_{t+1}-h*C_t\right)^{-\sigma_C}}{\left(C_t-h*C_{t-1}\right)^{-\sigma_C}}*\left(\pi_{t+1}^{*X}-\left(\pi_t^{*X}\right)^{\iota_X}*\left(\pi_*^{*cpi}\right)^{1-\iota_X}\right)*\left(\frac{\pi_{t+1}^{*X}}{\pi_{t+1}}\right)^2*\frac{\varepsilon_{t+1}}{\varepsilon_t}*\pi_{t+1}*\frac{Q_{t+1}^X}{Q_t^X}\right]=0.$$

(8)

Importers

Importers will buy products abroad and resell them with a mark-up domestically, maximising profits:

$$E_t \sum_{i=0}^{\infty} \lambda_{t,t+i} \left(\frac{P_{t+i}^F(k)}{P_{t+i}^{cpi}} \left(\frac{P_{t+i}^F(k)}{P_{t+i}^F} \right)^{-\frac{1}{\nu_F}} Im_{t+i} - \mathcal{E}_{t+i} * \frac{P_{t+i}^{*cpi}}{P_{t+i}^{cpi}} * \left(\frac{P_{t+i}^F(k)}{P_{t+i}^F} \right)^{-\frac{1}{\nu_F}} Im_{t+i} - \frac{k_F}{2} * \left(\frac{P_{t+i}^F(k)}{P_{t+i-1}^F(k)} - (\pi_{t-1}^F)^{\iota_F} * (\pi_*)^{1-\iota_F} \right)^2 * Im_{t+i} * \frac{P_{t+i}^F}{P_{t+i}^{cpi}} \right),$$

where $P_t^F(k)$ is the price being set by *k*-importer; Im_t is the volume of imports in the economy; P_t^{*cpi} is the level of prices on foreign products; $\pi_t^F = \frac{P_t^F}{P_{t-1}^F}$, $Im_{t+i}(k) = \left(\frac{P_{t+i}^F(k)}{P_{t+i}^F}\right)^{-\frac{1}{\nu_F}} Im_{t+i}$ is the demand for a product imported by an individual firm.

The first order condition:

$$\left(1 - \frac{1}{\nu_F}\right) + \frac{1}{\nu_F} * rer_t * \frac{1}{p_t^F} - k_F * \left(\pi_t^F - (\pi_{t-1}^F)^{\iota_F} * (\pi_*)^{1-\iota_F}\right) * \pi_t^F +$$
$$+ k_F * \beta * E_t \left(\frac{e^{z_{t+1}^{pref}}}{e^{z_t^{pref}}} \frac{(C_{t+1} - h * C_t)^{-\sigma_C}}{(C_t - h * C_{t-1})^{-\sigma_C}} * \left(\pi_{t+1}^F - (\pi_t^F)^{\iota_F} * (\pi_*)^{1-\iota_F}\right) * \left(\frac{\pi_{t+1}^F}{\pi_{t+1}}\right)^2 * \pi_{t+1} * \frac{Im_{t+1}}{Im_t} \right) = 0 \quad (9)$$

where rer_t is the real exchange rate determined by the formula $rer_t = \frac{\mathcal{E}_t * P_t^{*cpi}}{P_t^{cpi}}$.

Investment firms

Investment firms will maximise profits:

$$\max_{K,I} E_t \sum_{i=0}^{\infty} \lambda_{t,t+i} \frac{ZK_{t+i} K_{t+i} - P_{t+i}^I I_{t+i}}{P_{t+i}^{cpi}}$$

on condition

$$K_{t+1} = (1 - \delta) K_t + e^{\varepsilon_t^I} * \left(1 - \frac{k_I}{2} * \left(\frac{I_t}{I_{t-1}} - f_t\right)^2\right) I_t. \quad (10)$$

The first order conditions ($q_t = \frac{Q_t}{P_t^{cpi}}; zk_{t+1} = \frac{ZK_{t+1}}{P_{t+1}^{cpi}}; p_t^I = \frac{P_t^I}{P_t^{cpi}}$):

$$q_t = E_t \left(\beta \frac{e^{z_{t+1}^{pref}}}{e^{z_t^{pref}}} \frac{(C_{t+1} - h * C_t)^{-\sigma_C}}{(C_t - h * C_{t-1})^{-\sigma_C}} * \left(zk_{t+1} + q_{t+1} * (1 - \delta) \right) \right) \quad (11)$$

$$p_t^I = q_t e^{\varepsilon_t^I} * \left(-k_I \left(\frac{I_t}{I_{t-1}} - f_t\right) \frac{I_t}{I_{t-1}} + \left(1 - \frac{k_I}{2} \left(\frac{I_t}{I_{t-1}} - f_t\right)^2\right) \right) +$$

$$+E_t \quad \beta \frac{e^{z_{t+1}^{pref}}}{e^{z_t^{pref}}} \frac{C_{t+1}-h*C_t \quad ^{-\sigma_c}}{C_t-h*C_{t-1} \quad ^{-\sigma_c}} q_{t+1} \varepsilon_{t+1}^I k_I \quad \frac{I_{t+1}}{I_t} - f_t \quad \frac{I_{t+1}}{I_t} \quad ^2 \tag{12}$$

Trade balance

The dynamics of net foreign assets are recorder as follows:

$$R_t^* * rp \quad b_t^*, p_t^{oil}, e^{RP} \quad ^{-1} * \mathcal{E}_t * B_t^* = \mathcal{E}_t * B_{t-1}^* + \mathcal{E}_t * p_t^{X*} * P_t^{*cpi} * Q_t^X + \mathcal{E}_t * p_t^{oil} * P_t^{*cpi} * X_t -$$

$$- \mathcal{E}_t * P_t^{*cpi} * Im_t - \mathcal{E}_t * P_t^{*cpi} * dRes_t \quad , \tag{13}$$

$$d_t = -\frac{\mathcal{E}_t * B_t^*}{F_t * P_t^{cpi}},$$

where X_t is the volume of exports; $dRes_t$ is the variable responsible for change in the reserves.

Aggregation

$$C_t = \frac{H_t^{cons} \quad ^{\gamma^{cons}} * \ Im_t^{cons} \quad ^{1-\gamma^{cons}}}{\gamma^{cons} \quad ^{\gamma^{cons}} * \ 1-\gamma^{cons} \quad ^{1-\gamma^{cons}}} \tag{14}$$

$$P_t^H * H_t^{cons} = P_t^{cpi} * C_t \tag{15}$$

$$P_t^F * Im_t^{cons} = P_t^{cpi} * C_t \tag{16}$$

$$I_t = \frac{H_t^{inv} \quad ^{\gamma^{inv}} * \ Im_t^{inv} \quad ^{1-\gamma^{inv}}}{\gamma^{inv} \quad ^{\gamma^{inv}} * \ 1-\gamma^{inv} \quad ^{1-\gamma^{inv}}} \tag{17}$$

$$P_t^H * H_t^{inv} = P_t^I * I_t \tag{18}$$

$$P_t^H * H_t^{inv} = P_t^I * I_t \tag{19}$$

Market clearing

$$H_t = H_t^{inv} + H_t^{cons} \tag{20}$$

$$Im_t = Im_t^{inv} + Im_t^{cons} \tag{21}$$

$$Y_t^E = H_t + Q_t^X \tag{22}$$

Others

$$\pi_t^H = \frac{p_t^H}{p_{t-1}^H} * \pi_t \tag{23}$$

$$\pi_t^F = \frac{p_t^F}{p_{t-1}^F} * \pi_t \tag{24}$$

$$\pi_t^{X*} = \frac{p_t^{X*}}{p_{t-1}^{X*}} * \pi_t^{F*} \tag{25}$$

$$rer_t = \frac{\mathcal{E}_t * P_t^{*cpi}}{P_t^{cpi}} = \mathcal{E}_t * p_t^{*cpi} \tag{26}$$

Monetary policy

The Taylor rule:

$$\frac{R_t}{R_*} = \left(\frac{R_{t-1}}{R_*}\right)^{\phi_R} \left(\left(\frac{\pi_t}{\pi_*}\right)^{\phi_\pi}\right)^{1-\phi_R} * exp^{e_t^R}. \tag{27}$$

Real exchange rate targeting is of the following form:

$$rer_t = e^{e_t^{\mathcal{E}}} * rer_{ss,t}. \tag{28}$$

Demand for exports

$$Q_t^X = e^{z_t^X} \left(p_t^{X*}\right)^{-\eta_X} Y_t^* \tag{29}$$

Exogenous processes

$$logz_t^A = \rho_z * log\, z_{t-1}^A + e_t^z$$
$$z_t^{RP} = \rho_{RP} * z_{t-1}^{RP} + e_t^{RP}$$
$$z_t^X = \rho_X * z_{t-1}^X + \varepsilon_t^X$$
$$z_t^{\mathcal{E}} = \rho_{\mathcal{E}} * z_{t-1}^{\mathcal{E}} + e_t^{\mathcal{E}}$$
$$z_t^{\mathrm{I}} = \rho_{\mathcal{E}} * z_{t-1}^{\mathrm{I}} + e_t^{\mathrm{I}}$$
$$z_t^{\mathrm{pref}} = \rho_{\mathrm{pref}} * z_{t-1}^{\mathrm{pref}} + e_t^{\mathrm{pref}}$$

$$log\frac{X_t}{A_t} = \rho_X * log\frac{X_{t-1}}{A_{t-1}} + e_t^X$$
$$\pi_t^* = \rho_\pi * \pi_{t-1}^* + e_t^{\pi*}$$
$$y_t^* = \rho_{y*} * y_{t-1}^* + e_t^{y*}$$
$$R_t^* = 0$$
$$p_t^{oil} = 0$$

The model includes no additional equations in an explicit form:

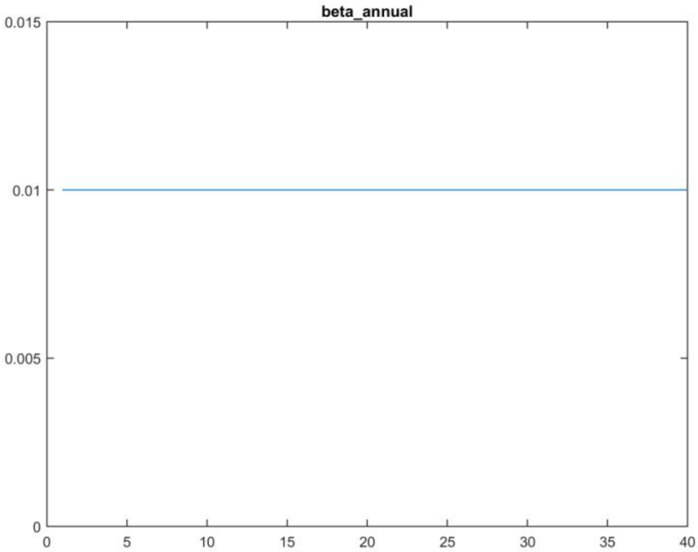
$$logA_t = logg + logA_{t-1} + logz_t^A$$
$$g = g^{\frac{1}{1-\alpha}}$$

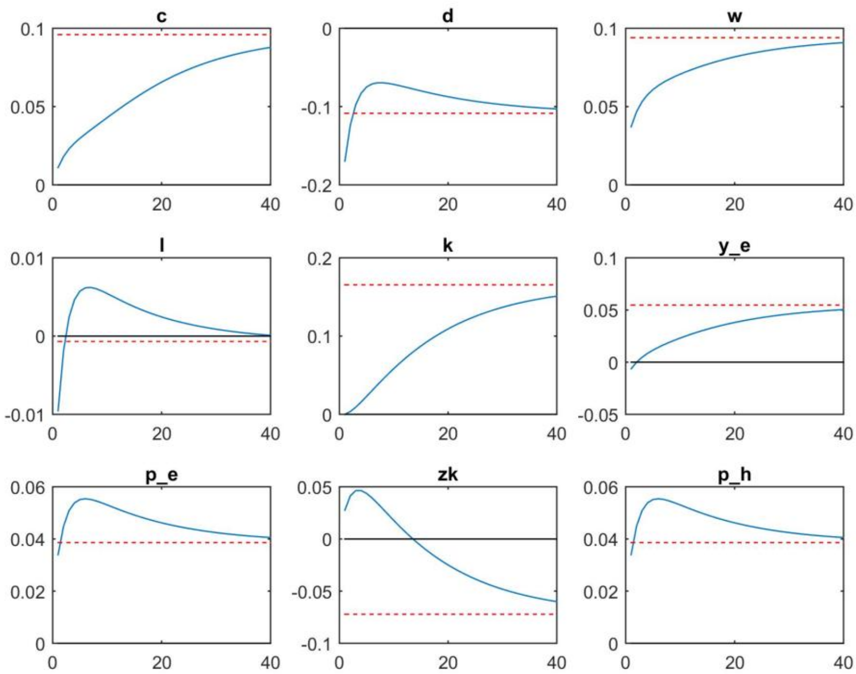
$z = \frac{z_t^A}{1-\alpha}$.

APPENDIX 3

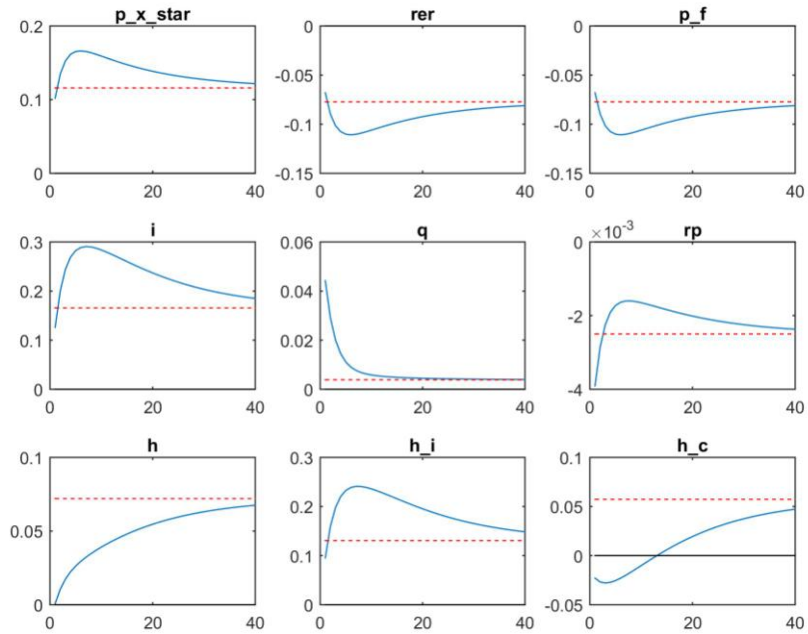
Below are calculations of equilibrium levels and impulse response for all three shocks in the model under study[37]:

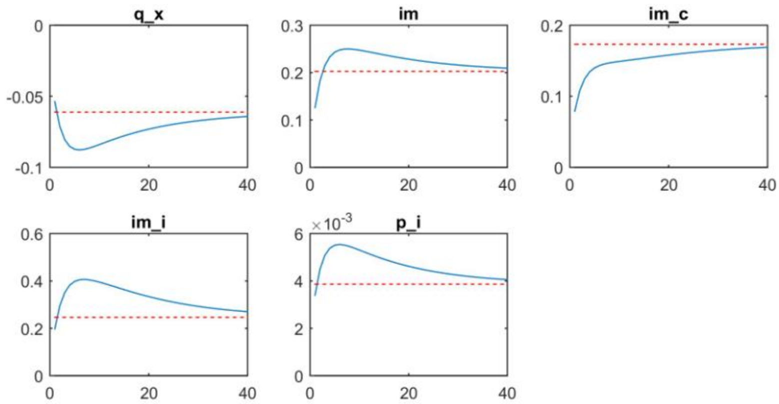
1. A 0.01 growth in the subjective future discount ratio

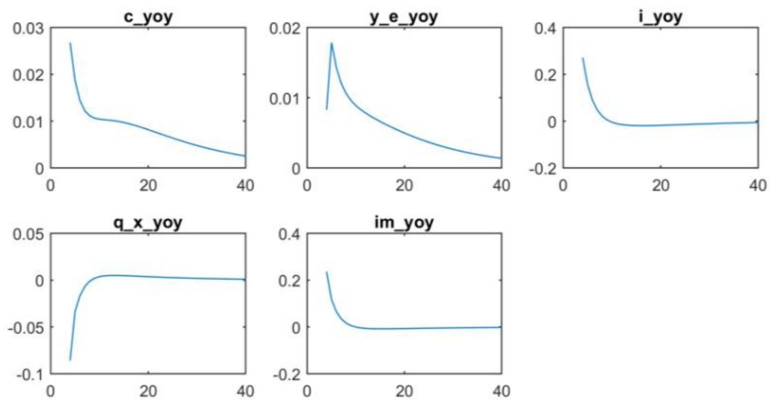




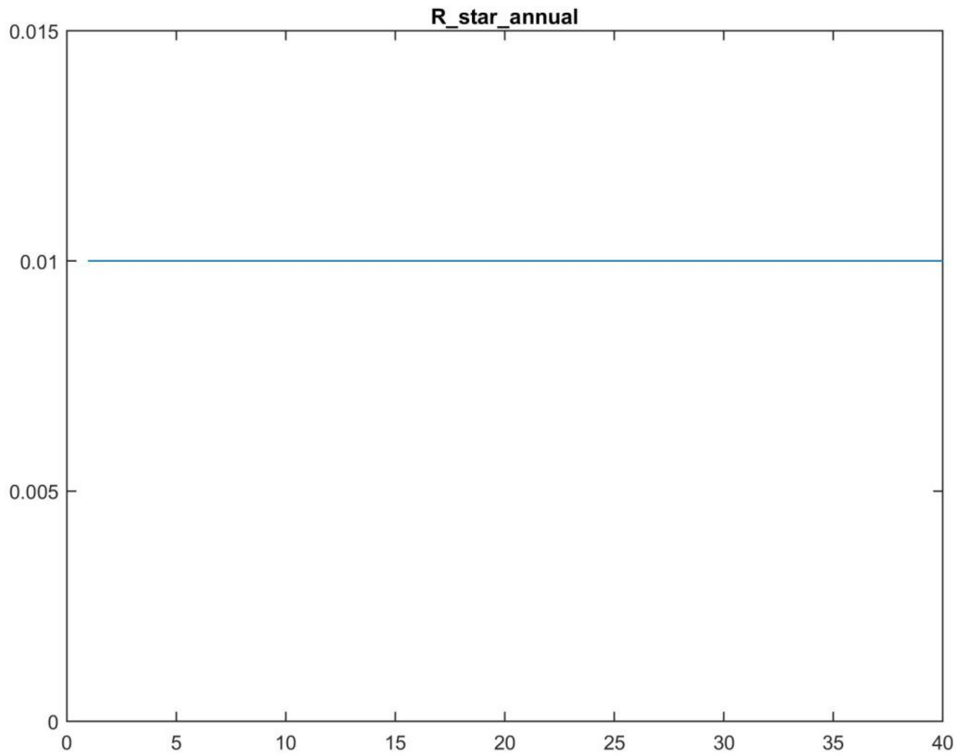
[37] The starting point of diagrams with the response of variable to shocks in terms of quarter-on-quarter is quarter 4 (the shock occurs in quarter 1)

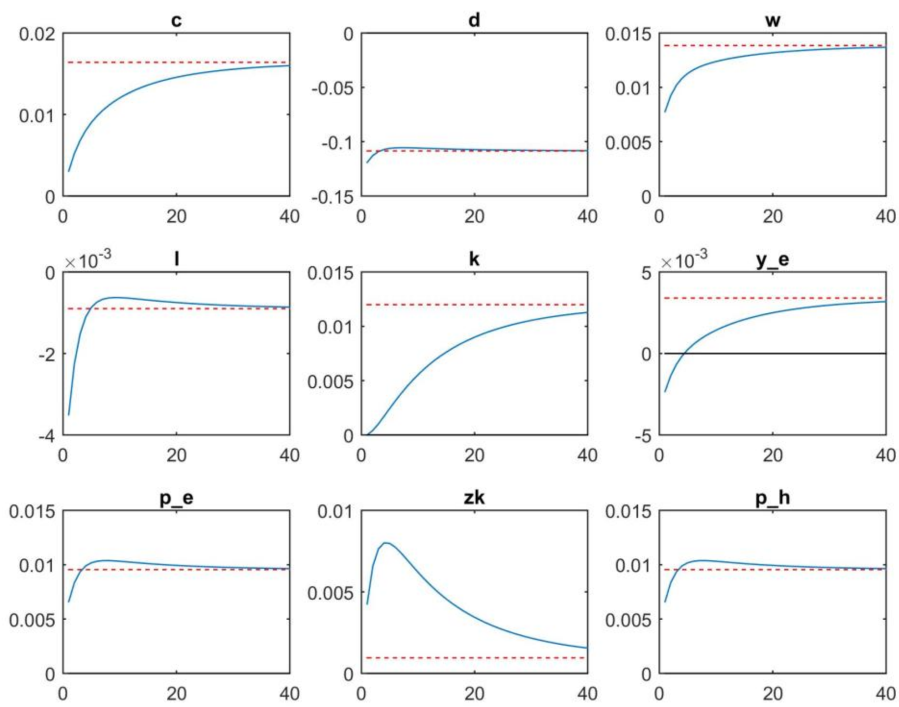

p_x_star
rer
p_f
i
q
rp
×10^-3
h
h_i
h_c
0
20
40

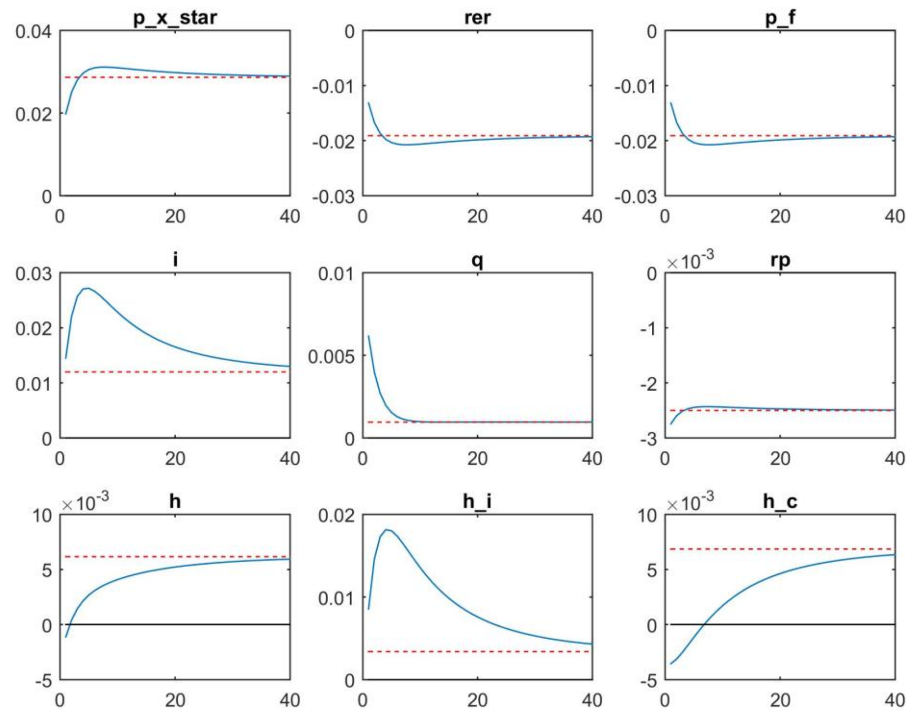

q_x
im
im_c
im_i
p_i
×10^-3
0
20
40

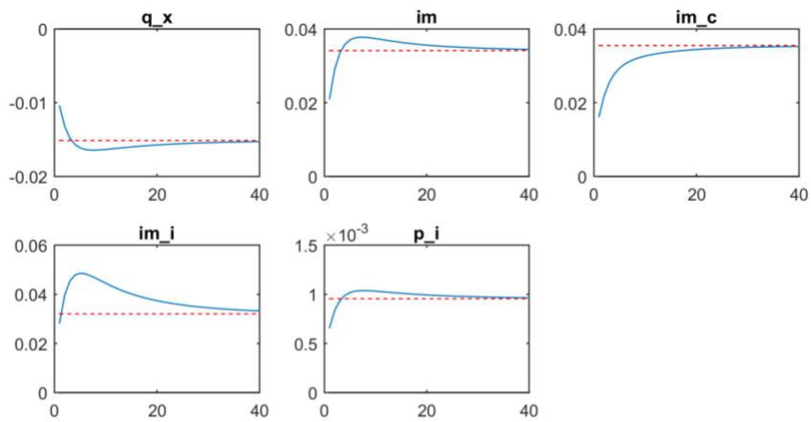

c_yoy
y_e_yoy
i_yoy
q_x_yoy
im_yoy
0
20
40

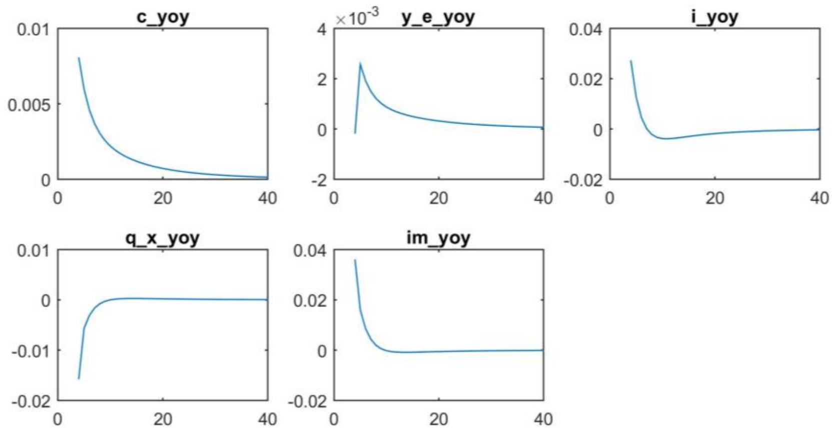
2. A 1 pp growth in the global interest rate



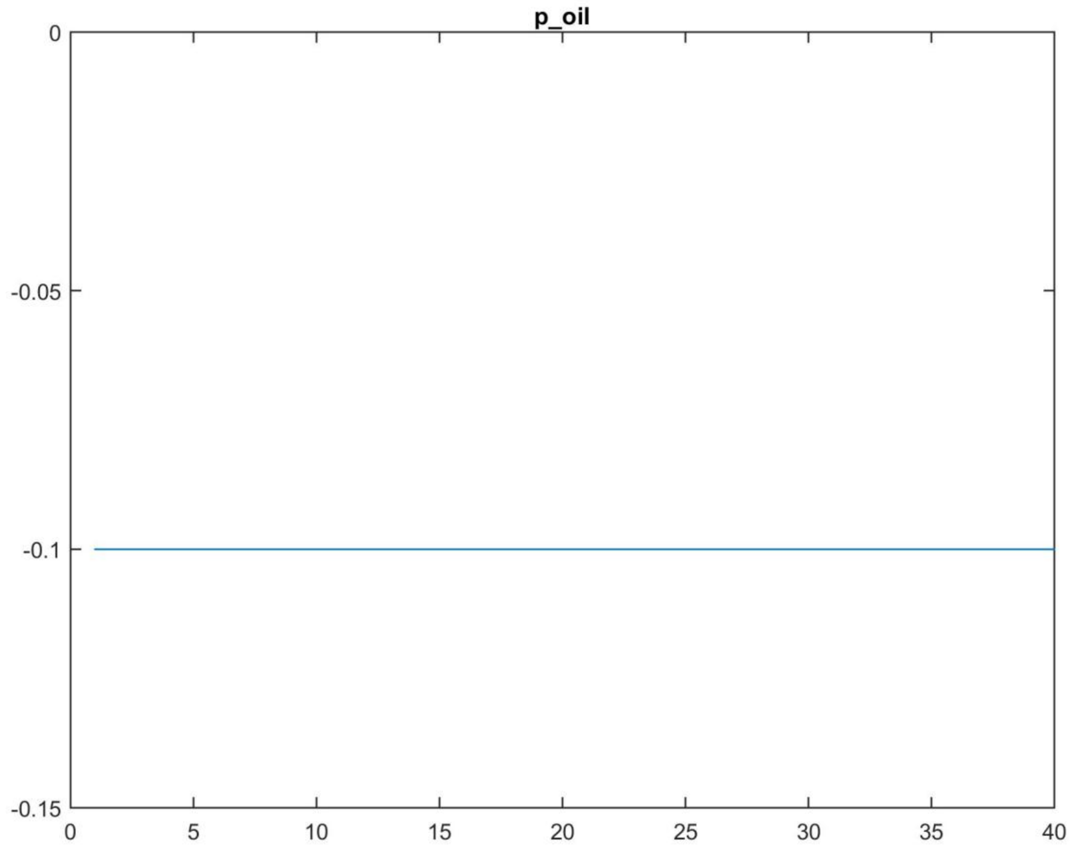


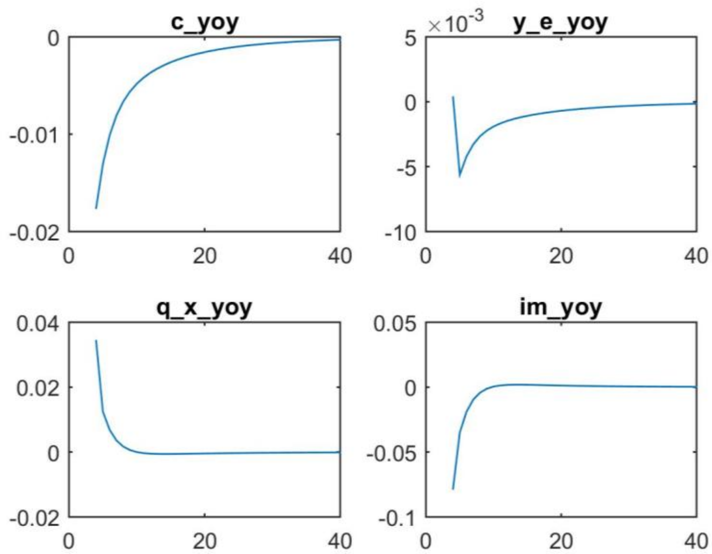


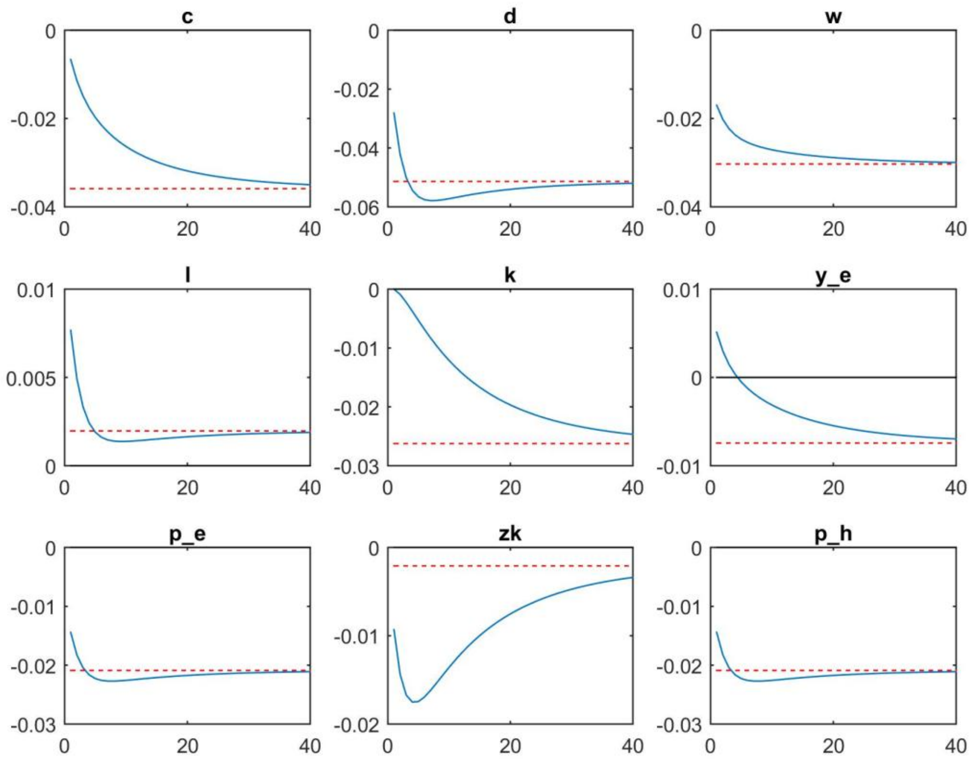


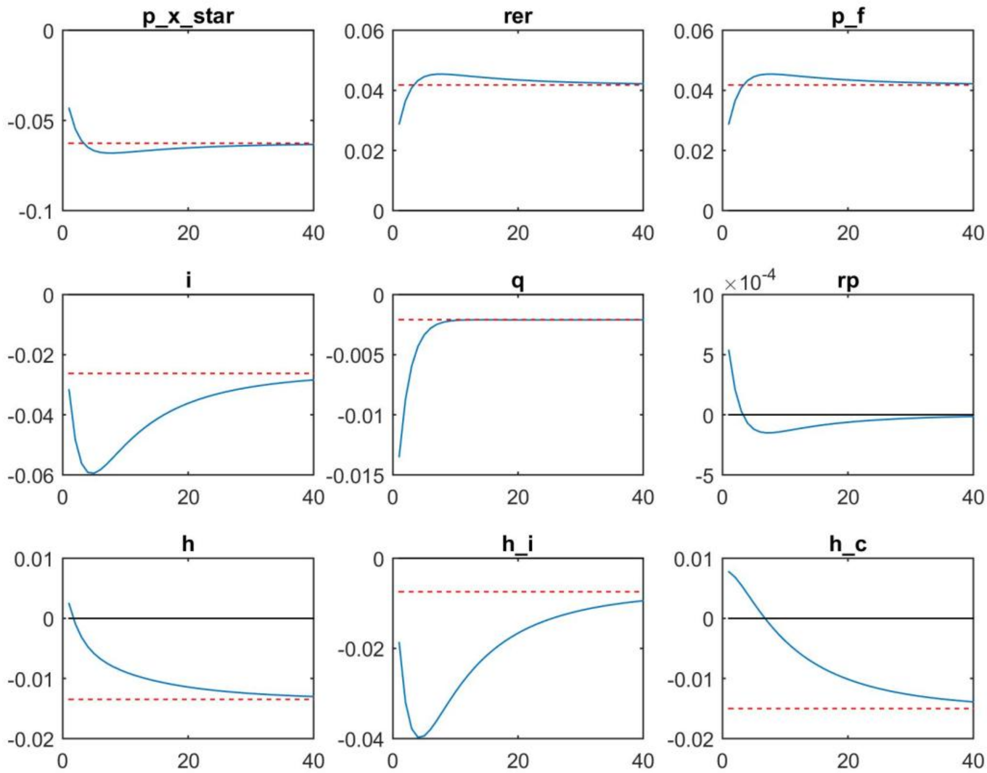


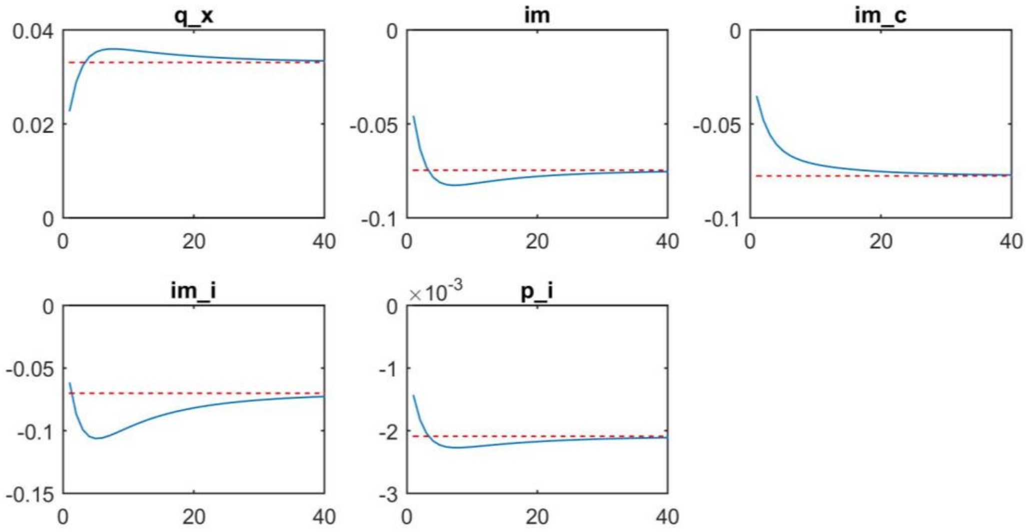
3. A 10% drop in the price of oil


p_oil
0
-0.05
-0.1
-0.15
0
5
10
15
20
25
30
35
40


c_yoy
y_e_yoy
×10-3
i_yoy
q_x_yoy
im_yoy
0
-0.01
-0.02
5
0
-5
-10
0.05
0
-0.05
-0.1
0.04
0.02
0
-0.02
0
20
40


c
d
w
l
k
y_e
p_e
zk
p_h
0
-0.02
-0.04
-0.06
-0.01
-0.03
0.01
0.005
20
40


p_x_star
rer
p_f
i
q
rp
h
h_i
h_c
×10-4
0.06
0.04
0.02
-0.05
-0.1
-0.005
-0.015
10
5
-5
20
40


q_x
im
im_c
im_i
p_i
×10^-3

Appendix 4

Table. Description of statistical data (model and instrumental variables) for estimating the paned data-based equilibrium rate model

Variable	Indicator name	Data source	Measuring unit
r_{it}	Real interest rate in the money market	IFS IMF Indicators:' Money Market Rates' and "consumer prices, YoY, end of the year'; alternative indicator 'real rates'	% p.a.
g_{it}	Per capita GDP growth in constant prices	World Bank data GDP per capita (constant LCU), in terms of YoY	% p.a.
s_{it}	Gross savings in GDP %	World Bank data Gross domestic savings (% of GDP)	%
$KAopennes_{it}$	Capital account openness indicator, Chinn-Ito (2006)	http://web.pdx.edu/~ito/Chinn-Ito_website.htm	ratio
CA_{it}	Current account of the balance of payments, % of GDP	World Bank data Current account balance (% of GDP)	%
N_{it}	Share of 16-64-year-olds in the total population	World Bank data Population, ages 15–64 (% of total)	%

Countries: Argentina, Australia, Brazil, Canada, Chile, China, Hong Kong, Czech Republic, France, Germany, India, Indonesia, Ireland, Italy, Japan, Mexico, Netherlands, New Zealand, Poland, Russia, Singapore, South Africa, Spain, Sweden, Switzerland, Turkey, USA, Great Britain, Colombia, Peru.

Model for selection from developed and developing countries:

Ratio	Value
Constant	8.71***
GDP per capita	1.15*
Gross savings	-0.35***
Chinn-Ito index	-0.08

*Significance: *** – 1-% level; ** – 5-%-level; * – 10-% level.*

Model for selection only from developing countries:

Ratio	Value
Constant	9.28**
GDP per capita	1.15
Gross savings	-0.39*
Chinn-Ito index	-0.03

*Significance: *** – 1% level; ** – 5%-level; * – 10% level.*