Bank of Russia

What measures of real economic activity slack are helpful for forecasting Russian inflation?

WORKING PAPER SERIES

No. 50 / October 2019

Ramis Khabibullin

Ramis Khabibullin

Bank of Russia. Email: KhabibullinRA@cbr.ru

Author is grateful to anonymous referees, Gergely Galics, Artem Prokhorov, Alexey Ponomarenko, Arina Sapova, Sergei Seleznev, Konstantin Styrin, Andrey Sinyakov, Nikolay Arefiev, Oxana Malakhovskaya, Grigory Kantorovich, Alexander Isakov, Sofiya Saul, participants of the META 2018 conference, participants of the 22nd Central Bank Macroeconomic Workshop and participants of the Bank of Russia seminar for their helpful comments and suggestions. I am also grateful to Arina Sapova, Dmitriy Chernyadiev and Alexandra Zhivaykina for their methodological recommendations regarding data handling and primary data analysis. All errors and mistakes are responsibility of the author.

Bank of Russia Working Paper Series is anonymously refereed by members of the Bank of Russia Research Advisory Board and external reviewers.

Cover image: Shutterstock.com

Address: 12 Neglinnaya street, Moscow, 107016
Tel.: +7 495 771-91-00, +7 495 621-64-65 (fax)
Website: www.cbr.ru

Abstract

This paper investigates inflation forecasting accuracy of several real activity slack measures for the Russian economy. Several Bayesian unobservable-components models using several real activity variables were considered. I show that real-activity slacks gain no improvement in Russian inflation forecasting. This is true for the monthly and for the quarterly data. The estimation was made in the period from the beginning of 2003 to the end of 2018 for monthly data and from the beginning of 1999 to the end of 2018 for the quarterly data. Moreover, their real-times estimates are unreliable in the sense of the magnitude of their revisions.

JEL-classification: C32, C53, E31, E32, E37

Keywords: Phillips curve, factor model, unobserved components model, output gap, real activity slack, Bayesian estimation

Contents

1. Introduction ... 6

2. Data ... 8

3. Several state-space models ... 9

3.1 Inflation equation specification ... 9

3.2 Real variable trends and slacks specifications ... 10

3.3 Time-varying parameters and regularization ... 12

3.4 Exogenous variables forecasting ... 13

3.5 All model specifications at a glance ... 13

4. Forecasting results ... 14

5. Robustness check: comparison to Hodrick-Prescott Filter ... 17

6. Reliability of real activity slack measures ... 18

7. Conclusion ... 23

References ... 24

A. Data, priors and initialization ... 26

A.1. Data sources ... 26

Monthly data ... 26

Quarterly data ... 27

A.2. The necessity of stochastic volatility assumption ... 27

A.3. Priors and initialization ... 29

B. Estimation Details ... 31

B.1. Real variable trends ... 32

Common trend case ... 33

Trivariate model case ... 33

B.2. Inflation trend ... 33

B.3. Parameters ... 33

B.4. Variances ... 34

B.5. Hierarchical LASSO parameters ... 34

B.6. SSVS probabilities ... 34

C. Marginal Likelihood calculation ... 35

C.1. Step1. Integrating with respect to trends 35

C.2. Step 2. Rao-Blackwellization with respect to other hidden states 36

C.3. Step 3. Integration with respect to parameters 36

D. Smoothed trend and cycle estimates for DFM model specification 37

E. Diebold–Mariano test P-values. Monthly Data. 37

1. Introduction

Phillips curve dependence is one of the most commonly used assumptions in the inflation forecasting literature. On the basis of macroeconomic theory one can a priori treat real economic activity slacks (such as output gap or unemployment slack) as good inflation predictors. For instance, it is usually assumed that ceteris paribus the higher the output gap is, the higher is the inflationary pressure and vice versa.

However, there is no consensus in the literature on the methodology of the real activity variables trend and gap (slack) estimation. Despite the fact that there are plenty of methods to estimate gaps (slacks), none of them provides suitable economic interpretation (Orphanides, Norden (2002)). In this paper I'll concentrate on unobservable components models (UC) of trend-cycle decomposition.

In this paper I estimated several Bayesian UC models for Russian economy with different real economic activity slack measures and show that the majority of them provides little improvements in inflation forecasting compared to standard benchmark models in terms of RMSE and marginal predictive likelihood. Moreover, I show unreliability of such measures: the magnitude of the revisions is significantly large. It means that at the moment of forecasting no one can be sure that current gap estimates would not change after adding new data points in the sample[1]. I show this for two data sets: monthly data from 2002 to 2018 and quarterly data from 2000 to 2018. I also show that standard Bayesian regularization techniques, such as stochastic specification search and LASSO hierarchical priors do not help to improve the forecasting performance.

This paper is mainly in line with the literature on the output gap and unemployment slack estimation using the UC models. While early versions of output gap estimation are presented with only univariate UC models (see Harvey (1985), Watson (1986) and Clark (1987)), recently authors have estimated bivariate and multivariate models. Since the seminal paper of Kuttner, (1994), it became a common practice to include output gap into the inflation equation under assumption of the Phillips curve relationship (see, among others, Harvey, Trimbur, (2007); Planas, Rossi, (2008); Faust, Wright (2013); Chan, Grant, (2017a)). Sinclair (2009) estimated bivariate system, in which unemployment slack is included in the output gap equation (Okun's dependence). Up-to-date practice in the UC model literature is to estimate trivariate models using various additional macroeconomic variables (Berger, Kempa (2011) use the system of output gap, inflation and exchange rate; Benes, N'Diaye (2004) use output gap, unemployment slack and inflation). Grant, Chan, (2017b) provided the stochastic specification search algorithm for the system which consisted of output gap, unemployment slack and inflation equations.

It is well known (see Planas, Rossi (2004); Jarocinsky, Lenza (2018)), that the specification of the UC system can significantly affect smoothed and filtered output gap estimates. Moreover, the cycle estimates of UC filters depend on the inclusion of the nonzero correlation between trend and cycle components in the model (see Morley et.al (2003), Basistha (2007), Grant, Chan (2017a) and Li, Mendieta-Munos, (2019, forthcoming) for more general case)

Although the real activity slack estimates are sensitive to the model specification, it is common practice to treat their real-time estimates as leading indicator of high inflation. It is usually said that the higher the output gap today is, the stronger is the inflationary pressure and, ceteris paribus, the higher inflation is predicted to be. However, there is evidence that output gap measures can be treated as bad inflation predictors. Orphanides, Norden (2002) studied several commonly used methods to estimate U.S. output gap estimates and evaluated their ability to

[1] If one compares filtered output gap in period t using parameters, estimated in the sample up to the period t with respective output gap value in period t estimated using the full-sample parameters estimates, there will be high variance of revisions.

forecast inflation. Authors made both pseudo-real time forecasts and forecasts based on full sample output gap estimates. They pay attention on two major issues. Firstly, output gap measures are unreliable for the U.S. data: there are large ex-post revisions of the output gap estimates, which make them useless for the real-time forecasting. Secondly, the majority of models considered indicate that output gap estimates have low inflation forecasting accuracy.

Similar exercises were performed using the European union (E.U.) data as well (see Marcellino, Musso (2011) and Jarocinsky, Lenza (2017)). It was shown in both papers that uncertainty of data revisions for E.U. data is much lower compared to the U.S. data. Marcellino, Musso (2011) reported low forecasting power of standard output gap measures. Jarocinsky, Lenza (2017) proposed several specifications of dynamic factor model. They constructed output gaps as common factors extracted from several real activity variables. They showed high forecasting performance of such real economic activity measures in both forecasting and nowcasting exercises. Basistha, et al, (2007) considered inflation equation specification, which includes leads of the inflation as well as lags. In this paper I use inflation specification, similar to (Chan, (2018)).

Several authors estimate the output gap measures for Russian economy using univariate and multivariate UC models. In papers (Kloudova (2015) and Zubarev and Trunin (2017)) authors estimated bivariate models, including output gaps into the inflation equation. However, in both papers only in-sample model characteristics were taken into account. Current paper focus not on the in-sample properties of real activity slacks but on their ability to improve out-of-sample inflation forecasts.

Polbin (2019, forthcoming) estimated output gap adjusting for the oil price dynamics. Since oil price change is significantly correlated with CPI inflation, this feature can improve forecasting performance of output gap. However, I did not find significant improvement in terms of RMSE for this type of model specification.

Instead of forecasting inflation levels, inflation surprises (the deviance between inflation level and inflation forecast) can be predicted. Phillips–curve dependence can be considered under assumption that output gap affects the inflation surprises (see Coibion, Gorodnichenko (2015)). Firstly, I use the Phillips curve dependence with the use of the common trend with inflation expectations using Bloomberg and InFOM survey forecasts. Secondly, inflation expectations can be considered to be specific function of several observable variables. The main problem is the specification of this function. But due to the absence of proper criterion to distinguish between models, I didn't estimate this type of assumptions.

All UC models considered in this paper were estimated using Gibbs sampler. All unobservable states, such as trends, gaps and time-varying variances were estimated using simulation smoother, proposed in Chan, Jeliazkov (2009). This smoother has an advantage over widely-used Carter-Kohn (1994) and Durbin-Koopman (2000) smoothers: integrated likelihood[2] with respect to unobservable states has a closed-form solution. Detailed descriptions of all Gibbs-sampler steps can be found in appendix B[3].

The remaining of the paper is organized as follows. In the section 2 the data is described. Section 3 is devoted to the model specifications. Main forecasting results are presented in section 4 and 5 (including comparisons with Hodrcick–Prescott filter). Section 6 consists of results on real variable slack revisions.

[2] Throughout this paper I define the 'integrated likelihood' to be the density $p(Y|\theta, X, \psi)$, while the 'marginal likelihood' to be the density $p(Y|X, \psi)$. Here θ — parameters, Y, X — endogenous and exogenous variables, ψ — prior hyperparameters. See appendices A and B for notations and definitions.

2. Data

In this paper I adopted a quasi-real-time approach, considered by Orphanides, Norden, (2002). I use 2 datasets: the quarterly data and the monthly data. All variables are seasonally adjusted using X-13ARIMA-SEATS. I divided each dataset into 3 blocks: real variables block, inflation-related block and exogenous variables block. Unfortunately, due to the absence of Russian data vintages, I have no possibility to consider data revisions.

Quasi-real time approach means that each time when forecast is made, I need to forecast not only inflation, but gaps and exogenous variables as well[4]. This adds scenario uncertainty and resulting forecasts can be even worse than simple competitor model without additional regressors.

I used two datasets to produce quasi-real time forecasts: monthly data and quarterly data. Short description of both datasets can be found in appendix A.

Quarterly dataset. Real variables block consists of 0). Real GDP y_t^0, 1). real investment y_t^1, 2). real exports y_t^2, 3). real Imports y_t^3, 4). business confidence y_t^4, 5). industrial production index (IPI) y_t^5, 6). capacity utilization y_t^6. 7). unemployment rate y_t^7, The majority of these variables are transformed into logarithms, except unemployment rate. Unemployment rate was taken in percentages.

Inflation measure is based on the Rosstat's CPI p_t. Inflation was calculated as $\pi_t = 100 \times (\log(p_t) - \log(p_{t-1}))$. Instead of using annualized inflation rates I use quarterly inflation. The aim of the study is to forecast annual inflation using models on higher frequency data. To proxy inflation expectations I used quarter-ahead Bloomberg Consensus Forecasts in the case of quarterly data and quarter-ahead InFom Survey forecasts in the case of monthly data.

I use only two exogenous variables: Brent oil price inflation $(\pi_t^{oil} = 100 \times \left(\log\left(p_t^{oil}\right) - \log\left(p_{t-1}^{oil}\right)\right)$. and the change in log Broad effective nominal exchange rate .

All quarterly data series are available from the beginning of 1999. However, inflation expectations start only from the 1st quarter of 2008. The quarterly data ends at the 4th quarter of 2018. All models were estimated starting from the 1st quarter of 2000. Instead of using inflation expectations themselves, I use the common trend of inflation expectations and inflation itself.

Monthly dataset. Monthly dataset consists of larger number of real variables. However, the big share of them starts from the 1st quarter of 2003. Monthly real variables block consists of all time series, included in quarterly real variables set, except real GDP. Several variables were added: 8). cargo index, 9). retail index, 10). construction index. All these series were transformed into logarithms.

Monthly data inflation is also based on Rosstat's CPI measure and was calculated analogously. I used InFoM survey data on households' inflation projections to measure inflation expectations. However, these time series are available only from the 1st quarter of 2011. All models were estimated on the sample from the beginning of 2003 year.

In this paper recursive forecasts were considered. To make recursive forecasts I iteratively separated the whole sample into learning and test sample. The models are estimated using the data in the learning sample. The smallest learning sample starts from the beginning of available data series (January 2003 or 1st quarter 1999) and ends in the August 2007 (3rd quarter of 2007). Then I recursively add additional month (quarter) to the learning sample and generate 12-month ahead forecast. I run this procedure within the whole test sample.

I calculate RMSEs within the test sample, which starts in the August 2007 (4rd quarter of 2007) and ends in the December 2018(4th quarter of 2018). For each model estimated in the test sample I calculate 1,3,6,12-month (1,2,3,4-quarter) ahead inflation forecast and calculate RMSEs

[4] I forecast gaps using the specified AR-process for gaps and exogenous variables using simple VAR with automated lag choice based on the BIC criterion.

with respect only to inflation itself (ignoring the forecasting accuracy of other variables in the model). To mitigate the lag selection problem, I calculated several lag specifications and selected the one with maximum log marginal likelihood.

3. Several state-space models

In the spirit of the flexible factor gap model outlined by Jarocinsky, Lenza (2017), I estimated several versions of UC models, that include real activity slack measures as predictors of the inflation. I construct each model by combination of several assumptions listed below. For each of these assumptions I introduce abbreviation to distinguish proposed models. Appendices A and B provide information on all parameters and variables notations.

3.1 Inflation equation specification

To forecast inflation, I used the extended version of inflation UC models with stochastic volatility (Chan (2013)). The observation equation for the inflation is following:

$$z_t = \pi_t - \widetilde{\pi_t} - \sum_{r=0}^{R} (\lambda_r g_{t-r} + X'_{t-r}\beta_r) \tag{1}$$

$$z_t = \sum_{j=1}^{q} \varphi_j z_{t-j} + \exp(0.5 \times h_t^{\pi})\, \varepsilon_t^{\pi}, \tag{2}$$

where $\widetilde{\pi_t}$ — inflation trend, g_t — real activity slack measure, defined below, X_t — exogenous variables, defined below, h_t^{π} — log time-varying inflation variance, q — number of lags for the MA-part of the model, R — number of lags of real activity slack measures and exogenous variables, $\varepsilon_t^{\pi} \sim i.i.d.\ N(0,1)$ is the inflation observation error term.

Inflation trend is defined as follows:

$$\widetilde{\pi_t} = \sum_{s=1}^{p} \rho_s^{\pi} \tilde{\pi}_{t-s} + \exp(0.5 \times q_t^{\pi})\, v_t^{\pi tr} \tag{3}$$

where q_t^{π} — log time-varying inflation trend variance, p — lag length for the AR-part of the inflation trend. If $p = 1, q = 0$ and $\rho_1 = 1$, this trend-cycle specification is the same as in the Stock, Watson (2007) model. However, as I will show below, the proposed trend specification with estimated parameters ρ_s^{π} has better forecasting performance for Russian inflation.

Vector X_t contains oil price inflation, change in nominal exchange rate and expectation trend. Expectation trend for monthly data was estimated as the common trend of two expectation variables provided by InFOM agency. Quarterly data expectations consist only of Bloomberg Consensus one-quarter-ahead forecasts. To estimate the common inflation expectation trend the simplest specification was chosen:

$$\pi_k^e = \widetilde{\pi_t^e} + \sigma_{ek}\varepsilon_t^{ek}, k = 1 \dots K \tag{4}$$

$$\widetilde{\pi_t^e} = \widetilde{\pi_{t-1}^e} + \sigma_{e.tr} v_t^{\pi e} \tag{5}$$

where $K = 2$ for monthly data and $K = 1$ for quarterly data. Note that expectations are available only from 2011 for monthly data and from 2008 for quarterly data. It leads to two problems. Firstly, due to the lack of data points expectation trend could be put in the model only starting with the 10th period after the beginning of the expectations series. Secondly, expectation trend estimation is very unstable when few data points are used. To mitigate this problem, I set very low prior variance $\sigma_{e.tr}$ for the trend process (see appendix A for details).

One of the crucial issues of the proposed inflation specification is the choice of lag length hyperparameters (q, R and p). Following the practice of standard UC models estimation (Chan (2013)), I selected maximum values of these hyperparameters to be $p_{max} = 2$, $q_{max} = 11$, $R_{max} = 12$ for monthly data and $p_{max} = 2$, $q_{max} = 3$, $R_{max} = 3$ for quarterly data.

Throughout the paper the log stochastic variance is defined as a standard random-walk process:

$$h_t^j = h_{t-1}^j + \sigma_{h.j}\xi_t^{h.j}; \qquad q_t^j = q_{t-1}^j + \sigma_{q.j}\xi_t^{q.j} \tag{6}$$

where h_t^j — log variance for observation equation j, q_t^j — log variance for state equation j, $\xi_t^{h.j}$, $\xi_t^{q.j} \sim \chi^2$. The smoothed states are estimated using (Kim, et. al, (1998)) normal mixture approximation. To test whether the inclusion of the stochastic volatility is necessary, I compared Bayes factors[5] for several model specifications, estimated using (Chan, (2018)) method (see details in appendix B). The model with stochastic volatility both in observation and state equations has highest Bayes factor. Hence, this model is preferable to others.

Recall that to calculate RMSE values each model was estimated 120 times for monthly data and 32 times for quarterly data. To select lag length, I compared models on the basis marginal likelihood (in-sample property)[6]. To estimate marginal likelihood values, I used the approximation proposed by Chib (1995) for the Gibbs Sampler output (see appendix B for details).

I considered only 3 different specifications of inflation equation:

1. **UCSV.** This is the standard Stock,Watson (2007) inflation UC specification. It is the case when there are no exogenous variables, $q = 0$; $\beta_r = 0$; $p = 1$; $\rho_1 = 1$. For this class of specifications, I will only add or remove real activity slacks, defined in the next section.
2. **UCSVMA**. This is the specification, defined in equations (1-3) with restriction of non-inclusion of exogenous variables: $\beta_r = 0$.
3. **UCSVMAX**. This is the specification, defined in equations (1-5) without any restrictions.

3.2 Real variable trends and slacks specifications

3.2.1 Benchmark model

The main model used in this paper is the quazi-Dynamic Factor Model (QDFM) specification, similar to Jarocinsky, Lenza (2015). The idea of this model is to estimate the real variable gaps common factor. This gap is defined as the gap of one of the variables (reference variables) y_t^0 and at the same time as the common factor for all other real variables $y_t^n, n = 1..N$. Jarocinsky, Lenza (2015) use real GDP as the reference variable and interpret the gap as 'output gap' measure. In this paper I treat as reference variables real GDP for quarterly data and real industrial production index for monthly data[7].

The reference variable is modelled as the standard trend–gap decomposition:

$$y_t^0 = \mu_t^0 + g_t, \tag{6QDFM1}$$

where μ_t^0 — reference variable trend, g_t — common gap factor. All other real variables are defined as:

$$y_t^n = \mu_t^n + \sum_{r=0}^{R} B_r^n g_{t-r} + \varepsilon_t^n, \;\; n = 1..N \tag{6QDFM2}$$

where $\varepsilon_t^n \sim N(0, \exp(h_t^n))$, h_t^n — time-varying log volatility, μ_t^n — real variable n's trend, N — number of real variables except reference variable. Parameters B_r^n can be treated as factor

[5] Bayes factor is the fraction of posterior probabilities of two models. In practice it is more convenient to estimate log Bayes factor: $\log(BF(1,2)) = \log\big(p(M_1|Data)\big) - \log\big(p(M_2|Data)\big)$. The higher the Bayes factor is, the more probable the model 1 is compared to the model 2.

[6] Bayesian information criterion and Deviance information criteria (see Grant, Chan (2016)) can also be applied to select the lag length. However, there is no consensus which type of DIC criterion should be used in the forecasting exercise, so I used the standard criterion. This exercise is done to mimic the real time forecasting problem without knowledge of future forecasting accuracy. The alternative way is to use cross-validation techniques to select model hyperparameters. However, the sample length is too short to implement it.

[7] However, one can choose other variables to be reference variables. In my forecasting exercise I use specifications with different reference variables.

loadings for the common gap factor. Recall that R lags of the common gap is included in the equation (QDFM2), where R is the lag length of all exogenous variables in the model.

Following (Planas, Rossi, (2008)), I used the following gap specification:

$$g_t = \rho_1^g g_{t-1} + \rho_2^g g_{t-2} + \nu_t^g \quad (7a)$$

$$\rho_1^g = 2A\cos\left(\frac{2\pi}{\omega}\right);\ \rho_2^g = -A^2 \quad (7b)$$

where $\nu_t^g \sim N(0, \sigma_g^2)$, parameters (ρ_1^g, ρ_2^g) are functions of parameters A and ω. [8],

Gaps by definition depend on the trend specifications. In this paper we use several popular trend specifications (Grant, Chan, (2018) and Jarocinsky, Lenza (2015)). For each of the real variables $s = 0..N$:

Random Walk with Drift: $$\mu_t^s = \alpha_0^s + D_t'\zeta + \mu_{t-1}^s + \nu_t^{\mu.s} \quad (8RWD)$$

Integrated Random Walk: $$\mu_t^s = \alpha_t^s + \mu_{t-1}^s \quad (8IRW1)$$

$$\alpha_t^s = \alpha_{t-1}^s + \nu_t^{\mu.s} \quad (8IRW2)$$

Simple Linear Trend: $$\mu_t^s = \alpha_0^s + \alpha_{tr}^s t + D_t'\zeta + \nu_t^{\mu.s} \quad (8SLT)$$

where α_0^s — constant drift, α_t^s — time-varying drift, μ_0^s — constant term, $\nu_t^{\mu.s} \sim N(0, \sigma_{\mu.s}^2)$, $\nu_t^{\alpha.s} \sim N(0, \sigma_{\alpha.s}^2)$. D_t — matrix of gummy variables, including additive and innovative outliers for the crises of August, 2008 and December, 2014[9].

In the spirit of model specification, presented in (Polbin, 2019), for IPI I also considered additional model specification (GOIL), adding oil price inflation to the definition of output gap:

$$\mu_{t\,t}^s = \alpha + \mu_{t-1}^s + \sum_{s=0}^{r} \beta_s^{goil} \pi_{t-s}^{oil} + \nu_t^{\mu.s} \quad (7TOIL)$$

The block of real variables includes all variables, described in the section 2. The choice of the reference variable significantly affects the resulting common gap measure. For the purpose of inflation forecasting any variable can be chosen to be reference one. On the one hand, real GDP and IPI are natural candidates to be reference variables.

On the other hand, Russian real GDP and IPI are low-volatile in the period considered. It means that other real variables could either gain additional information useful for the forecasts or generate additional noise. Thus, I estimated several quasi-DFMs with alternative reference variables.

The other problem is the choice of the number of factors. In the benchmark model specification only one common gap is considered. Since the seminal paper of (Bei, Ng, (2003)), it became a common practice to choose the number of factors using information criteria. However, one of the crucial assumptions of this approach is the large number of periods and (or) variables. Under the settings of this paper this assumption doesn't hold.

3.2.2 Common trend model

Economic theory predicts that all real variables can be characterized not only by the common gap component, but by the same real variables trend μ_t^{com}. Under this assumption one can define the model in the following way:

$$y_t^0 = \mu_t^{com} + g_t \quad (9QDFMCT1)$$

---

[8] Instead of using such specification, Jarocinsky, Lenza (2018) and Grant, Chan (2018) simply restrict parameters (ρ_1, ρ_2) to generate stationary process. However, I found that this approach lead to unstable forecasts in the beginning of the sample. The other way to define cyclical process is to use permutation matrix specification (Harvey, et.al). I use adaptive Metropolis step to estimate parameters (A, ω). See details in appendix B.

[9] Due to the fact that I made recursive forecasts, estimation sample length differs at each step of the forecasting exercise. Dummy variable for the particular event is included only if at least 6 points after the event are present in the sample.

$$y_t^n = A^n \mu_t^{com} + \sum_{r=0}^{R} B_r^n g_{t-r} + \varepsilon_t^n, \;\; n = 1..N \qquad \text{(9QDFMCT2)}$$

where N real variables set includes all variables as in the previous specification except business confidence and unemployment rate[10]. Common trend μ_t^{com} dynamics is defined similar to the specification in the previous section (equations 8).

3.2.3 Trivariate model

One of the most popular state-space models for the trend-cycle decomposition is the system of three equations: real GDP (IPI), unemployment rate and inflation equations. I will use the TVM abbreviation for this model specification. The model is stated as follows. Unemployment rate u_t is decomposed into non-accelerating rate of unemployment u_t^* (NAIRU) and cycle component u_t^g:

$$u_t = u_t^* + u_t^g \qquad \text{(10TVM)}$$

$$u_t^* = \alpha_t^u + u_{t-1}^* \qquad \text{(11TVM)}$$

$$\alpha_t^u = \alpha_{t-1}^u + v_t^{u*} \qquad \text{(12TVM)}$$

$$u_t^g = \rho_1^u u_{t-1}^g + \rho_2^u u_{t-2}^g + v_t^{ug} \qquad \text{(12TVM)}$$

where α_t^u — time-varying NAIRU intercept, parameters (ρ_1^u, ρ_2^u) are restricted to generate stationary process, $v_t^{ug} \sim N(0, \sigma_{ug}^2)$; $v_t^{u*} \sim N(0, \sigma_{u*}^2)$.

Real GDP (IPI) y_t^0 is decomposed into trend and cycle components under the assumption of Okun's Law dependence:

$$y_t^0 = \mu_t^0 + \delta_u(u_t - u_t^*) + g_t, \qquad \text{(13TVM)}$$

where μ_t^0 is defined by equations (8) and gap g_t is defined as AR(2)-process (7).

Inflation state and observation equations are the same as in section 3.1.

3.2.4 Bivariate models

Instead of using several real variables in the model simultaneously, one can consider the simplest possible case: the case with single real variable and inflation. For this type of models I iterated several real variables and used only integrated random walk trend specification (8IRW1-2). Hence, for his specification:

$$y_t^s = \mu_t^s + g_t, \qquad s \in \{1,2 \ldots, N\} \qquad \text{(14BVM)}$$

This simple model is one of the natural benchmarks to the proposed quasi-DFM specification.

3.3 Time-varying parameters and regularization

There are two features of Russian real variables and inflation dynamics. It is usually said that there were several structural shocks within the sample related to the change in the monetary policy and oil price trends. This could lead to the change in parameters. To account for this difference, I add time-varying parameters in the inflation equation only for bivariate models and consider this case as a special case. Hence, the model is equivalent to the model, specified in section 3.2.4, except that the inflation gap equation is the following:

$$z_t = \pi_t - \widetilde{\pi_t} - \sum_{r=0}^{R} \left(\lambda_r^{(t)} g_{t-r} + X_{t-r}' \beta_r\right) \qquad \text{(16TVP1)}$$

$$\lambda_r^{(t)} = \lambda_r^{(t-1)} + v_t^r \qquad \text{(16TVP2)}$$

In the case of quasi-DFM model the number of parameters is large. This can lead to the model overfit, especially in the beginning of the test sample. Moreover, it is not clear what real variables should be included in the common gap factor estimation. In the case of quarterly data this problem is especially severe. So I tried to regularize models for the quarterly data.

[10] This is due to the fact that individual trends for these variables differ significantly from the trends of other real variables.

Thus, following (Korobilis, (2013)), I included hierarchical LASSO prior for inflation equation parameters λ_r and β_r and to the factor loadings parameters B_r^n. Let $Hpar = (\lambda_r, \beta_r, B_r^n)$. Adopting this approach, one treats prior distribution parameters not as strict model hyperparameters, but as random variable as well. In this paper I focus mainly on the prior variance parameters. Under LASSO-type shrinkage they have exponential prior distribution:

$$p\big(var(Hpar_i)\big) \sim \exp(0.5 \times (\overline{Hpar})^2)$$

This model specification shrinks parameter values to zero depending on the value of hyperparameter $\overline{Hpar}$.

Following (Grant, Chan, (2018)), I also considered stochastic specification search (SSVS) algorithm with respect to parameters variance $var(Hpar)$ based on the (George, et. al (2008)) procedure. The procedure is based on the sampling of discrete random variable u_i, which indicates the switch between two proposed prior variances. It means that if $u_i = 1$, the prior variance is low (this is equivalent to restricted parameter case), else if $u_i = 0$ the prior variance is high (this is equivalent to unrestricted parameter case). See appendix B for more details.

3.4 Exogenous variables forecasting

To forecast oil price inflation and the change in log nominal exchange rate I use the simple VAR model with 2 variables and automatic lag length choice using BIC criterion. It was applied iteratively each time the forecast is made.

3.5 All model specifications at a glance

Combinations of assumptions above gain different model specifications. In this paper I consider only part of them. The characteristics of each estimated model are listed in the table 2.

Table 1. Characteristics of all considered models

MODEL	TREND TYPE	OUTPUT	BLOCK OF REAL VARIABLES	REGULARIZATION
UCSV	—	—	—	—
UCSVMA	—	—	—	—
UCSVMAX	—	—	—	—
UCSVMAX DFM	RWD, IRW, SLT	Several reference variables	All	—
UCSVMAX DFMCT	IRW	Real GDP, IPI	All	—
UCSVMAX TVM	RWD	X	Only unemp-loyment rate	—
UCSVMAX BVM	RWD, IRW, SLT, TOIL	Several reference variables	—	—
UCSVMAX DFM HIER, BMA	RWD, IRW, SLT	X	—	LASSO, BMA

The first 3 models are standard competitor models in the state-space modelling literature (Chan, (2013)). UCSVMAX DFM specification uses all the available information from the real variables block to predict inflation and I treat this specification as the main benchmark. The most natural competitor to this model is Bivariate one (UCSVMAX BVM). To test if popular in the literature specifications improve forecasting accuracy, I included UCSVMAX TVM, UCSVMAX CYC and UCSVMAX DFMCT specifications.

It can be seen that some models are nested, some are overlapping with each other. This fact is crucial for out-of-sample tests procedures and interpretation of results. The relationships between these models are depicted in figure 1.

Each model was estimated using the Gibbs Sampler procedure, described in appendix B. At each forecasting step I used 100000 iterations with burn-in equal to 40000 iterations.

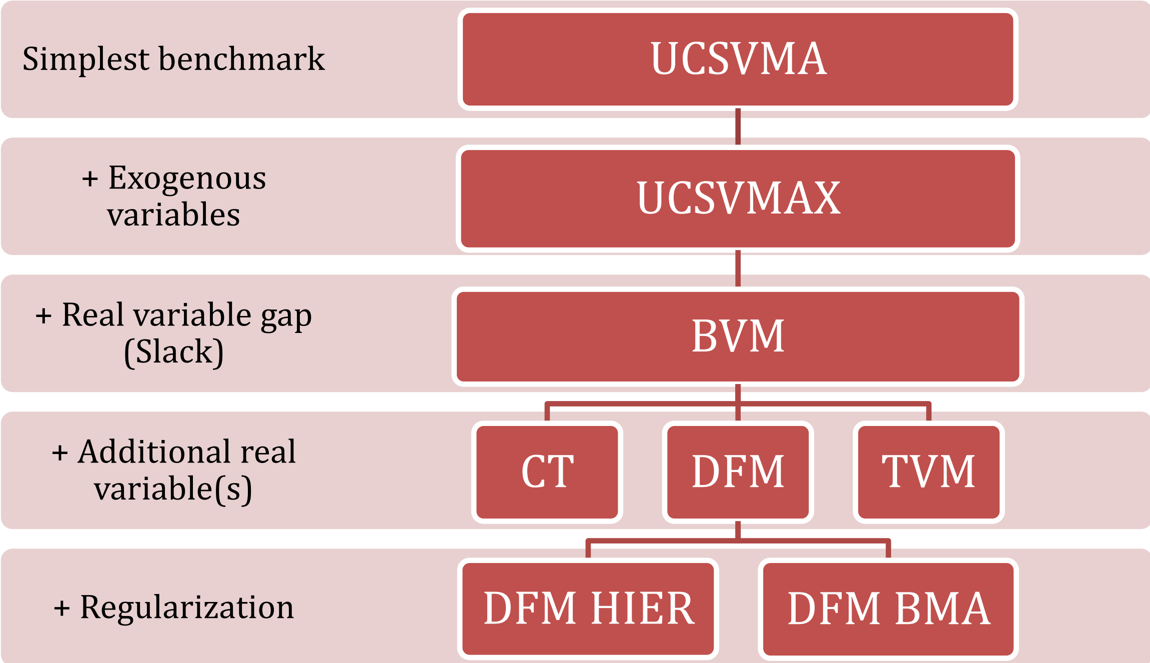


Figure 1. The relationship between considered models

4. Forecasting results

Monthly data. Since my main benchmark model is quasi-dynamic factor model, I show trend and cycle decomposition of several reference real variables in appendix D. The RMSEs for each of the model shown in the table 3. For all of the BVM and DFM specifications I report the best in terms of RMSEs trend specification (for each of reference variables IRW, RWD, RW were estimated). The length of the test sample is 120 points and starts from the August, 2008.

Table 2. RMSE estimates for monthly data models (P-values of Diebold–Mariano (1995) test compared to UCSVMAX are presented in Appendix E).

	1M	3M	6M	9M	12M	YOY
UCSV	0.266	0.443	0.477	0.494	0.581	4.417
UCSVMA	0.323	0.390	0.411	0.450	0.512	3.669
UCSVMAX	0.299	0.388	0.408	0.434	0.467	3.435
UCSVMAX DFM IPI IRW	0.328	0.396	0.418	0.462	0.482	3.469
UCSVMAX DFM EXP IRW	0.308	0.398	0.406	0.434	0.467	3.468
UCSVMAX DFM IMP IRW	0.310	0.403	0.405	0.438	0.474	3.518
UCSVMAX DFM EMP RWD	0.300	0.392	0.414	0.460	0.493	3.634
UCSVMAX DFM CONSTR RWD	0.299	0.386	0.405	0.443	0.482	3.446
UCSVMAX DFM CARGO IRW	0.312	0.392	0.412	0.444	0.479	3.560
UCSVMAX DFM BUSCONF RW	0.304	0.393	0.398	0.437	0.476	3.426
UCSVMAX DFM RETAIL RWD	0.306	0.388	0.402	0.447	0.484	3.590
UCSVMAX DFM UNEMP RW	0.313	0.380	0.426	0.467	0.500	3.467
UCSVMAX CT IPI IRW	0.306	0.405	0.419	0.463	0.504	3.765
UCSVMAX TVM	0.380	0.390	0.402	0.453	0.473	3.431
UCSVMAX BVM IPI IRW	0.299	0.397	0.406	0.450	0.473	3.461
UCSVMAX BVM EXP IRW	0.291	0.395	0.410	0.461	0.477	3.579
UCSVMAX BVM BUSCONF RWD	0.403	0.520	0.545	0.541	0.633	4.969
UCSVMAX BVM UNEMP RW	0.270	0.389	0.406	0.465	0.481	3.434
UCSVMAX BVM CARGO RW	0.241	0.412	0.391	0.444	0.463	3.431**
UCSVMAX BVM IPI TOIL	0.327	0.441	0.486	0.495	0.522	4.487

Recall that inflation measure was in percentages. The first 5 columns report the RMSE of 1,3,6,9,12 month ahead forecasts. The last column represents the cumulative inflation within the year forecast. These forecasts were calculated as $\sum_{j=1}^{12} \hat{\pi}_{t+j}$ and were compared to the actual cumulative inflation $\sum_{j=1}^{12} \pi_{t+j}$ at each point of time t.

The results are separated into 3 blocks. The first block is the standard benchmark models UCSV (Stock, Watson, (2007)) and its modification UCSVMAX (Chan, (2013)). The second bock consists of quasi-DFM models with different reference variables. The third block consists of other competitive models.

It can be seen that oil price inflation and the change in nominal exchange rate significantly improve the quasi-real time forecasts: RMSE for UCSVMAX is lower than RMSE for UCSV in all cases.

Neither of the reference variables improves the forecasting accuracy of these benchmarks. This is true for both DFM and BVM specifications. Moreover, the difference in RMSE between models is not significant. To investigate the reason for such RMSE behavior, I calculated the 2-year recursive RMSE estimates. It means that I calculated RMSEs within the two-year window in the test sample and recursively move this window one month forward. The results for BVM specifications are shown in the figure 2. The results for the DFM specifications are approximately the same.

Moreover, it can be seen that inclusion of oil price dynamics into the trend equation (specification BM TOIL) doesn't improve forecasting accuracy, but worsens it. This can be due to the fact that oil price inflation is difficult to forecast and the change in oil prices significantly affects the slope of the Industrial production trend.

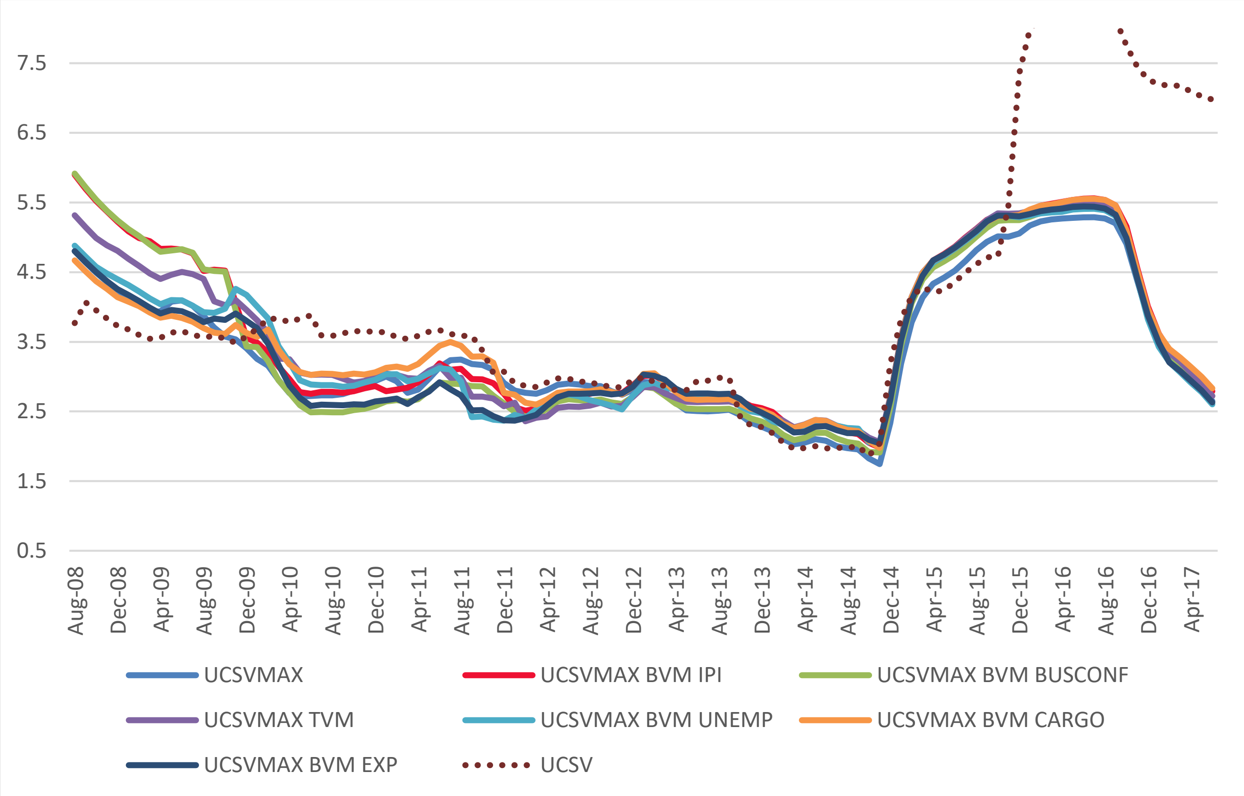


Figure 2. Recursive RMSEs for several models

Each point of time in the figure 2 is the last point of 2-year window of RMSE calculation. It can be seen that the main problem of the forecasts of bivariate models is the crisis in 2008. It can be explained by the low volatility of Russian real variables before the crisis. Due to the abrupt change in real variables prior to crisis, the gap forecasts were quite volatile, which lead to the large RMSE values.

In the period of crisis in 2014 all models considered have approximately the same forecasting accuracy, except unemployment rate and cargo production index. It can be seen also that IPI has no improvement in the forecast accuracy at all. At the same time the inclusion of unemployment rate helps to predict the inflation near the crisis in 2014. Trivariate model does not improve this forecast. Real export series on the one hand improve the forecasting accuracy near crisis periods, but significantly worsen it in stable periods.

Moreover, the usage of common factor gap gains no improvement in terms of RMSE forecast accuracy.

This result can be explained for several variables. The main problem of the Russian macroeconomic real activity data is the fact that available time series are very short and there are only 3 large identifiable recessions after the 1998 crisis. However, several variables are much more persistent. Appendix D depicts Industrial production and unemployment trend-cycle decomposition. It can be seen that there is no identifiable cycles in industrial production dynamics except the downturn in 2008. Unemployment rate is also persistent. This finding is in line with findings that the unemployment rate is inelastic to the change in wages. This could lead to the low volatility of unemployment rate in the periods of recessions and booms. It could result in the non-identifiable change in the unemployment slack in near crisis periods and low forecasting performance.

Quarterly data. In the case of quarterly data there is a problem of small sample: only 48 points are available since the 1st quarter of 1999. I use 32 points for quarterly data in the test sample, which leads to two issues. Firstly, the crisis of 2008 is not included in the testing sample (in contrast to monthly data). Secondly, in the beginning of the testing sample the inclusion of large lag length leads to the model overfit. So, for the first 10 points of the test sample I restricted lag length parameters to be $(p = 1, q = 0, R \leq 3)$. For all other test sample points I set $(p \leq 2; q \leq 3; R \leq 3)$. The forecasting results are presented in the table 3. It can be seen, that the performance of real activity slack improves forecasts.

Table 3. RMSE estimates for quarterly data models.

	1Q	2Q	3Q	4Q	YOY
UCSV	1.556	1.562	1.599	1.739	5.030
UCSVMA	1.251	1.295	1.409	1.387	3.648
UCSVMAX	1.227	1.227	1.318	1.351	3.377
UCSVMAX BVM GDP	1.247	1.202	1.323	1.322	3.284
UCSVMAX BVM IPI	1.210	1.211	1.285	1.304	4.579
UCSVMAX BVM REAL INV	1.166	1.214	1.320	1.342	3.040
UCSVMAX BVM REAL IMP	1.220	1.240	1.328	1.388	5.245
UCSVMAX BVM REAL EXP	1.209	1.231	1.293	1.328	3.114
UCSVMAX BVM EMP	1.267	1.250	1.311	1.354	3.635
UCSVMAX BVM UNEMP	1.179	1.308	1.373	1.354	3.448
UCSVMAX DFM GDP	1.230	1.244	1.353	1.370	3.367
UCSVMAX DFM IPI	1.198	1.294	1.384	1.333	3.938
UCSVMAX DFM RINV	1.136	1.263	1.364	1.348	3.373
UCSVMAX DFM IMP	1.210	1.290	1.373	1.323	3.806
UCSVMAX DFM EXP	1.203	1.295	1.368	1.340	3.368
UCSVMAX DFM EMP	1.210	1.296	1.389	1.334	3.858
UCSVMAX DFM UNEMP	1.216	1.301	1.382	1.330	3.298
UCSVMAX DFM HIER GDP	1.288	1.293	1.270	1.202	4.089
UCSVMAX DFM BMA GDP	1.698	1.690	1.689	1.665	5.247

It can be seen, that DFM specification worsens forecasting accuracy compared to the BVM model. For employment, IPI and import DFM specification forecasting accuracy is even worse than the one for UCSVMAX benchmark. It can be either due to low gaps volatility at the beginning of the test sample (as in the case of monthly data) or due to the model overfit.

If the later problem is crucial, HIER and BMA specifications should help to mitigate it. However, RMSEs for hierarchical DFM model does not outperform neither BVM, nor DFM specifications. The same is true for BMA specification.

Nevertheless, real investment and real export gaps slightly improve forecasting accuracy for quarterly data in contrast to the monthly data.

5. Robustness check: comparison to Hodrick-Prescott Filter

To check the robustness of the main result, it is natural to compare it to other gaps and slacks estimation methods.

Two well-known alternatives to the unobservable components models are Hodrick-Prescott (HP) and Hamilton filters. The first was criticized for three reasons (Hamilton, (2018)):

1. It is based on assumptions, different from assumed underlying data-generating process;
2. Trends estimated at the end of the sample are different from those in the middle;
3. Typical procedures to elucidate smoothing hyperparameter are not based on the statistical properties of filtered series.

Hamilton proposed another filter, based on local projections approach. However, in forecasting exercise there is no consensus in filtering method choice. For instance, it was shown that credit gaps filtered by HP filter outperforms those estimated via Hamilton filter in accuracy of predicting crises (Drehmann, Yetman. (2018)).

Due to these facts I compare forecasts obtained from UCSV models with forecasts based on the gaps obtained using Hodrick–Prescott (HP) filter. To handle the first issue of the standard HP–filter, I estimate HP–AR filter alternative, specified in (Grant, Chan, (2017c)) paper. To forecast gaps, I used standard AR(p) model with lag order choice made on the basis on the AIC measure on the real-time basis. RMSEs for these models are shown in the table 2.

To correctly compare model with HP filter with UCSV DFM specification, I also included principal component of all gaps into the UCSVMAX inflation equation (specification UCSVMAX HP PCA).

Table 4. RMSE estimates for UCSVMAX specification with inclusion of HP-filtered gaps. Monthly data.

	1M	3M	6M	9M	12M	YOY
UCSVMAX HP IPI	0.351	0.420	0.426	0.438	0.466	3.678
UCSVMAX HP EXP	0.302	0.382	0.399	0.435	0.469	**3.413**
UCSVMAX HP IMP	0.328	0.435	0.442	0.453	0.484	3.841
UCSVMAX HP EMP	0.290	0.385	0.421	0.448	0.479	3.568
UCSVMAX HP CONSTR	0.308	0.384	0.402	0.423	0.452	**3.260**
UCSVMAX HP CARGO	0.318	0.407	0.423	0.449	0.480	3.638
UCSVMAX HP RETAIL	0.309	0.390	0.433	0.450	0.476	3.537
UCSVMAX HP BUSCONF	0.334	0.394	0.406	0.439	0.477	3.488
UCSVMAX HP UNEMP	0.307	0.401	0.433	0.453	0.479	3.683
UCSVMAX HP PCA	0.342	0.393	0.416	0.423	0.457	**3.388**

It can be seen, that the forecasting accuracy of the UCSV–type trends are higher than the one for HP–filter trends. Moreover, only real export gap, construction gap and factor gap, obtained using PCA generate inflation forecasts with RMSE lower than the one for UCSVMAX specification. It means that neither gaps specified by UCSV, nor HP-filtered gaps significantly improve forecasting accuracy.

6. Reliability of real activity slack measures

Another issue of the trend-cycle decompositions, mentioned by (Orphanides, Norden, (2002)) and (Jarocinsky, Lenza, (2018)), is the reliability of one-step ahead forecasts of the real economic activity slacks or output gaps. Output gaps and real activity slack measures are often used as the indicator of inflationary pressure. However, it seems like for the US and EU data this measure has the same magnitude of revisions as the magnitude of the smoothed gap estimates.

The same result is true for the Russian economy as well. To illustrate this thesis, I show the IPI gap, Export gap and unemployment slack estimates on the basis of DFM specification in figures 3-5. Figures compare the smoothed estimates of the gap (black lines) and one-month ahead forecast (read lines and grey shaded areas)[11].

[11] All gaps presented here, are percentage change from the trend value

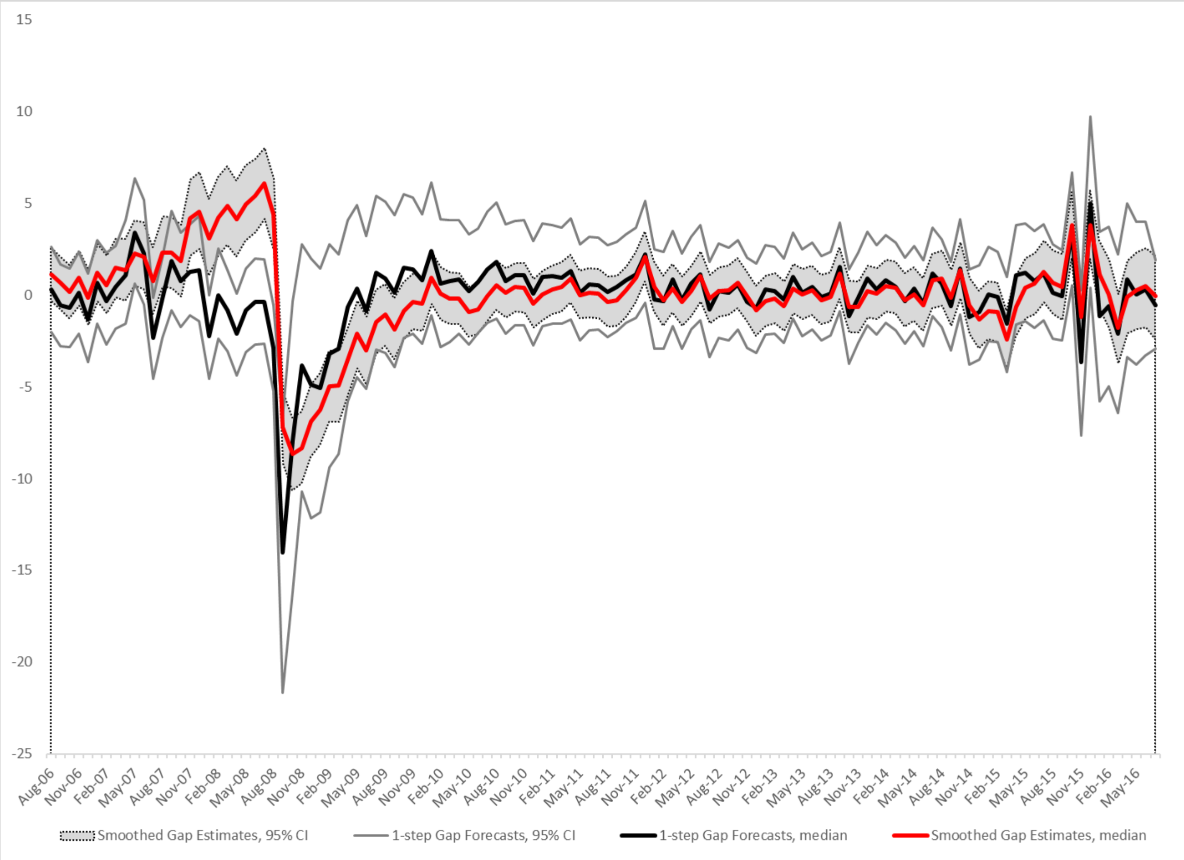


Figure 3. IPI gap, DFM specification. Smoothed estimates and 1-step ahead forecasts

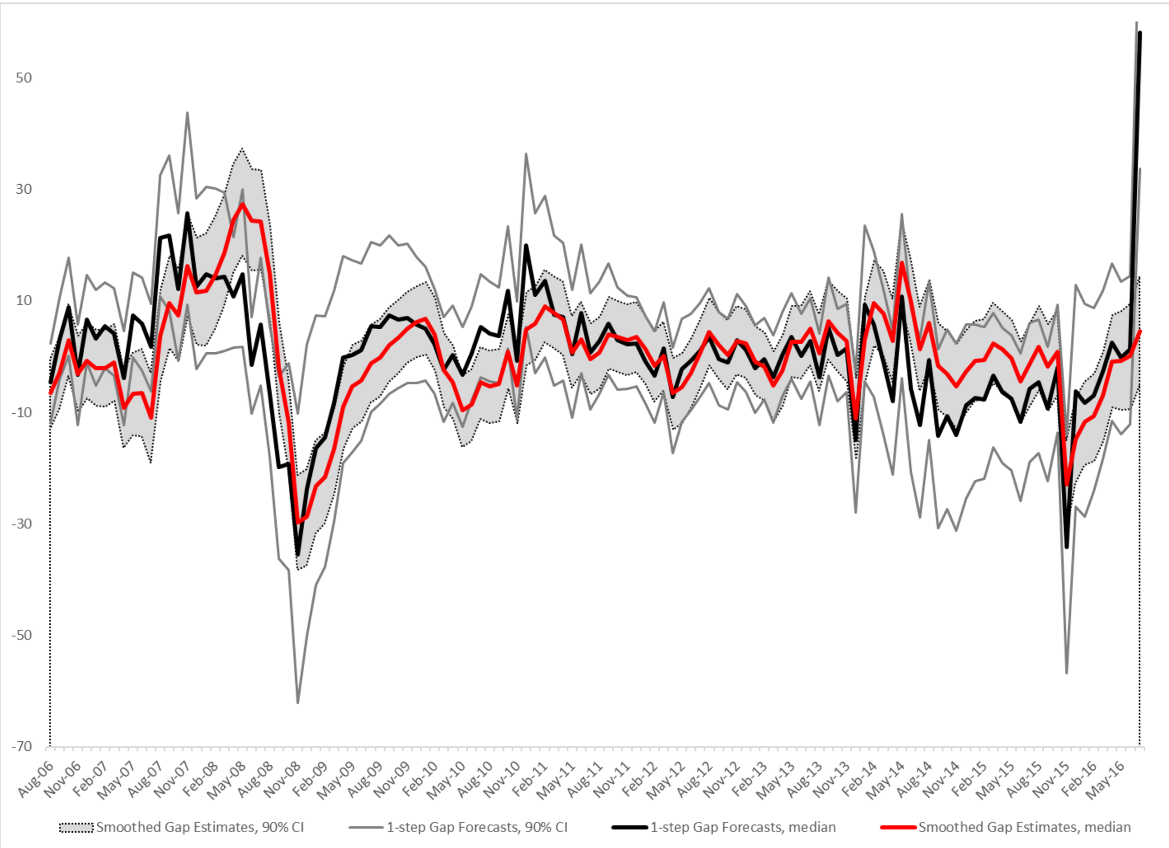


Figure 4. Real Export gap, DFM specification. Smoothed estimates and 1-step ahead forecasts

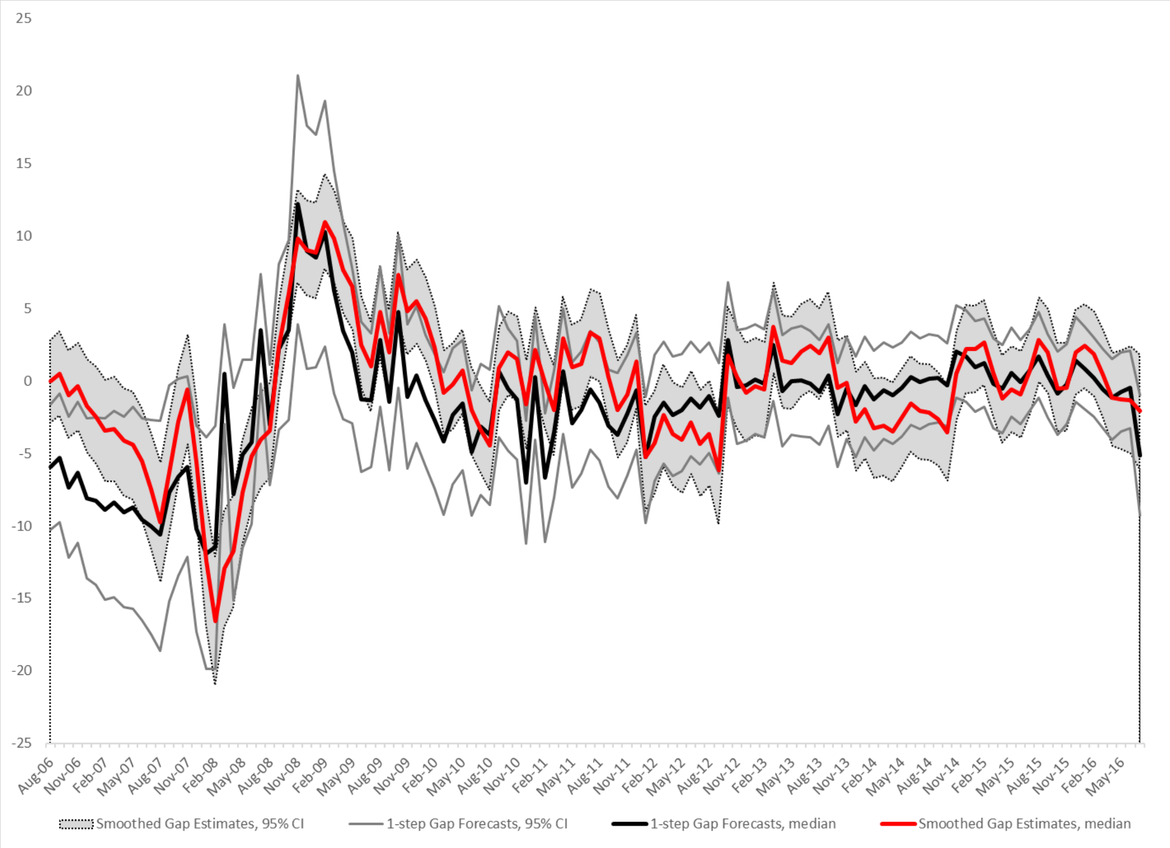


Figure 5. Unemployment slack, DFM specification. Smoothed estimates and 1-step ahead forecasts

It can be seen that there is a significant reevaluation of trend values at the end of the sample, especially at the moment of 2008 crisis. The reevaluation is lower than in the case of European data set (Jarocinski, Lenza, (2015)). However, it can be seen that smoothed estimates do not always lie in the smoothed estimates confidence interval. It can be treated as an evidence of significant reevaluation, inflation forecasts can be biased.

Another problem is smoothed real variables slacks revisions. Figures 6-8 depict selected slack revisions. The solid red line represents the final smoothed estimates, dotted lines — the evolution of in-sample (smoothed) estimates of real variables slacks for each sample in the training set. It can be seen that the magnitude of the revisions is high, especially in the beginning of the train sample. It means that at each point of time one cannot be sure that the dynamics of real activity slacks would not change after addition of new data points.

Therefore, neither one-step real-time slack forecast, nor in-sample slack dynamics can be reliable for the variables considered in this paper.

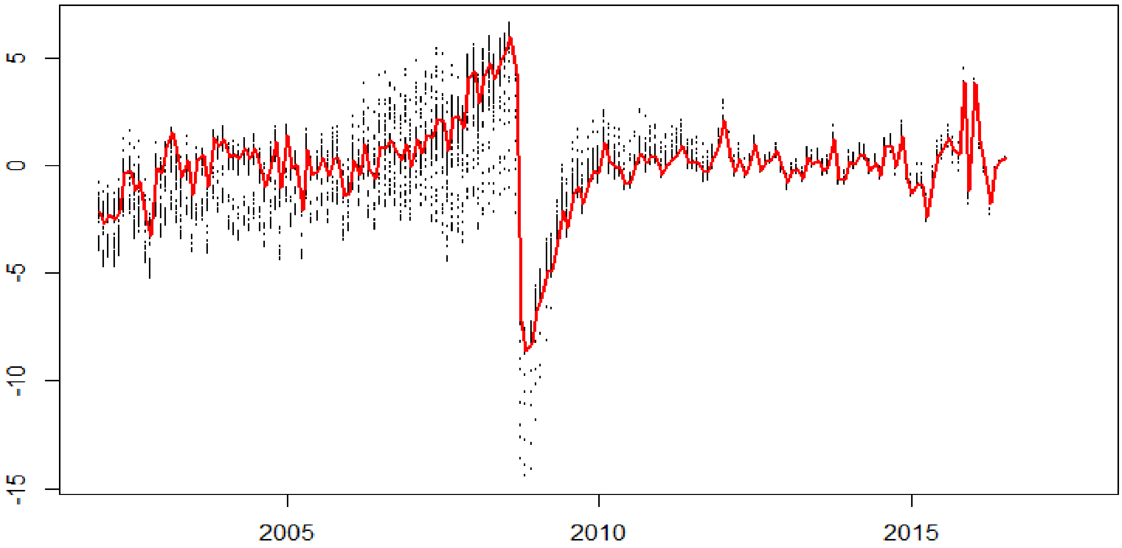


Figure 6. Smoothed Industrial Production gap revisions, DFM specification. Red line refers to the whole sample estimate. Black dots refer to the recursive revisions at each point of time.

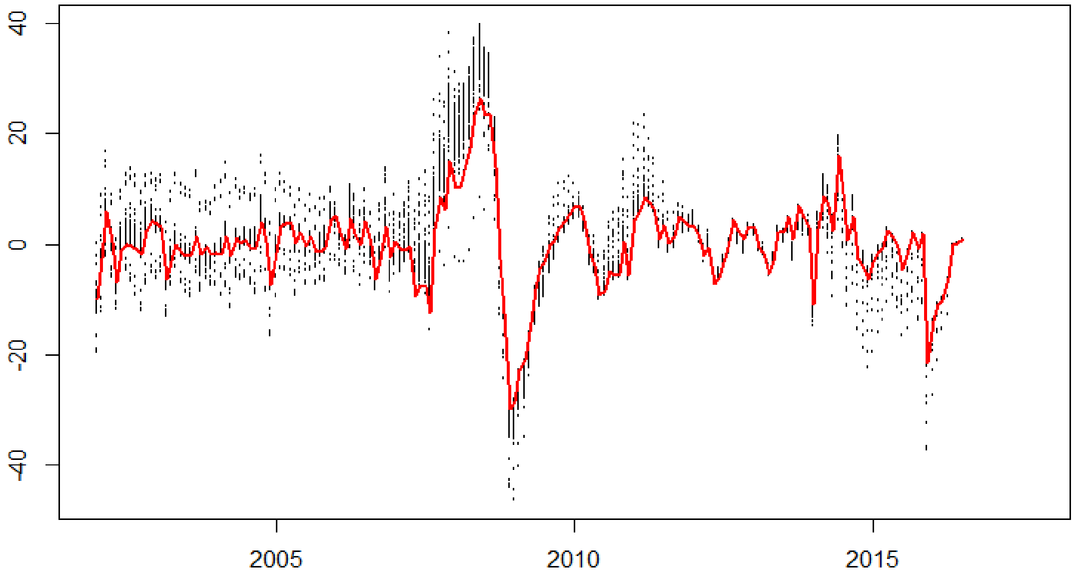


Figure 6. Smoothed Real Exports gap revisions, , DFM specification. Red line refers to the whole sample estimate. Black dots refer to the recursive revisions at each point of time.

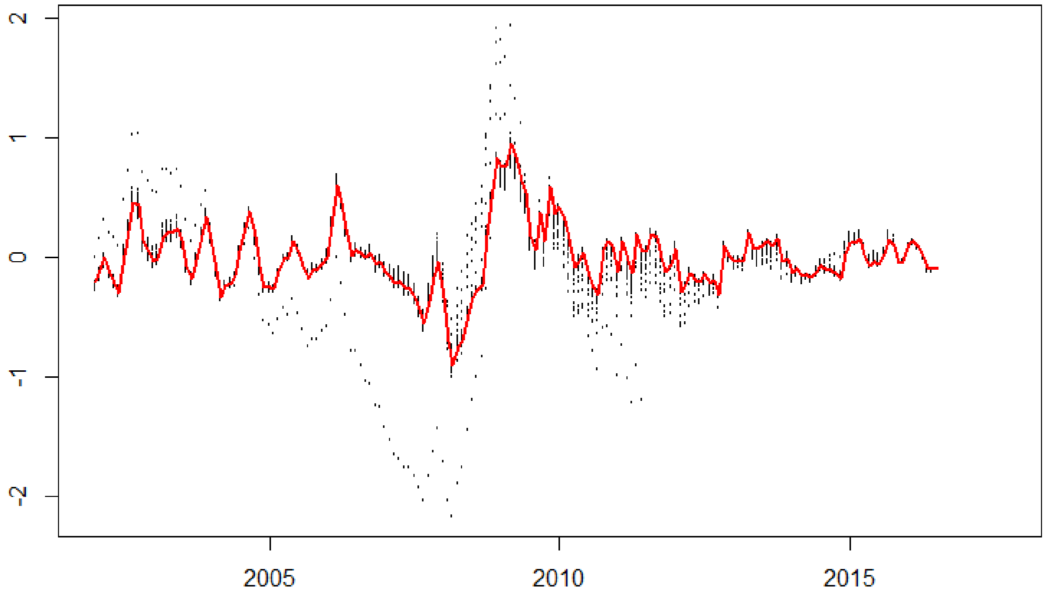


Figure 8. Smoothed Unemployment slack revisions, DFM specification. Red line refers to the whole sample estimate. Black dots refer to the recursive revisions at each point of time.

The other issue is the interpretation of the connection between output gap estimates and related to them inflation forecasts. It is usually said that the higher the output gap is, the higher is inflation forecast is supposed to be. This can be true if one compares gap measure for the same model specification. However, if one compares gap measures between models, this is not true. To demonstrate this fact, for each of the reference variables I compared DFM specification with corresponding BVM specification. For each of the reference variables I calculate one-step inflation forecast difference between DFM and BVM specifications and difference in one-step gap forecasts between specifications.

The most counter-intuitive results are for employment and real cargo production index reference variables. Results for both variables are depicted in figures 9 and 10.

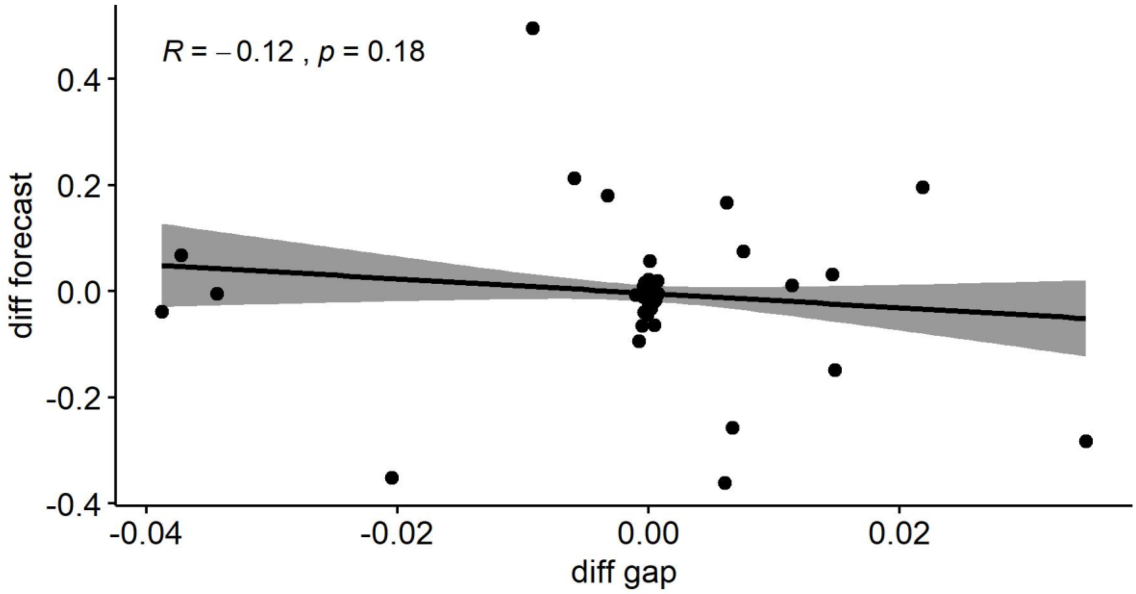


Figure 9. Correlation between differences in gaps and inflation forecasts between DFM and BVM specifications. Employment reference variable.

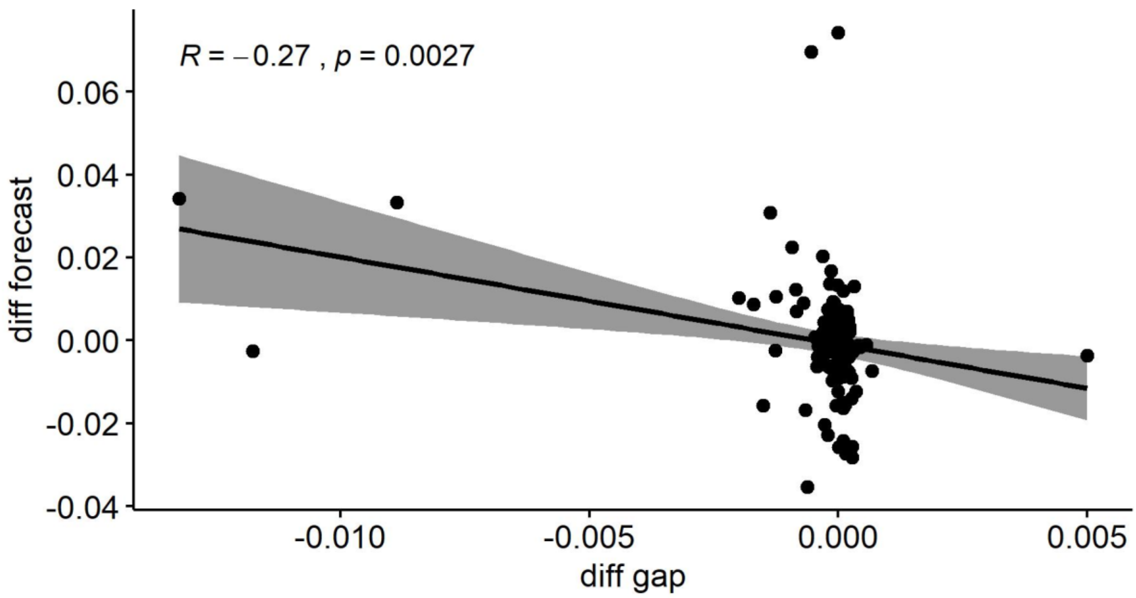


Figure 10. Correlation between differences in gaps and inflation forecasts between DFM and BVM specifications. Employment reference variable.

In both cases the correlation between forecast and gap differences is negative (though, in the case of employment the correlation is not significant). This fact contradicts with the usual way of interpretation of such types of models: the model with higher gap does not produce higher inflation forecasts. This can be so due to the fact that the difference in model specifications leads not only to the change in gap estimates, but also to the change in the posterior parameter distributions. This leads to the violation of positive dependence between these two measures.

7. Conclusion

I estimate the Russian real activity slack measure on the basis of unobservable components model and find that these measures do not help to improve inflation forecasts. Moreover, the magnitude of revisions of these slack measures is high, which signals of unreliability of real-time estimates of such measures. This result is in line with the evidence from the US data (Orphanides, Norden (2002)). Bivariate, trivariate models and models with oil price affecting trends do not help to improve the forecasting accuracy as well. This can be explained by several facts. Firstly, by the low number of real activity variables cycles in Russia within the period considered. Secondly, by the nature of recent crises in Russia, which differs from the 2008 crisis. So it is difficult to forecast in real-time 2014 crisis using the information of 2008 crisis. Thirdly, the Russian inflation can be characterized by several structural breaks, which could affect the parameters estimates and, as a result, the forecasting accuracy of the whole model.

In this paper I show that real activity slack measures are characterized by high magnitude of in-sample and out-of-sample revisions. I suppose that the revisions might be much higher due to the presence of data revisions. However, due to the absence of the long history of these revisions there is no possibility to check this hypothesis.

References

Basistha, A., & Nelson, C. R. (2007). New measures of the output gap based on the forward-looking new Keynesian Phillips curve. *Journal of Monetary Economics*, *54*(2), 498-511.

Basistha, A. (2007). Trend-cycle correlation, drift break and the estimation of trend and cycle in Canadian GDP. *Canadian Journal of Economics/Revue canadienne d'économique*, *40*(2), 584-606.

Benes, J., & N'Diaye, P. M. B. (2004). *A Multivariate Filter for Measuring Potential Output and the NAIRU Application to the Czech Republic* (No. 4-45). International Monetary Fund.

Carter, C. K., & Kohn, R. (1994). On Gibbs sampling for state space models. *Biometrika*, *81*(3), 541-553.

Chan, J. C., & Jeliazkov, I. (2009). Efficient simulation and integrated likelihood estimation in state space models. *International Journal of Mathematical Modelling and Numerical Optimisation*, *1*(1-2), 101-120.

Chan, J. C. (2013). Moving average stochastic volatility models with application to inflation forecast. *Journal of Econometrics*, *176*(2), 162-172.

Chan, J. C., & Grant, A. L. (2016). Fast computation of the deviance information criterion for latent variable models. *Computational Statistics & Data Analysis*, *100*, 847-859.

Chan, J. C. (2018). Specification tests for time-varying parameter models with stochastic volatility. *Econometric Reviews*, *37*(8), 807-823.

Clark, Peter K. (1987); "The Cyclical Component of US Economic Activity", *Quarterly Journal of Economics*, 102, 797-814.

Coibion, O., & Gorodnichenko, Y. (2015). Is the Phillips curve alive and well after all? Inflation expectations and the missing disinflation. *American Economic Journal: Macroeconomics*, *7*(1), 197-232.

Diebold, F. X. and Mariano, R. S. (1995). *Comparing Predictive Accuracy.* Journal of Business & Economic Statistics, 13(3):253.

Durbin, J., & Koopman, S. J. (2012). *Time series analysis by state space methods*. Oxford university press.

Giordani, P., & Kohn, R. (2010). Adaptive independent Metropolis–Hastings by fast estimation of mixtures of normals. *Journal of Computational and Graphical Statistics*, *19*(2), 243-259.

Grant, A. L., & Chan, J. C. (2017a). A Bayesian Model Comparison for Trend-Cycle Decompositions of Output. *Journal of Money, Credit and Banking*, *49*(2-3), 525-552.

Grant, A., & Chan, J. C. (2017b). Measuring the output gap using stochastic model specification search.

Grant, A. L., & Chan, J. C. (2017c). Reconciling output gaps: Unobserved components model and Hodrick–Prescott filter. *Journal of Economic Dynamics and Control*, *75*, 114-121.

Harvey, A. C. (1985); "Trends and Cycles in Macroeconomic Time Series", *Journal of Business and Economic Statistics*, 3, 216-27

Harvey, A. C., Trimbur, T. M., & Van Dijk, H. K. (2007). Trends and cycles in economic time series: A Bayesian approach. *Journal of Econometrics*, *140*(2), 618-649.

Jarociński, M., & Lenza, M. (2018). An inflation-predicting measure of the output gap in the euro area. *Journal of Money, Credit and Banking*, *50*(6), 1189-1224.

Kim, S., Shephard, N., & Chib, S. (1998). Stochastic volatility: likelihood inference and comparison with ARCH models. *The review of economic studies*, *65*(3), 361-393.

Kloudova, D. (2015). Estimating output gap and potential output for Russia and its usefulness by forecasting inflation. *International Journal of Economic Sciences*, *4*(1), 45-59.

Korobilis, D. (2013). Hierarchical shrinkage priors for dynamic regressions with many predictors. *International Journal of Forecasting*, *29*(1), 43-59.

Marcellino, M., & Musso, A. (2011). The reliability of real-time estimates of the euro area output gap. *Economic Modelling*, *28*(4), 1842-1856.

Morley, J. C., Nelson, C. R., & Zivot, E. (2003). Why are the Beveridge-Nelson and unobserved-components decompositions of GDP so different?. *Review of Economics and Statistics*, *85*(2), 235-243.

Li M. & Mendieta-Muñoz, I. (2019, forthcomming) Identifying interactions between permanent and transitory shocks via multivariate simultaneous unobserved components models with heteroskedasticity

Neal, R. M. (2011). MCMC using Hamiltonian dynamics. *Handbook of markov chain monte carlo*, *2*(11), 2.

Nelson, C., & Zivot, E. (2000, August). Why are Beveridge-Nelson and Unobserved-Component Decompositions of GDP so Different?. In *Econometric Society World Congress 2000 Contributed Papers* (No. 0692). Econometric Society.

Orphanides, A., & Norden, S. V. (2002). The unreliability of output-gap estimates in real time. *Review of economics and statistics*, *84*(4), 569-583.

Planas, C., & Rossi, A. (2004). Can inflation data improve the real-time reliability of output gap estimates?. *Journal of Applied Econometrics*, *19*(1), 121-133.

Planas, C., Rossi, A., & Fiorentini, G. (2008). Bayesian analysis of the output gap. *Journal of Business & Economic Statistics*, *26*(1), 18-32.

A.V. Polbin (2019) Estimation of the business cycle component of the Russian GDP with unobserved component model under high dependence on the terms of trade

Sala-i-Martin, X., Doppelhofer, G., & Miller, R. I. (2004). Determinants of long-term growth: A Bayesian averaging of classical estimates (BACE) approach. *American economic review*, 813-835.

Stock, J. H., & Watson, M. W. (2007). Why has US inflation become harder to forecast?. *Journal of Money, Credit and banking*, *39*, 3-33.

Zubarev, A. V., & Trunin, P. V. (2017). The analysis of the dynamics of the Russian economy using the output gap indicator. *Studies on Russian Economic Development*, *28*(2), 126-132.

A. Data, priors and initialization

A.1. Data sources

Monthly data

SERIES	AVAILABLE PERIOD	TRANSFOR-MATION	SOURCE
Industrial Production Index, y_t^0 (IPI)	2003M1:2018M9	Log, SA	CEIC
Inflation, π_t	2003M1:2018M9	SA, 100 times Log difference	Rosstat
Inflation expectations, π_t^e (инФОМ)	20011Q1:2104Q1; 2014M3:2018M9		инФОМ
Real exports (EXP), $	2003M1:2018M9	Log, SA	CEIC
Real imports (IMP), $	2003M1:2018M9	Log, SA	CEIC
Construction (CONSTR)	2003M1:2018M9	Log, SA	CEIC
Cargo (CARGO)	2003M1:2018M9	Log, SA	CEIC
Retail; (RETAIL) Retail (Food Goods); Retail (Nonfood Goods)	2003M1:2018M9	Log, SA	CEIC
Business Confidence (BUSCONF)	2003M1:2018M9	Log, SA	OECD Data
Total employment by professional status (EMP)	2003M1:2018M9	Log, SA	OECD Data
Unemployment rate (UNEMP)	2003M1:2018M9	-	Rosstat
Oil prices	2003M1:2018M9	Log difference,	Bank of Russia
Broad Effective Exchange Rate	2003M1:2018M9	Log difference,	Bank of Russia

Quarterly data

SERIES	AVAILABLE PERIOD	TRANSFORMATION	SOURCE
Real GDP, y_t^0 (GDP)	1999Q1:2018Q4	Log, SA	Rosstat
Inflation, π_t	1999Q1:2018Q4	SA, 100 times Log difference	Rosstat
Consensus Inflation forecast, π_t^e	2008Q1:2018Q4		Bloomberg
Real investment (INV)	1999Q1:2018Q4	Log, SA	Rosstat
Real imports (IMP)	1999Q1:2018Q4	Log, SA	Rosstat
Real exports (EXP)	1999Q1:2018Q4	Log, SA	Rosstat
Industrial Production (IPI)	1999Q1:2018Q4	Log, SA	Rosstat
Business Confidence (BUSCONF)	1999Q1:2018Q4	Log, SA	OECD Data Base
Total employment by professional status (EMP)	1999Q1:2018Q4	Log, SA	OECD Data Base
Unemployment rate (UNEMP)	1999Q1:2018Q4	-	Rosstat
Oil prices	1999Q1:2018Q4	Log difference, SA	Bank of Russia
Gas prices	1999Q1:2018Q4	Log difference, SA	Bank of Russia
Broad Effective Nominal Exchange rate	1999Q1:2018Q4	Log difference, SA	Bank of Russia

A.2. The necessity of stochastic volatility assumption

To test the assumption of the inclusion of stochastic volatility for the state-space model of inflation, I used Bayesian model comparison approach based on the log Bayes factors comparison using (Chan, (2018)) approximation method. The log Bayes Factor for models M_1 and M_2 comparison is the following fraction:

$$\log(BF_{12}) = \log\big(p(M_1|D,\psi_1)\big) - \log\,(p(M_2|D,\psi_2)),$$

where $p(M_i|D,\psi_i) = \int p(M_i|D,\theta_i,\psi_i)d\theta_i$ — marginal likelihood, θ_i — model M_i 's parameters, ψ_i — prior hyperparameters.

The higher the log Bayes factor is, the more probable is the model 1 compared to the model 2. I used the standard Stock–Watson UCSV model specification with stochastic volatility in both trend and cycle for the inflation. I use this model to be M_1 in BF_{12} notation. I compare this model to the model with constant trend variance, constant cycle variance and constant trend and cycle variance. The results are present in table 5.

Table 5. Bayes Factors for different specifications of the standard UCSV model for inflation

LOG BF COMPARED TO UCSV MODEL (NUMERICAL S.E.)	h_t^{π}-CONST	q_t^{π}- CONST	BOTH VARIANCES ARE CONSTANT
Quarterly data	9.0 (8.52)	0.0 (0.30)	171 (3.97)
Monthly data	3.0 (0.22)	1.5 (0.15)	221.3 (4.20)

It can be seen that for UCSV model is more probable that the UC model without time-varying variance in both trend and cycle. On the other hand, in Jeffrey's classification (Zellner, (1989)) UCSV model is strongly preferable to the model with constant h_t^{π} variance. On the other hand, UCSV model and the model with constant q_t^{π} are equivalent in the sense of Bayes factor.

In the BMA literature (see, for example, Sala-i-Martin, et al (2004)) instead of model selection it is more theoretically relevant to use these Bayes factors to compute posterior model probabilities and then calculate weighted average of the parameter of interest with weights proportional to posterior model probabilities. However, in this paper due to large amount of model specifications I focus on the standard UCSV specification with both trend and cycle time-varying variance.

Estimated half-log time varying variances are depicted in figure 8.

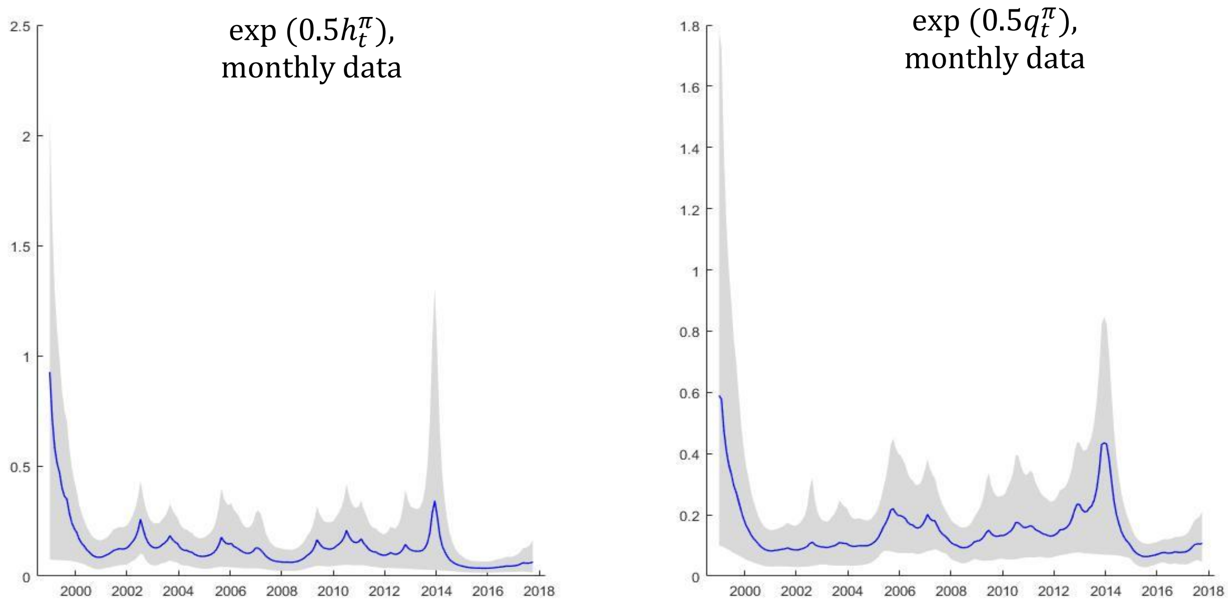


Figure 7. UCSV time-varying log inflation trend (right) and gap (left) variance. Monthly data.

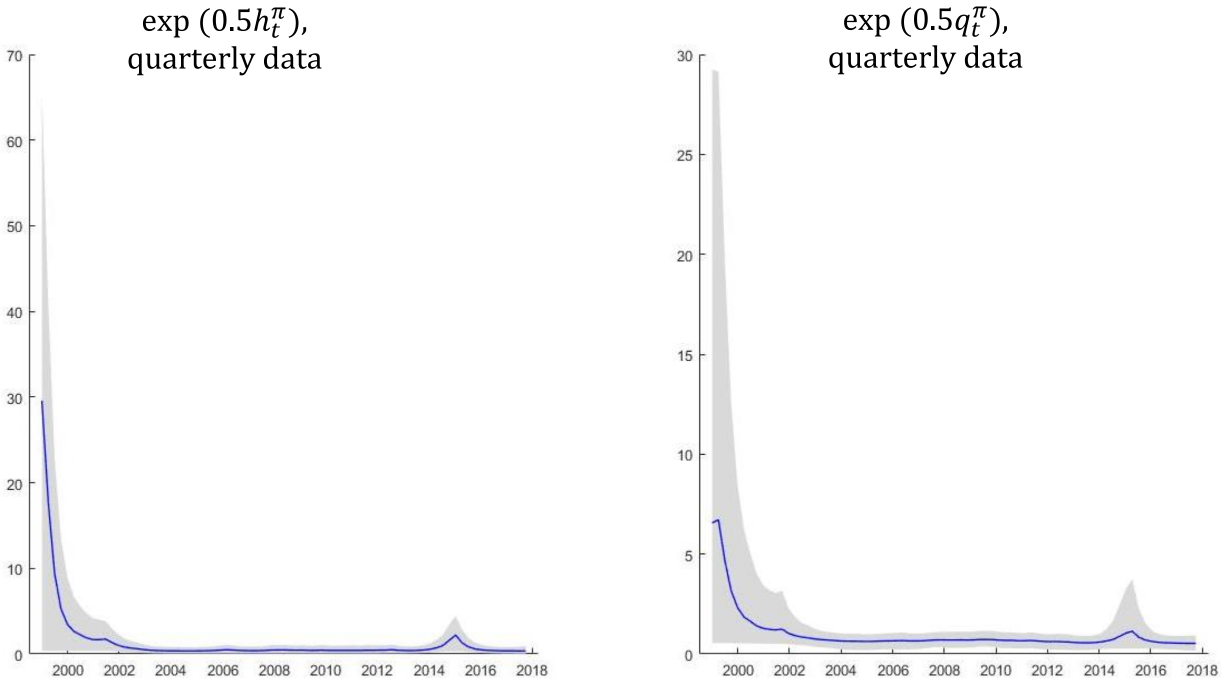


Figure 8. UCSV time-varying log inflation trend (right) and gap (left) variance. Quarterly data.

A.3. Priors and initialization

Gap AR-process parameters. Following Planas, Rossi, (2008), I defined parameters for the AR-process using parameters $(A,\ \omega)$ (see equations (7a-7b)). Parameter ω stands for periodicity of cycles and A reflects gap persistence. I restrict (A, ω) to vary between (L_A, L_ω) and (H_A, H_ω). Both hyperparameters have beta prior distribution:

$$\frac{A-L_A}{H_A-L_A} \sim Beta\big(\widetilde{\alpha_A}, \widetilde{\beta_A}\big)\ ; \qquad \frac{\omega-L_\omega}{H_\omega-L_\omega} \sim Beta\big(\widetilde{\alpha_\omega}, \widetilde{\beta_\omega}\big)$$

For both data sets I choose the prior means (m_A, m_ω) and variances $(\sigma_A^2, \sigma_\omega^2)$ of both parameters and calculate prior hyperparameters using the following formulae:

$$\widetilde{\alpha_s} = Sh_s \times \widetilde{\beta_s};\ Sh_s = (m_s - L_s)(H_s - m_s)^{-1};\ \widetilde{\beta_s} = (1 - Sh_s \times \sigma_s^2)(\sigma_s^2(1 + Sh_s) \times Sh_s)^{-1}$$

For both monthly and quarterly datasets, I set $L_A = 0; H_A = 1$ with mean $m_A = 0.5$ and variance $\sigma_A^2 = 0.1$. Following Jarocinsky, Lenza, (2015), for quarterly data I choose bounds for periodicity parameter to be $(L_\omega, H_\omega)^{quarter} = (10, 50)$ with mean value of $m_\omega^{quarter} = 32$. It means that on average each cycle has periodicity of 32 quarters. Respectively, for monthly data I set $(L_\omega, H_\omega)^{month} = (30, 150)$ and $m_\omega^{month} = 96$.

Loadings matrices and inflation parameters. Due to the short time series, in contrast to Jarocinsky, Lenza, (2015), I used more 3 types of prior covariance matrices definitions. Firstly, in the standard case without regularization I set standard Minnesota–type prior. The variance of loading parameter for the l-th lag $(l = 0,1, \ldots, R)$ of the gap for the j-th variable is set to: $\sigma_j^2 \sigma_0^{-2} W_{tight}^B \big(W_{lag}^B\big)^{-l}$. Here σ_0^2 —reference variable's variance, σ_j^2 — variance of the j-th real variable, W_{tight}^B — tightness prior hyperparameter, W_{lag}^B— lag tightness parameter.

For inflation equation parameters $\theta^{infl} = [\lambda, \beta]$ the prior variance for j-th parameter is $\sigma_j^2 \sigma_0^{-2} W_{exog}^{infl} W_{tight}^{infl} \big(W_{lag}^{infl}\big)^{-l}$. Analogously, W_{tight}^{infl} — tightness prior hyperparameter, W_{lag}^{infl}— lag tightness parameter, W_{exog}^{infl} — exogenous variables diffusion parameter (it's equal to 1 for gap and gap lags).

Prior mean values for all parameters are equal to zeros.

Loadings matrix is initialized as the VAR coefficients matrix for the first differences of all real variables included in the DFM specification. Inflation equation coefficients are initialized using the standard ARIMAX specification with gaps equal to first differences of the reference variable.

Trend and gap variances. Following Jarocinsky, Lenza, (2015), for each real variable $n = 0 \dots N$ I set trend and cycle variances to be equal to $0.25 \times var(\Delta y_t^n)$, where Δy_t^n — first differences. The prior trend and gap variance is equal to these values as well.

The prior for the first trend values are equal to the first values of the real variable itself. The prior variance for the first trend μ_0^n /cycle $g_0^n = (y_0^n - \mu_0^n)$ values is equal to 10 times the corresponding overall trend/cycle variance. Initial trend and gap values for $t = 1 \dots T$ are set to be equal to Hodrick-Prescott decomposition with smoothing parameter of 14400 for monthly data and 1600 for quarterly data.

B. Estimation Details

In this appendix I focus on the DFM UCSVMAX specification derivations. UCSVMA, UCSVMAX, BVM UCSVMAX and DFM UCSVMA are just restricted cases of this model.

All derivations are based on the (Chan, Jeliazkov, (2009)) paper. To proceed, some additional notations should be made. Firstly, define all lag polynomials in the form of Toeplitz matrices. To do this, note, that for some variable $x = [x_0, x_1, \dots, x_T]'$ the AR-process of the form $x_t = \sum_{j=1}^{p} \rho_j^x x_{t-j} + \varepsilon_t$ can be represented as $H_\rho^x x = \varepsilon$, where H_ρ^x is a Toeplitz matrix with zeros below the main diagonal, ones on the main diagonal, $-\rho_j^x$ on the j-th upper diagonal and zeros everywhere else. I define the operation of construction of such matrices as $H_v = TM(v)$, where H_v — Toeplitz matrix with element v_j on its j-th upper diagonal (including the main diagonal):

$$H_v = TM(v) = \begin{bmatrix} v_1 & v_2 & v_3 & \cdots & v_k \\ 0 & v_1 & v_2 & \ddots & \vdots \\ 0 & 0 & v_1 & v_2 & v_3 \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ 0 & 0 & 0 & 0 & v_1 \end{bmatrix}$$

Define:

$H = TM([1, -1, \mathbf{0}_{T-3}])$	$H_\varphi = TM\big([1, -\varphi', \mathbf{0}_{T-q-1}]\big)$	$H_\lambda = TM([\lambda', \mathbf{0}_{T-r-1}])$
$H_\rho = TM\big([1, -\rho_\pi', \mathbf{0}_{T-p-1}]\big)$	$H_g = TM\big([1, -\rho_g', \mathbf{0}_{T-q-1}]\big)$	$H_B^n = TM([(B^n)', \mathbf{0}_{T-r-1}])$

All covariance matrices are diagonal. However, several disturbances are characterized by time-varying variances. I distinguish covariance matrices for observation equation and those for the state equations:

$S_j = diag\big(\exp(h_j)\big),\ j = 0..N$	$D_j = diag\big(\sigma_j^2 \times i_T\big) + I_T(1,1)W_{\mu j},\ j = 1..N$	$D_{\pi^e} = diag\big(\sigma_{\pi^e}^2 \times i_T\big) + I_T(1,1)W_{\pi e}$
$S_\pi = diag(\exp(2 \times h_\pi))$	$D_\pi = diag(\exp(q_\pi)) + I_T(1,1)W_\pi$	$S_{ek} = diag(\sigma_{ek}^2 \times i_T)$

Here $I_T(1,1)$ is $T \times T$ zero matrix with $(1,1)$ element equal to 1, $W_{\mu j}$ — j-th real variable trend initial value prior variance, W_π — inflation trend initial value prior variance, $W_{\pi e}$ — inflation expectations trend initial value prior variance.

All these matrices are banded matrices. Following (Chan, Jelizakov, (2017)), I use MATLAB sparse matrices routines to work with these matrices.

Hence, the full posterior of the model DFM UCSVMAX can be written in the following way (here θ — all model parameters vector, Y — endogenous variables, X — exogenous variables):

$$p(\theta, h, Y|X, \psi) \propto \exp\big(-0.5 \times (P + E)\big),$$

The log-likelihood kernel is:

$$\begin{aligned} E = {} & (y_0 - \mu_0)' H_g' S_0^{-1} H_g (y_0 - \mu_0) \\ & + \sum_{j=1}^{n} \big(y_j - \mu_j - H_{Bj}(y_0 - \mu_0)\big)' S_j^{-1} \big(y_j - \mu_j - H_{Bj}(y_0 - \mu_0)\big) \\ & + (\hat{\pi} - \tilde{\pi} - H_\kappa^0 \tilde{\pi}^e - H_\lambda(y_0 - \mu_0))' H_\varphi' S_\pi^{-1} H_\varphi (\hat{\pi} - \tilde{\pi} - H_\kappa^0 \tilde{\pi}^e - H_\lambda(y_0 - \mu_0)) \\ & + \sum_{k=1}^{K} \big(\widehat{\pi_k^e} - \kappa_k \tilde{\pi}^e\big)' S_{ek}^{-1} \big(\widehat{\pi_k^e} - \kappa_k \tilde{\pi}^e\big) \\ & + (H_{\alpha 1}\mu_0 - H_{\alpha 2}\tilde{\mu}_0)' D_0^{-1} (H_{\alpha 1}\mu_0 - H_{\alpha 2}\tilde{\mu}_0) + \sum_{j=1}^{n} \big(H_{\alpha 1}\mu_j - H_{\alpha 2}\tilde{\mu}_j\big)' D_j^{-1} \big(H_{\alpha 1}\mu_j - H_{\alpha 2}\tilde{\mu}_j\big) \\ & + \tilde{\pi}' H_\rho' D_\pi^{-1} H_\rho \tilde{\pi} + +\tilde{\pi}^{e\prime} H' D_{\pi^e}^{-1} H \tilde{\pi}^e, \end{aligned}$$

where $\hat{\pi} = \pi - X\beta$. The joint density for each of the models in subsection 3.1 can be represented by the formula above. In the case of LLT trend specification $H_{\alpha 1} = H^2$, $H_{\alpha 2} = H$, $\tilde{\mu}_0 = e_T^1 \mu_{00}$; in the case of RWD trend specification $H_{\alpha 1} = H$, $H_{\alpha 2} = I_T$, $\tilde{\mu}_0 = e_T^1 \mu_{00} + i_T \alpha$; and finally, in the case of RW trend: $H_{\alpha 1} = H$, $H_{\alpha 2} = I_T$, $\tilde{\mu}_0 = e_T^1 \mu_{00}$.

And parameters prior kernels are:

$$P = \beta' W_\beta^{-1} \beta + \sum_{n=1}^{N} B_n' W_{Bn}^{-1} B_n - 2\ln\left(p_{Beta}\left(\frac{A - L_A}{H_A - L_A}, \widetilde{\alpha_A}, \widetilde{\beta_A}\right)\right) - 2\ln\left(p_{Beta}\left(\frac{\omega - L_\omega}{H_\omega - L_\omega}, \widetilde{\alpha_\omega}, \widetilde{\beta_\omega}\right)\right)$$
$$+ \sum_{s=0}^{K} \kappa_s' W_\kappa^{-1} \kappa_s + \lambda' W_\lambda^{-1} \lambda + \varphi' W_\varphi^{-1} \varphi + \rho_\pi' W_{\rho\pi}^{-1} \rho_\pi + \rho_g' W_{\rho g}^{-1} \rho_g$$

where W_s — prior covariance matrix. Prior hyperparameters elucidation is discussed in the previous appendix section.

The Gibbs Sampler consists of the following steps:

1. Sample $\tilde{\pi}, \tilde{\pi}_k^e | \{\mu_k\}_{k=0}^N, \theta_{par}, \rho_g, h_{1:T}, \sigma^2$
2. Sample $\mu_0 |\, \tilde{\pi}, \tilde{\pi}_k^e, \{\mu_k\}_{k=0}^N, \theta_{par}, \rho_g, h_{1:T}, \sigma^2$
3. Sample $\mu_n |\, \tilde{\pi}, \tilde{\pi}_k^e, \{\mu_k\}_{k=0;k\neq n}^N, \theta_{par}, \rho_g, h_{1:T}, \sigma^2$
4. Sample $\theta_{par} = (\varphi, \beta, \lambda, B, \rho_\pi, \kappa) |\, \tilde{\pi}, \tilde{\pi}_k^e, \{\mu_k\}_{k=0}^N, \rho_g, h_{1:T}, \sigma^2$
5. Sample $\rho_g |\, \tilde{\pi}, \tilde{\pi}_k^e, \{\mu_k\}_{k=0}^N, \theta_{par}, h_{1:T}, \sigma^2$
6. Sample $h_{1:T} = (h_{1:T}^0, \dots h_{1:T}^N, h_{1:T}^\pi, q_{1:T}^\pi) |\, \tilde{\pi}, \tilde{\pi}_k^e, \{\mu_k\}_{k=0}^N, \theta_{par}, \rho_g, \sigma^2$
7. Sample $\sigma^2 = (\sigma_0^2, \dots \sigma_N^2, \sigma_{ek}^2, \sigma_{\pi e}^2) |\, \tilde{\pi}, \tilde{\pi}_k^e, \{\mu_k\}_{k=0}^N, \theta_{par}, \rho_g, h_{1:T}$
8. Sample
   a. ψ in the case of HIER specification
   b. sw_j in the case of BMA specification.

B.1. Real variable trends

The DFM–type reference variable trend can be sampled as normal vector with precision matrix V_{μ_0} and mean m_{μ_0}:

$$V_{\mu_0} = H_g' S_0^{-1} H_g + H_{\alpha 1}' D_0^{-1} H_{\alpha 1} + \sum_{j=1}^{N} H_{Bj}' S_j^{-1} H_{Bj} + H_\lambda' S_\pi^{-1} H_\lambda$$
$$m_{\mu_0} = V_{\mu_0}^{-1} \left(H_g' S_0^{-1} H_g y_0 + H_{\alpha 1}' D_0^{-1} H_{\alpha 2} \tilde{\mu}_0 + \sum_{j=1}^{N} H_{Bj}' S_j^{-1} \left(H_{Bj} y_0 - (y_j - \mu_j) \right) + H_\lambda' H_\varphi' S_\pi^{-1} H_\varphi (H_\lambda y_0 - (\hat{\pi} - \tilde{\pi})) \right)$$

Also set $g = y_0 - \mu_0$. Then each individual real variable trend apart from reference variable, can be sampled as normal random variable with precision matrix V_{μ_n} and mean m_{μ_n}, $n = 1..N$:

$$V_{\mu_n} = S_n^{-1} + H_{\alpha 1}' D_n^{-1} H_{\alpha 1}$$
$$m_{\mu_n} = V_{\mu_n}^{-1} (S_n^{-1} (y_n - H_{Bn} g) + H_{\alpha 1}' D_n^{-1} H_{\alpha 2} \tilde{\mu}_n)$$

Real variables trends for other considered models can be sampled in the following way:

Common trend case

In the case when $\mu_0 = \mu_n$ for $n = 1..N$, the common trend is sampled as normal random variable with following parameters:

$$V_\mu = H_g' S_0^{-1} H_g + H_{\alpha 1}' D_0^{-1} H_{\alpha 1} + \sum_{j=1}^{N} (I_T + H_{Bj}') S_j^{-1} (I_T + H_{Bj}) + H_\lambda' S_\pi^{-1} H_\lambda$$

$$m_\mu = V_\mu^{-1} \left(H_g' S_0^{-1} H_g y_0 + H_{\alpha 1}' D_0^{-1} H_{\alpha 2} \tilde{\mu}_0 + \sum_{j=1}^{N} (I_T + H_{Bj}') S_j^{-1} (H_{Bj} y_0 - y_j) + H_\lambda' H_\varphi' S_\pi^{-1} H_\varphi (H_\lambda y_0 - (\hat{\pi} - \tilde{\pi})) \right)$$

Trivariate model case

In the case of trivariate model with real output (real GDP or IPI) and unemployment rate, output trend is normal random variable with parameters:

$$V_{\mu_0} = H_g' S_0^{-1} H_g + H_{\alpha 1}' D_0^{-1} H_{\alpha 1}$$

$$m_{\mu_0} = V_{\mu_0}^{-1} \left(H_g' S_0^{-1} H_g \big(y_0 - \delta_u (u - u^*)\big) + H_{\alpha 1}' D_0^{-1} H_{\alpha 2} \tilde{\mu}_0 + H_\lambda' H_\varphi' S_\pi^{-1} H_\varphi (H_\lambda y_0 - (\hat{\pi} - \tilde{\pi})) \right)$$

NAIRU u^* is normal random variable as well with parameters:

$$V_{u*} = \delta_u^2 H_g' S_0^{-1} H_g + H_{u*}' S_0^{-1} H_{u*} + H_{\alpha 1}' D_{u*}^{-1} H_{\alpha 1}$$

$$m_{\mu_0} = V_{\mu_0}^{-1} \left(\delta_u H_g' S_0^{-1} H_g \big(\delta_u u - (y_0 - \mu_0)\big) + H_{\alpha 1}' D_{u*}^{-1} H_{\alpha 2} \widetilde{u^*}_0 + H_\lambda' H_\varphi' S_\pi^{-1} H_\varphi (H_\lambda y_0 - (\hat{\pi} - \tilde{\pi})) \right)$$

B.2. Inflation trend

Inflation trend itself can be sampled as normal random variable with parameters:

$$V_{\tilde{\pi}} = H_\varphi' S_\pi^{-1} H_\varphi + H_\rho' D_\pi H_\rho$$

$$m_{\tilde{\pi}} = V_{\tilde{\pi}}^{-1} \left(H_\varphi' S_\pi^{-1} H_\varphi (\hat{\pi} - H_\kappa^0 \tilde{\pi}^e - H_\lambda g) \right)$$

Inflation expectations trend is characterized by the following parameters:

$$V_{\tilde{\pi}e} = (H_\kappa^0)' H_\varphi' S_\pi^{-1} H_\varphi H_\kappa^0 + H' D_{\tilde{\pi}e}^{-1} H + \sum_{k=1}^{K} \kappa_k^2 S_{ek}^{-1}$$

$$m_{\tilde{\pi}e} = V_{\tilde{\pi}e}^{-1} \left((H_\kappa^0)' H_\varphi' S_\pi^{-1} H_\varphi (\hat{\pi} - \tilde{\pi} - H_\lambda g) + \sum_{k=1}^{K} \kappa_k S_{ek}^{-1} \hat{\pi}^e \right)$$

B.3. Parameters

All posterior conditional parameter values $(\varphi, \beta, \lambda, \kappa, \rho_\pi)$ can be sampled as a simple Bayesian posterior parameter values with normal prior (with zero means) and likelihood functions:

$$V_{\theta_s} = (Z_s' V_s^{-1} Z_s + W_s^{-1});\ m_{\theta_s} = V_{\theta_s}^{-1} (Z_s' V_s^{-1} x_s), \qquad \text{(E3)}$$

where $\theta_s \in \{\varphi, \beta, \lambda, \kappa\}$, Z_φ consists of q lags of $\breve{\pi} = \big(\hat{\pi} - \tilde{\pi} - H_\kappa^0 \tilde{\pi}^e - H_\lambda (y_0 - \mu_0)\big)$, x_φ — of $\breve{\pi}$ itself; $Z_\beta = X$ and $x_\beta = \big(\pi - \tilde{\pi} - H_\kappa^0 \tilde{\pi}^e - H_\lambda (y_0 - \mu_0)\big)$; Z_λ consists of r lags of g and its contemporaneous values, and $x_\lambda = \hat{\pi} - \tilde{\pi} - H_\kappa^0 \tilde{\pi}^e$. $V_\varphi = S_\pi$, $V_\lambda^{-1} = V_\beta^{-1} = H_\varphi' S_\pi^{-1} H_\varphi$. $Z_{\rho\pi}$ consists of the p lags of $\tilde{\pi}$, $x_{\rho\pi} = \tilde{\pi}$, $V_{\rho\pi}^{-1} = D_\pi^{-1}$.

Factor loadings B are sampled in the same way as VAR coefficients in the case of Minnesota-type prior. However, since I assumed zero correlation between real variables disturbances, I can sample each column of matrix B separately using equation (E3), where Z_{Bn} consists of R lags of g, $x_{Bn} = y_n - \mu_n$ and $V_{Bn}^{-1} = S_n^{-1}$.

To sample ρ_g, I need to sample (A, ω). To do this I add Adaptive Metropolis–Hastings step (Giordani, Kohn, (2010)) to the Gibbs-sampler procedure with random-walk normal proposal distribution. The main issue is the fact that parameters (A, ω) are bounded, while the proposal distribution is not. Following (Neal, (2011)), for each sample before acceptance–rejection step I restate proposal parameters pr_i using the following formulae:

$$pr_i = \begin{cases} pr_i^u - (pr_i - pr_i^u), if \quad pr_i > pr_i^u \\ pr_i^l + (pr_i^l - pr_i), if \quad pr_i < pr_i^l \end{cases},$$

where pr_i^u and pr_i^l — upper and lower bounds of pr_i respectively. This procedure repeats until $\forall i$: $pr_i \in [pr_i^l, pr_i^u]$.

B.4. Variances

For all of the proposed models there are 2 types of variances: time–varying and time-constant. Time–varying log variances are sampled using (Chib, et al, (1998)). Time-constant parameters are sampled from inverse-gamma distribution.

B.5. Hierarchical LASSO parameters

To sample parameters prior variance an approach of (Koop, Korobilis, (2011)) was implemented. To gain LASSO–type hierarchical model, exponential prior family for parameter prior variance should be considered:

$$W_i | \widetilde{W} \sim Exp\left(0.5 \times \widetilde{W}\right)$$
$$\widetilde{W} \sim Gamma(aw_1, aw_2)$$

where $\widetilde{W}$ — standard LASSO penalty coefficient analogue, W_i — prior variance of the ith parameter. Hence, the posteriors for these parameters are:

$$\frac{1}{W_i} | \tilde{\pi}, \tilde{\pi}_k^e, \{\mu_k\}_{k=0}^N, \theta_{par}, \rho_g, h_{1:T}, \sigma^2, \mathrm{Y}, \mathrm{X} \sim invGauss\left(\sqrt{\frac{\widetilde{W}}{\theta_i^2}}, \widetilde{W}\right)$$

$$\widetilde{W} | \tilde{\pi}, \tilde{\pi}_k^e, \{\mu_k\}_{k=0}^N, \theta_{par}, \rho_g, h_{1:T}, \sigma^2, \mathrm{Y}, \mathrm{X} \sim Gamma\left(k + aw_1, aw_2 + \sum_{j=1}^{K} W_j\right)$$

B.6. SSVS probabilities

Another popular way to regularize parameters is to use the stochastic search variable selection (SSVS) using (George. et al, (2008)) algorithm. For each parameter θ_i this algorithm distinguishes between 2 possible prior variance values: peak $\widetilde{W}_{i.H} \to \infty$ with probability p_i^W and plateau $\widetilde{W}_{i.L} \to 0$ with probability $1 - p_i^W$. The probability is sampled as additional step of Gibbs Sampler:

$$p_i^W \mid \tilde{\pi}, \tilde{\pi}_k^e, \{\mu_k\}_{k=0}^N, \theta_{par}, \rho_g, h_{1:T}, \sigma^2, \mathrm{Y}, \mathrm{X} = (1 + u_i)^{-1}$$
$$u_i = \exp\left(-0.5 \times \theta_i \{ \widetilde{W}_{i.L}^{-1} - \widetilde{W}_{i.H}^{-1} \}\right)$$

C. Marginal Likelihood calculation

For each of the state-space models in-sample marginal likelihood was calculated using the approach, presented in the (Chib, 1995) paper. The method is based on the following formula:

$$\ln\big(p(Y|X,\psi)\big)=\ln\big(p(Y|\theta^*,X)\big)+\ln\big(p(\theta^*|\psi)\big)-\ln\big(p(\theta^*|Y,X,\psi)\big)$$

where θ^* — MAP or MLE parameter estimate (see Chib, 1995).
The main issue to be resolved is the calculation of the $\ln\big(p(\theta^*|Y,X,\psi)\big)$ term. It was done in several steps.

C.1. Step1. Integrating with respect to trends

Following the approach of (Grant, Chan, 2017), I integrated the joint density with respect to all stochastic trends using the fact that the smoothed trends posterior conditional on other parameters is normal:

$$p(\theta,h,Y|X,\psi)=\int p(\theta,hY,s|X,\psi)\,d\boldsymbol{s}=p(\theta,h|\psi)\int \exp(-0.5\times(C_1+E))\,d\boldsymbol{s}$$

$$C_0=\big((n+1)+K\big)T\ln(2\pi)+\ln\left(|S_0||S_\pi||D_0||D_\pi|\prod_{j=1}^{n}|S_j|\,|D_j|\prod_{k=1}^{K}|S_{ek}|\right)$$

$$C_1=(n+1)T\ln(2\pi)+C_0$$

$$\begin{aligned}E=\;&(y_0-\mu_0)'H_g'S_o^{-1}H_g(y_0-\mu_0)\\&+\sum_{j=1}^{n}\big(y_j-\mu_j-H_{Bj}(y_0-\mu_0)\big)'S_j^{-1}\big(y_j-\mu_j-H_{Bj}(y_0-\mu_0)\big)\\&+(\hat{\pi}-\kappa_0\tilde{\pi}-H_\lambda(y_0-\mu_0))'H_\varphi'S_\pi^{-1}H_\varphi(\hat{\pi}-\kappa_0\tilde{\pi}-H_\lambda(y_0-\mu_0))\\&+\sum_{k=1}^{K}\big(\widehat{\pi_k^e}-\kappa_k\tilde{\pi}\big)'S_{ek}^{-1}\big(\widehat{\pi_k^e}-\kappa_k\tilde{\pi}\big)\\&+(H_{\alpha1}\mu_0-H_{\alpha2}\tilde{\mu}_0)'D_0^{-1}(H_{\alpha1}\mu_0-H_{\alpha2}\tilde{\mu}_0)+\sum_{j=1}^{n}\big(H_{\alpha1}\mu_j-H_{\alpha2}\tilde{\mu}_j\big)'D_j^{-1}\big(H_{\alpha1}\mu_j-H_{\alpha2}\tilde{\mu}_j\big)\\&+\tilde{\pi}'H_\rho'D_\pi^{-1}H_\rho\tilde{\pi}\end{aligned}$$

where $\hat{\pi}=infl-X\beta$. The joint density for each of the models in subsection 3.1 can be represented by the formula above. In the case of LLT trend specification $H_{\alpha1}=H^2$, $H_{\alpha2}=H$, $\tilde{\mu}_0=e_T^1\mu_{00}$; in the case of RWD trend specification $H_{\alpha1}=H$, $H_{\alpha2}=I_T$, $\tilde{\mu}_0=e_T^1\mu_{00}+i_T\alpha$; and finally, in the case of RW trend: RW $H_{\alpha1}=H$, $H_{\alpha2}=I_T$, $\tilde{\mu}_0=e_T^1\mu_{00}$.
Define:

$$V_0^{-1}=H_g'S_o^{-1}H_g+\sum_{j=1}^{n}H_{Bj}'S_j^{-1}H_{Bj}+H_\lambda'H_\varphi'S_\pi^{-1}H_\varphi H_\lambda+\sum_{k=1}^{K}S_{ek}^{-1}$$

$$K_{\mu_0\mu_0}=H_{\alpha1}'D_0^{-1}H_{\alpha1}+V_0^{-1}$$

$$K_{\mu_j\mu_j}=H_{\alpha1}'D_0^{-1}H_{\alpha1}+H_j'S_j^{-1}H_j$$

$$K_{\tilde{\pi}\tilde{\pi}}=H_\rho'D_\pi^{-1}H_\rho+\kappa_0^2H_\varphi'S_\pi^{-1}H_\varphi+\sum_{k=1}^{K}\kappa_k^2S_{ek}^{-1}$$

$$K_{\mu_0\mu_j}=H_{Bj}'S_j^{-1}$$

$$K_{\mu_0\tilde{\pi}}=\kappa_0H_\lambda'H_\varphi'S_\pi^{-1}H_\varphi+\sum_{k=1}^{K}\kappa_kH_{\lambda k}'S_{ek}^{-1}$$

$$m_{\mu_0} = H'_{\alpha 1} D_0^{-1} H_{\alpha 2} \tilde{\mu}_0 + V_0^{-1} y_0 - \sum_{j=1}^{n} H'_{Bj} S_j^{-1} y_j - \sum_{k=1}^{K} H'_{\lambda k} S_{ek}^{-1} \; \hat{\pi}_k^e - H'_\lambda H'_\varphi S_\pi^{-1} H_\varphi \hat{\pi}$$

$$m_{\mu_j} = H'_{\alpha 1} D_0^{-1} H_{\alpha 2} \tilde{\mu}_j + S_j^{-1} H_{Bj} (y_j - y_0)$$

$$m_{\tilde{\pi}} = \kappa_0 H'_\varphi S_\pi^{-1} H_\varphi (\hat{\pi} - H_\lambda y_0) + \sum_{k=1}^{K} \kappa_k S_\pi^{-1} (\hat{\pi}_k^e - H_{\lambda k} y_0)$$

$$R = y_0' \left(\sum_{j=1}^{n} H'_{Bj} H'_j S_j^{-1} H_j H_{Bj} y_j + H'_\varphi S_\pi^{-1} H_\varphi \hat{\pi} + \sum_{k=1}^{K} S_{ek}^{-1} \; \widehat{\pi^e} \right) + y_0' V_0^{-1} y_0 + \sum_{j=1}^{n} y_j' S_j^{-1} y_j$$

$$+ \hat{\pi}' H'_\varphi S_\pi^{-1} H_\varphi \hat{\pi} + \sum_{k=1}^{K} \hat{\pi}'_k \; S_{ek}^{-1} \hat{\pi}_k$$

$$s = \begin{bmatrix} \mu_0 \\ \mu_{1:n} \\ \tilde{\pi} \end{bmatrix}; \; K = \begin{bmatrix} K_{\mu_0 \mu_0} & -K'_{\mu_0 \mu_{1:n}} & -K'_{\mu_0 \tilde{\pi}} \\ -K_{\mu_0 \mu_{1:n}} & K_{\mu_{1:n} \mu_{1:n}} & 0 \\ -K_{\mu_0 \tilde{\pi}} & 0 & K_{\tilde{\pi}\tilde{\pi}} \end{bmatrix}; m_s = \begin{bmatrix} m_{\mu_0} \\ m_{\mu_{1:n}} \\ m_{\tilde{\pi}} \end{bmatrix};$$

$$\mu_{1:n} = [\mu_1, \quad \dots \quad \mu_n]'; \; m_{\mu_{1:n}} = [m_{\mu_1}, \quad \dots \quad m_{\mu_n}]'; K_{\mu_{1:n} \mu_{1:n}} = \begin{bmatrix} K_{\mu_1 \mu_1} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & K_{\mu_n \mu_n} \end{bmatrix}$$

Hence:

$$p(\theta, h, Y | X, \psi) = p(\theta, h | \psi) \int \exp\big(-0.5 \times (C_1 + R + s'Ks - 2s'm_s)\big) \;\; d\boldsymbol{s}$$

$$= p(\theta, h | \psi) \int \exp\big(-0.5 \times (C_1 + R + s'Ks - 2s'm_s)\big) \;\; d\boldsymbol{s}$$

$$= p(\theta, h | \psi) \exp\big(-0.5 (C_1 + R - m_s' K^{-1} m_s)\big) \int \exp\big(-0.5 \, (s - K^{-1} m_s)' K (s - K^{-1} m_s)\big) \; d\boldsymbol{s}$$

$$= p(\theta | \psi) \exp(-0.5 \times (C_1 + R - m_s' K^{-1} m_s - (n+1) T \ln(2\pi) + |K|) \,)$$

Finally:

$$p(\theta, h, Y | X, \psi) = p(\theta | \psi) \exp(-0.5 \times (C_0 + R - m_s' K^{-1} m_s + |K|) \,)$$

C.2. Step 2. Rao-Blackwellization with respect to other hidden states

At the next step of the procedure the stochastic volatility and time-varying parameters should be integrated out. (Chib, (1995)) uses the following approximation using Rao-Blackwellization procedure:

$$p(\theta^s, Y | X, \psi) = \int p(\theta^s, h, Y | X, \psi) \, dh \approx \sum_{s=1}^{N_{smpl}} p(\theta^s, h^s, Y | X, \psi)$$

where θ^s and h^s are the Gibbs Sampler samples of parameters and hidden states respectively.

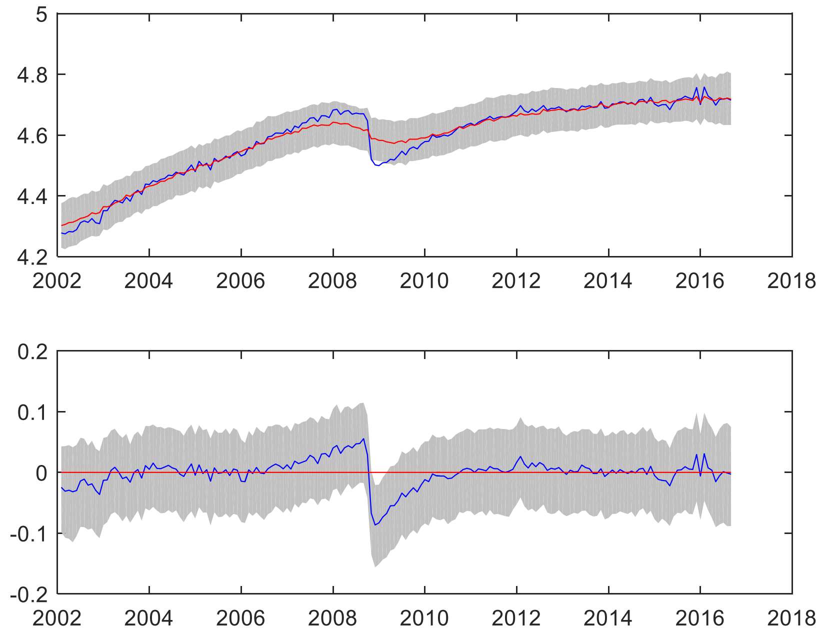
C.3. Step 3. Integration with respect to parameters

Recall that except cases of hierarchical LASSO shrinkage and BMA all parameter blocks in the model are independent conditional on stochastic volatility and other unobservable components. It means that the sampling parameters block-wise is equivalent to sampling all parameter vector at once.

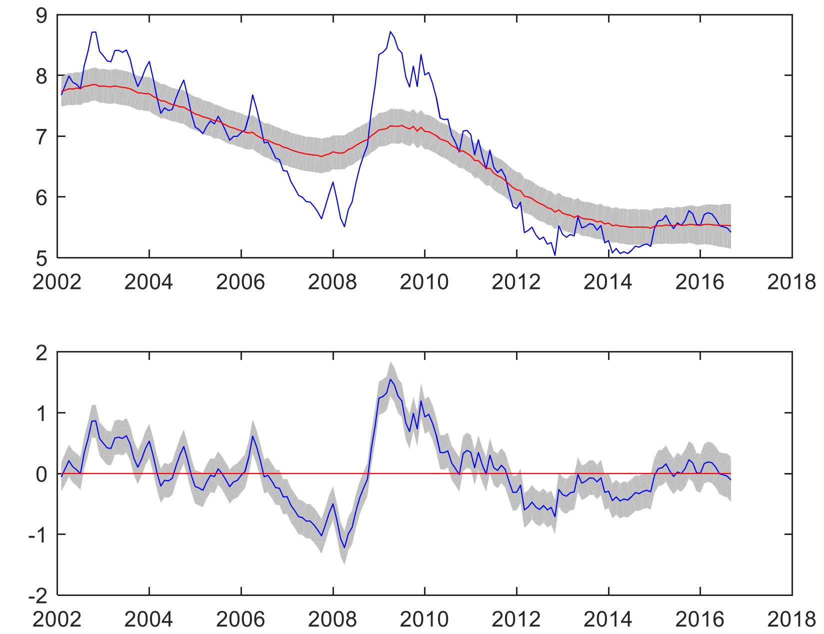
It means that (Chib, (1995)) in this case it is possible to be made with one-step sampling. In the case of BMA and HIER specifications lag lengths are restricted to be equal to 4 for quarterly data. Lags are regularized as well.

D. Smoothed trend and cycle estimates for DFM model specification

REAL INDUSTRIAL PRODUCTION INDEX(IPI)



UNEMPLOYMENT RATE (UNEMP)



E. Diebold–Mariano test P-values. Monthly Data.

	1m	3m	6m	9m	12m	YoY

UCSVMAX DFM IPI IRW	0.243084	0.623467	0.485165	0.454396	0.800326	0.551413
UCSVMAX DFM EXP IRW	0.313847	0.936402	0.715871	0.518548	0.799569	0.732703
UCSVMAX DFM IMP IRW	0.369225	0.691148	0.932712	0.701229	0.839599	0.818643
UCSVMAX DFM EMP RWD	0.330527	0.830042	0.884428	0.824101	0.69778	0.978125
UCSVMAX DFM CONSTR RWD	0.367426	0.315145	0.387986	0.990012	0.301387	0.406798
UCSVMAX DFM CARGO IRW	0.037776	0.504345	0.349615	0.318592	0.168596	0.256721
UCSVMAX DFM BUSCONF RW	0.037703	0.386058	0.461532	0.365877	0.170986	0.238683
UCSVMAX DFM RETAIL RWD	0.060317	0.587985	0.181586	0.259062	0.138794	0.154487
UCSVMAX DFM UNEMP RW	0.275173	0.402008	0.176482	0.189468	0.122339	0.186515
UCSVMAX CT IPI IRW	0.084826	0.698901	0.758127	0.629849	0.893617	0.932823
UCSVMAX TVM	0.099277	0.471157	0.962356	0.969763	0.063714	0.151712
UCSVMAX BVM IPI IRW	0.158088	0.594572	0.797228	0.772336	0.36613	0.930339
UCSVMAX BVM EXP IRW	0.003476	0.852879	0.750908	0.900599	0.716301	0.817526
UCSVMAX BVM BUSCONF RWD	0.012557	0.460671	0.866102	0.628565	0.295	0.350376
UCSVMAX BVM UNEMP RW	0.064172	0.897395	0.884676	0.925287	0.403025	0.755329
UCSVMAX BVM CARGO RW	0.131731	0.200466	0.120625	0.177953	0.055412	0.090657