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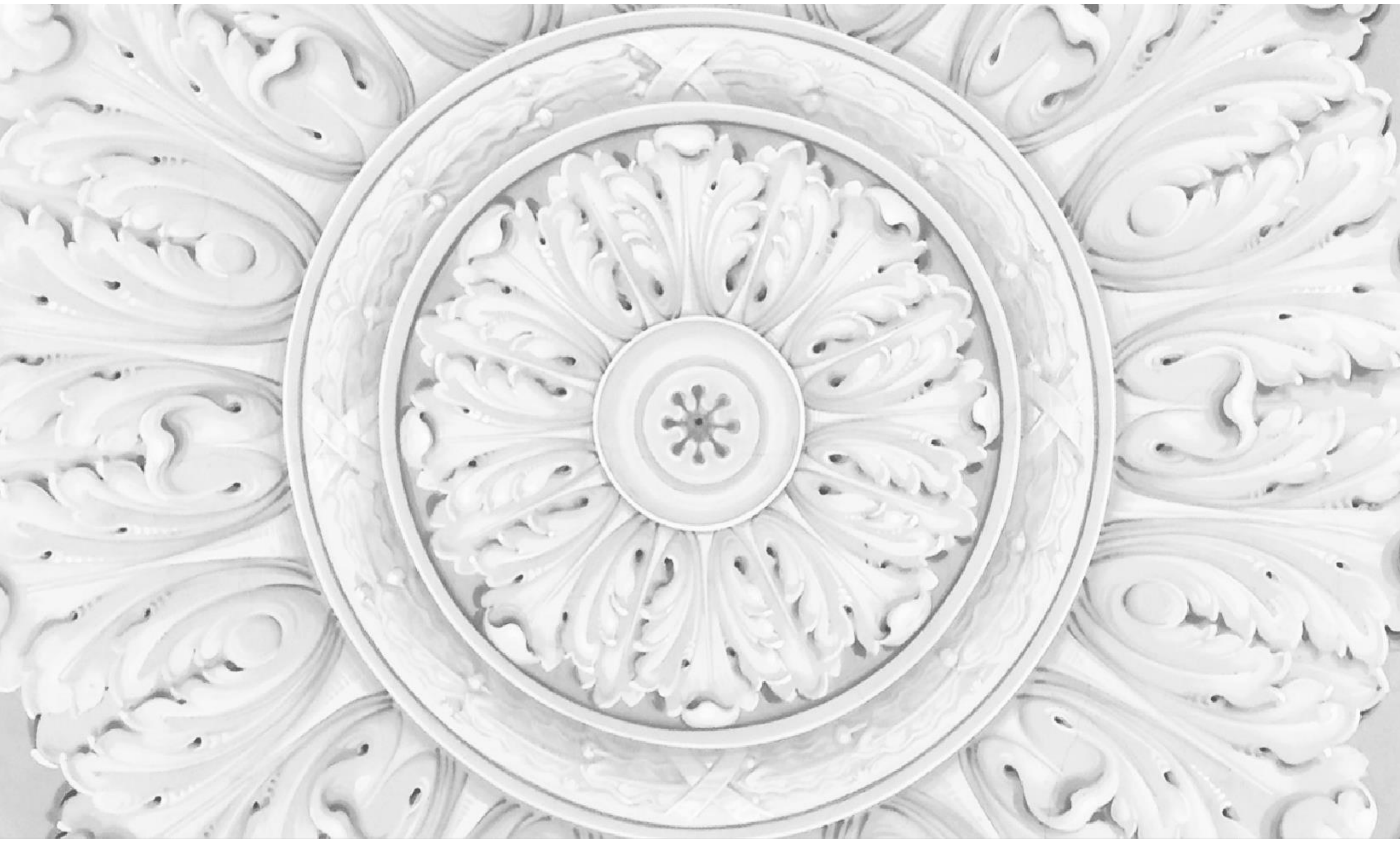

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Should Central Banks Prick Asset Price Bubbles? An Analysis Based on a Financial Accelerator Model with an Agent-Based Financial Market

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Abstract

This paper studies whether and how the central bank should prick asset price bubbles, if the effect of interest rate policy on bubbles can significantly vary across periods. For this purpose, I first construct a financial accelerator model with an agent-based financial market that can endogenously generate bubbles and account for their impact on the real sector of the economy. Then, I calculate the effect of different nonlinear interest rate rules for pricking asset price bubbles on social welfare and financial stability. The results demonstrate that pricking asset price bubbles can enhance social welfare and reduce the volatility of output and inflation, especially if asset price bubbles are caused by credit expansion. Pricking bubbles is also desirable when the central bank can additionally implement an effective communication policy to prick bubbles, for example, effective verbal interventions aimed at the expectations of agents in the financial market.

Keywords: *monetary policy; asset price bubble; New Keynesian macroeconomics; agent-based financial market.*
JEL classification: E44, E52, E58, G01, G02.

1. INTRODUCTION

The optimal response of monetary policy to asset price bubbles has been a subject of hot debate in the macroeconomic literature for a long time. This debate is known as the "*clean" versus "lean" debate*. Following the "clean" point of view, a central bank should not respond to an asset price bubble before the bubble bursts, beyond the necessary reaction for the stabilisation of inflation and employment, but merely clean up the consequences of the bubble. This approach prevailed in central banks and academia before the global financial crisis of 2008–2009. According to the opposite approach, the "lean against the wind" view, the central bank should try either to slow down the growth of asset price bubbles or to burst (or "prick") these bubbles. Nowadays, the focus of macroeconomic discussion has changed, from the question of *whether* central banks should respond to asset price bubbles to *how* they should respond.[1]

Many papers investigate how monetary policy should respond to asset prices (see e.g. Bernanke and Gertler (2000, 2001), Iacoviello (2005), Faia and Monacelli (2007), Nisticò (2012), Gelain et al. (2013), and Gambacorta and Signoretti (2014)), but almost all studies in this field, with rare exceptions, do not take into account the simultaneous effect of interest rate changes on the bubble component of asset prices. However, this effect is crucial for evaluating monetary policy response on asset price bubbles. If a tighter interest rate policy is not able to negatively affect asset price bubbles, it is unreasonable to implement the "leaning against the wind" policy, because an increasing interest rate will only slow down the economy, On the other hand, if this interest rate policy can significantly reduce asset price bubbles, it can be successfully used by central banks against bubbles.

Only a few papers, such as Kent and Lowe (1997), Filardo (2004), Gruen et al. (2005), and Fouejieu et al. (2014), consider the simultaneous effect of interest rate changes on asset price bubbles in their analysis, wherein they employ simple macroeconomic models comprising a small number of macroeconomic variables, such as output, inflation, interest rate, and asset prices. They generally assume that the probability of the bubble bursting and/or the size of the bubble are a linear function of the interest rate; therefore, in these models, the central bank may try to prick asset price bubbles by changing the interest rate. My paper significantly extends the existing

[1] For a more detailed discussion on the "clean" versus "lean" debate, see Mishkin (2011) or Brunnermeier and Schnabel (2015).

literature that explores how monetary policy should respond to asset price bubbles by making three important contributions.

First, I study how and whether the central bank should prick asset price bubbles by proposing a new theoretical explanation of the interest rate policy's impact on bubbles. There is no consensus in the macroeconomic literature regarding the impact of interest rate policy on asset price bubbles. According to the "conventional" view, a tighter interest rate policy may help reduce asset price bubbles. However, some papers (see e.g. Bordo and Wheelock (2007), Galí and Gambetti (2015), Blot et al. (2018)) cast doubts on the possibility of reducing bubbles by increasing the interest rate. The theoretical model constructed below combines the two views. It can endogenously generate different states, in some states, a tighter interest rate policy will not be able to reduce bubbles, whereas in other states, this policy will be quite successful in pricking bubbles. Thus, the effect of a tighter interest rate policy in the model may greatly vary across periods and cannot be represented by a linear function.

Second, I investigate the consequences of pricking asset price bubbles by the central bank for social welfare and financial stability assuming that the central bank should start pricking a bubble only when the latter has already grown to a significant size. This assumption corresponds to reality, because even if possible, it is very difficult to identify a bubble at the initial stage when it is small. Moreover, if the central bank can identify the bubble at the initial stage, a small deviation in the market price from the fundamental price may not grow in the future. Thus, at this stage, there may not be any significant threat to economic growth and financial stability, and it may be unreasonable to raise the interest rate at the expense of economic growth. If the central bank starts pricking only fairly large bubbles, it will cause a kind of nonlinear or piecewise reaction of the interest rate policy on asset price bubbles, as the policy ignores small deviations in the market price from the fundamental price but may very aggressively respond to large deviations. This type of monetary policy reaction related to pricking bubbles has previously received insufficient attention in the literature, because previous papers typically employ the Taylor rule with asset prices, in which the interest rate linearly responds to asset price bubbles.

Finally, in comparison to previous studies, which investigate the response of monetary policy on asset price bubbles and take into account the endogenous effect of monetary policy on bubbles, I propose a theoretical model that not only contains basic equations for inflation, output, interest rate, and the bubble but also includes consumption, the production function, sticky prices, and the financial accelerator mechanism. The proposed model enables to analyse the impact of pricking asset price bubbles on social welfare and the dynamics of other important macroeconomic variables, such as investment, capital, labour supply, and inflation.

This paper employs a novel theoretical framework based on the integration of an agent-based financial market in a financial accelerator model. The proposed model pertains to a recently emerged strand in the macroeconomic literature related to the synthesis of New Keynesian macroeconomics and agent-based financial market models.[2] The models in this field incorporate behavioural and speculative factors, which are the common features of asset price bubbles, in traditional macroeconomic frameworks. The theoretical model in this paper consists of two parts: the real sector, which is similar to the New Keynesian model with a financial accelerator and a bubble from Bernanke and Gertler (2000), and a financial market determined by the agent-based model in the spirit of Harras and Sornette (2011). The agent-based financial market is populated with bounded rational and heterogeneous traders, who trade futures contracts on capital from the real sector and therebyset the market price of capital in the model. The market price of capital can, sometimes, significantly deviate from the fundamental price of capital, implying the presence of a bubble in the financial market. Traders' decisions regarding trading operations are based on their opinions about future price movements and on the amount of available liquidity. The bubble bursts if many traders worry about the existence of this bubble, and the bursting of some bubbles may lead to a large decline in output, investment, and consumption. The amount of available liquidity significantly depends on the value of liquidity flow from the real sector to the financial market. The presence of high liquidity flow in the model enables to mimic situations in which bubbles are partially boosted by credit expansion, such as the global financial crisis of 2008–2009.

The central bank sets the interest rate following the interest rate rule that has a piecewise form. If the deviation of the market price of capital from the fundamental price is less than the threshold size, the interest rate rule corresponds to the standard Taylor rule based on the deviations of output and inflation from their steady state values. However, if this deviation is greater than the threshold size, the central bank raises the interest rate beyond the necessary reaction determined by the standard Taylor rule in each quarter until the bubble again becomes lower than the threshold size. After that, the

[2] The list of the earlier papers in this field includes but is not limited to Kontonikas and Ioannidis (2005), Bask (2012), and Lengnick and Wohltmann (2013). The review of more recent literature can be found in Lengnick and Wohltmann (2016).

central bank continues following the standard Taylor rule. An increasing interest rate reduces the fundamental price of capital and makes larger the deviation of the market price of capital from the fundamental price. The size of this deviation negatively affects traders' opinions on future price movements. If the bubble is not large enough and the central bank raises the interest rate, most likely such a tighter interest rate policy will not stop the growth of the bubble because many traders will not worry about the existence of the bubble yet. Therefore, in this case, the growing bubble and the tighter interest rate policy are observed together. Nevertheless, when the bubble reaches a significant size, many traders start to worry about the existence of the bubble, and the central bank can successfully prick the asset price bubble through the interest rate policy. Thus, the central bank's efforts to reduce the bubble's growth are inefficient when the bubble is not large; it can prick the bubble only when the bubble reaches a significant size. Another novel feature of my model is the possibility for the central bank to use communication policy, through which it may strengthen the worry of traders regarding the existence of the bubble, which will help prick the bubble.

To explore whether and how the central bank should prick asset price bubbles, I calculate the effect of several interest rate rules for pricking the bubble on social welfare and financial stability. Specifically, I calculate households' welfare and the volatility of output and inflation changing the threshold size of the bubble at which the central bank starts raising the interest rate beyond the necessary reaction determined by the standard Taylor rule in each quarter, until the bubble is again lower than the threshold size. In addition to different interest rate rules for pricking bubbles, I compute how households' welfare and the volatility of output and inflation depend on the effectiveness of the central bank's communication policy and on the value of liquidity flow from the real sector to the financial sector. The results of the analysis demonstrate that pricking asset price bubbles can enhance social welfare as well as reduce the volatility of output and inflation. This positive effect is greater when asset price bubbles are caused by credit expansion or when the central bank implements an effective communication policy to prick the bubble. However, pricking asset price bubbles only by raising the interest rate without an effective communication policy leads to negative consequences for social welfare and financial stability, because an increasing interest rate in this case may fail to burst asset price bubbles but slows down the economy. Thus, in my model, the effect of pricking asset price bubbles through the interest rate policy depends on the effectiveness of the central bank's communication policy. This theoretical result contributes to the recent discussion regarding the impossibility of reducing asset price bubbles using a tighter interest rate policy (see e.g. Galí (2014), Gali and Gambetti (2015), Beckers and Bernoth (2016), Allen et al. (2017), Blot et al. (2018)).

The paper is organised as follows. Section 2 describes the model. The calibration of the model is presented in Section 3. Section 4 discusses the model simulation results and their robustness. Section 5 analyses the effect of pricking asset price bubbles on social welfare and financial stability. Finally, Section 6 concludes the paper.

2. MODEL

The model consists of two parts: the real sector, which is similar to the New Keynesian model with a financial accelerator and a bubble from Bernanke and Gertler (2000), and the financial market, which is set by the agent-based model in the spirit of Harras and Sornette (2011). The real sector includes six types of agents: households, entrepreneurs, retailers, capital producers, the central bank, and the government. In contrast to the original paper of Bernanke and Gertler (2000), I do not consider money and exogenous shocks in the real sector for the sake of simplicity. The agent-based financial market is populated by bounded rational and heterogeneous traders. It is worth noting that the two parts of the model run on different time scales: the real sector operates quarterly, while the financial market operates weekly. Section 2.1 further presents the description of the real sector, while Section 2.2 provides the specification of the agent-based financial market. Finally, Section 2.3 explains in detail how the two parts are connected with each other.

2.1. Real Sector

2.1.1. Households

The model includes a continuum of households normalised to 1, who consume, work, and provide loans to entrepreneurs. The representative household solves the following standard utility maximisation problem:

$$maxE_0 \sum_{t=0}^{\infty} \beta^t U(C_t, L_t) = maxE_0 \sum_{t=0}^{\infty} \beta^t \left\{ \log C_t - \frac{{L_t}^{1+\sigma_l}}{1+\sigma_l} \right\}, \quad (1)$$

which depends on the current consumption C_t and the labour supply L_t, $0 < \beta < 1$ denotes the discount factor, and σ_l is the inverse elasticity of labour supply. The budget constraint of the representative household is given by:

$$C_t + B_t = W_t L_t + \frac{R_{t-1} B_{t-1}}{\pi_t} + \Pi_t - T_t, \quad (2)$$

where B_t and B_{t-1} denote loans to entrepreneurs at time t and $t-1$ respectively. Loan repayments at time $t-1$, $\frac{R_{t-1}B_{t-1}}{\pi_t}$ are adjusted for the inflation rate $\pi_t = \frac{P_t}{P_{t-1}}$ at time t, while the interest rate R_t is set by the central bank. The representative household receives wage W_t from entrepreneurs in exchange for its labour L_t, pays lump sum taxes T_t, and owns retail firms, obtaining firms' profit - Π_t. The first order conditions for the problem $(1) - (2)$ are standard and have the following form:

$$\frac{1}{C_t} = \beta \frac{1}{C_{t+1}} E_t\left(\frac{R_t}{\pi_{t+1}}\right) \quad (3)$$

$$\frac{W_t}{C_t} = L_t^{\sigma_l}, \quad (4)$$

where (3) and (4) are the Euler equation and the labour-supply condition respectively.

2.1.2. Entrepreneurs

Entrepreneurs manage perfectly competitive firms that produce intermediate goods using capital K_t and households' labour L_t. The production function of the representative entrepreneur is assumed to be of the Cobb-Douglas type:

$$Y_t = A K_t^{\alpha} L_t^{(1-\alpha)\Omega}, \quad (5)$$

where the parameter A represents technology process, α and $(1-\alpha)$ are the shares of capital and labour in the intermediate product respectively. Ω denotes the share of households' labour in the total labour. The amount of entrepreneurs' labour is normalised to 1, and the share of entrepreneurs' labour is equal $(1-\Omega)$. With the probability $(1-\upsilon)$ each entrepreneur can become bankrupt in any period. Under this assumption, entrepreneurs' net worth, N_t, will never be enough for the purchase of new capital K_{t+1}; therefore, entrepreneurs will always additionally borrow the amount B_t from households to finance capital acquisition:

$$B_t = Q_t K_{t+1} - N_t, \quad (6)$$

where Q_t is the fundamental price of capital at time t. Bernanke and Gertler (2000, 2001) introduce the "financial accelerator" mechanism from Bernanke, Gertler, and Gilchrist (1999), in which the interest rate for external financing, R_t^F, is greater than the interest rate, R_t, because of agency costs and asymmetric information, and depends on the ratio of the market value of capital to net worth:

$$E_t R_{t+1}^F = \frac{R_t}{\pi_{t+1}} \left(\frac{F_t K_{t+1}}{N_t} \right)^{\psi}, \tag{7}$$

where R_{t+1}^F denotes the expected rate of external financing, F_t is the market price of capital at time t , and ψ represents the parameter of financial accelerator mechanism. $lev = \frac{F_t K_{t+1}}{N_t}$ is the ratio of the market value of capital to the entrepreneurs' net worth, or in other words it is their financial leverage. The net worth of entrepreneurs is determined according to the following equation:

$$N_t = \nu[R_t^F F_{t-1} K_t - E_{t-1} R_t^F (F_{t-1} K_t - N_t)] + S_t^e, \tag{8}$$

where $S_t^e = (1-\alpha)(1-\Omega) A_t K_t^{\alpha} L_t^{(1-\alpha)(1-\Omega)}$ is the labour income of entrepreneurs. Entrepreneurs who become bankrupt at time t, consume the rest of the net worth in the amount C_t^e. The interest rate of external financing in (7) and the dynamics of entrepreneurs' net worth in (8) depend on the market price of capital F_t, which is changed as follows:

$$ln\left(\frac{F_t}{\bar{F}}\right) - \ln\left(\frac{Q_t}{\bar{Q}}\right) = ln\left(\frac{F_{t-1}}{\bar{F}}\right) - \ln\left(\frac{Q_{t-1}}{\bar{Q}}\right) + \tau_t^F, \tag{9}$$

where the variable τ_t^F is the exogenous market change impulse set by the interaction of traders in the financial market, who trade futures on capital. The calculation of τ_t^F will be described further in Section 2.3. $\bar{F} = 1$ and $\bar{Q} = 1$ are the steady state values of F_t and Q_t respectively. Equation (9) determines the size of the deviation of the market price of capital from the fundamental price of capital. I assume that without the market change impulse, τ_t^F, the size of this deviation remains the same over time. The following condition is fulfilled under the optimal demand on capital:

$$R_t^F = \frac{(R_t^k + (1-\delta)F_t)}{F_{t-1}}, \tag{10}$$

where R_t^k is the marginal return on capital. The first order conditions for entrepreneurs are as follows:

$$R_t^k = \frac{\alpha Y_t}{K_t} MC_t \tag{11}$$

$$S_t = \frac{(1-\alpha)Y_t}{L_t} MC_t \tag{12}$$

$$S_t^e = (1-\alpha)(1-\Omega)Y_t MC_t \ , \tag{13}$$

where $\frac{1}{MC_t}$ is the markup of retailers at time t, its description will be given further.

2.1.3. Capital Producers

The representative competitive capital producer purchases the amount of final goods I_t at the price P_t from retailers at the beginning of each period. Subsequently, she transforms the final goods into the equal amount of new capital and sells newly produced capital to the entrepreneurs at the price P_t^K. The representative capital producer maximises the following function:

$$\max_{I_t} \left[Q_t I_t - I_t - \frac{\chi}{2}\left(\frac{I_t}{K_t} - \delta\right)^2 K_t \right], \tag{14}$$

where $\frac{\chi}{2}\left(\frac{I_t}{K_t} - \delta\right)^2 K_t$ is quadratic adjustment costs, $Q_t = \frac{P_t^K}{P_t}$ denotes the relative fundamental price of capital at time t, while δ and χ represent the depreciation rate and the parameter of adjustment costs respectively. The first order condition is the standard Tobin's q equation:

$$Q_t - 1 - \chi\left(\frac{I_t}{K_t} - \delta\right) = 0 \tag{15}$$

The aggregate capital stock evolves according to:

$$K_t = (1-\delta)K_{t-1} + I_t \tag{16}$$

2.1.4. Retailers

I introduce nominal price rigidity in the model through the retail sector, populated by a continuum of monopolistic competitive retailers of mass 1 indexed by z. At time t

retailers purchase intermediate goods Y_t at the price P_t^w from entrepreneurs in a competitive market, differentiate them at no costs into $Y_t(z)$, and then sell to households and capital producers in the amount Y_t^f at the price $P_t(z)$ using a CES aggregation with the elasticity of substitution $\epsilon_y > 0$:

$$Y_t^f = \left(\int_0^1 Y_t(z)^{\frac{\epsilon_y - 1}{\epsilon_y}}\, dz\right)^{\frac{\epsilon_y}{\epsilon_y - 1}} \tag{17}$$

Each retailer faces the following individual demand curve:

$$Y_t(z) = \left(\frac{P_t(z)}{P_t}\right)^{-\epsilon_y} Y_t^f, \tag{18}$$

where P_t denotes the aggregate price index, which is determined as follows:

$$P_t = \left(\int_0^1 P_t(z)^{1-\epsilon_y}\, dz\right)^{\frac{1}{1-\epsilon_y}} \tag{19}$$

Following Calvo (1983), I assume that only the share of retailers $(1-\theta_p)$ can adjust their prices in each period to maximise the following profit function:

$$\Pi_t = \sum_{k=0}^{\infty} \theta_p^k \mathrm{E}_{t-1}\left[\Lambda_{t,k} \frac{P_t^* - P_{t+k}^w}{P_{t+k}} Y_{t+k}^*\right], \tag{20}$$

where $\Lambda_{t,k} \equiv \beta \frac{C_t}{C_{t+k}}$ denotes the discount factor of retailers, which is equal to the stochastic discount factor of households. P_t^* and $Y_t^*(z) = \left(\frac{P_t^*(z)}{P_t}\right)^{-\epsilon_y} Y_t$ are, respectively, the optimal price and optimal demand at time t. The first order condition for retailers is:

$$\sum_{k=0}^{\infty} \theta_p^k \mathrm{E}_{t-1}\left[\Lambda_{t,k}\left(\frac{P_t^*}{P_{t+k}}\right)^{-\epsilon_y} Y_{t+k}^*(R)\left[\frac{P_t^*}{P_{t+k}} - \left(\frac{\epsilon_y}{\epsilon_y - 1}\right)\frac{P_{t+k}^w}{P_{t+k}}\right]\right] = 0 \tag{21}$$

2.1.5. Central Bank

The central bank sets the interest rate according to the following interest rate rule:

$$\ln\left(\frac{R_t}{\bar{R}}\right) = \rho_r \ln\left(\frac{R_{t-1}}{\bar{R}}\right) + (1-\rho_r)\left(\rho_\pi \ln\left(\frac{\pi_t}{\bar{\pi}}\right) + \rho_y \ln\left(\frac{Y_t}{\bar{Y}}\right)\right) + \Delta r_t^{Bubble}, \tag{22}$$

where $\bar{R}, \bar{\pi}, \bar{Y}$ represent the steady state values of R_t, π_t, Y_t, respectively, while ρ_r, ρ_π, ρ_y denote their weights in the interest rate rule. Δr_t^{Bubble} is an additional increase in the interest rate related to pricking bubbles in the financial market by the central bank. The central bank sets Δr_t^{Bubble} following a piecewise rule that will be explained in more detail in Section 2.3. Moreover, the central bank may additionally affect the behaviour of traders in the financial market if it implements a certain communication policy (for example, verbal interventions). I will discuss the operation of this policy in the model in Section 2.2.

2.1.6. Government Sector

Government expenditures are financed by lump-sum taxes:

$$G_t = T_t \tag{23}$$

2.2. Financial Market

The agent-based model sets the operation of the financial market for futures contracts on capital and includes H traders. The interaction of these traders determines the market price of capital in the financial market in week w, $price_w$. The behavior of traders in the model is based on the certain number of rules, parameters of which vary from trader to trader and are drawn randomly from specified distributions. The agent-based financial market is constructed in the spirit of the Harras and Sornette (2011) model, with significant modifications. Specifically, I change the rules by which traders make decisions, add liquidity flows from the real sector to the financial market, and calibrate the model parameters to the real data. Under the considered calibration, the agent-based financial market can endogenously generate bubbles.

2.2.1. Trading Decisions

Each week w trader i makes one of three decisions: buy futures, sell futures, or refrain from participating in trading. It is worth noting that the model does not include the possibility of short positions. The trader's decision is based on her opinion on future price movements. The opinion of the trader i in week w - $\omega_{i,w}$ is based on the signal of three strategies: the individual strategy's signal, $IN_{i,w}$, the chartist strategy's signal, CH_w, and the fundamentalist strategy's signal, FU_w. Specifically, $\omega_{i,w}$ is set as follows:

$$\omega_{i,w} = x_{1i} * IN_{i,w} + x_{2i} * CH_w + x_{3i} * FU_w, \quad (24)$$

where x_{1i}, x_{2i}, and x_{3i} represent the coefficients that are unique for each trader and have a uniform distribution over the respective intervals $[0, X_1], [0, X_2]$, and $[0, X_3]$, where X_1, X_2, and X_3 are model parameters. The signal of individual strategy, $IN_{i,w}$, is also unique for each trader and has a simple standard normal distribution, $IN_{i,w} \sim N(0,1)$. The signals of fundamentalist strategy FU_w and chartist strategy CH_w are common for all traders in the market. The chartist strategy's signal is based on market sentiments and global news:

$$CH_w = LR + MR_w + \varepsilon_w^{CH}, \quad (25)$$

where LR is a model parameter which represents a fixed long-run component in the market sentiments, while ε_w^{CH} is a random global news shock in week w that has a standard normal distribution. MR_w refers to a changing medium-run component in the market sentiments and is equal, in week w, to the difference between two moving averages of the market price for the last 52 and 104 weeks, multiplied by the parameter $trend$:

$$MR_w = trend * \left(\sum_{j=w-52}^{w-1} price_j - \sum_{j=w-104}^{w-1} price_j\right) \quad (26)$$

The fundamental strategy's signal represents the worry of traders regarding the convergence of the market price of capital to the fundamental one. It depends on the deviation of the market price from the fundamental price, D_w, on the cumulative market return over the last m weeks, $\frac{price_{w-1} - price_{w-m}}{price_{w-m}}$, and on the increase in the interest rate by the central bank for pricking a bubble, Δr_t^{Bubble}:

$$FU_w = (-D_w + \varepsilon_w^{FU}) * \left(1 + \left|\min\left(0, \frac{price_{w-1} - price_{w-m}}{price_{w-m}}\right)\right|^{\xi 1} * \xi 2\right) * (1 + CPolicy * \Delta r_{t-1}^{Bubble}), \quad (27)$$

The first factor in equation (27), $(-D_w + \varepsilon_w^{FU})$, includes ε_w^{FU}, which is a normally distributed shock with zero mean and standard deviation σ^{FU}. This shock represents the noise in the expectations regarding the true value of the deviation of market price from the fundamental price. This deviation is calculated as follows:

$$D_w = \frac{price_w - Q_{t-1}}{Q_{t-1}}, \tag{28}$$

where $price_w$ denotes the market price of capital in the financial market in week w, and Q_{t-1} is the fundamental price of capital from the real sector of the model in the last quarter $t-1$. In reality, a larger deviation of an asset price from its fundamental value will make traders more sceptical about investments in this asset. Some of them may not open new positions, while others may even close the existing positions in their portfolios. The first factor in equation (27) enables to take this phenomenon into account in the model.

The second factor in equation (27), $\left(1 + \left|\min\left(0, \frac{price_{w-1} - price_{w-m}}{price_{w-m}}\right)\right|^{\xi 1} * \xi 2\right)$, includes some positive parameters $\xi 1, \xi 2 > 0$ and allows for occasional crashes and panics in the financial market. If the cumulative market return over the last m weeks, $\frac{price_{w-1} - price_{w-m}}{price_{w-m}}$, is positive, this factor will be equal to 1. However, when the cumulative market return over the last m weeks becomes negative, this factor will grow rapidly and increase the negative effect of an asset price misalignment on a trader opinion about future price movements. In this case, many traders may simultaneously close their position, which will lead to a collapse of the financial market.

The last factor in equation (27), $(1 + CPolicy * \Delta r_{t-1}^{Bubble})$, represents the influence of the central bank's communication policy (for example, verbal interventions) on the traders' opinions, where Δr_{t-1}^{Bubble} denotes an additional increase in the interest rate from the real sector (equation (22)) used by the central bank to prick a bubble. $CPolicy \geq 0$ is a parameter which determines the effectiveness of the communication policy. In Section 5, I explore consequences of changes in $CPolicy$ for households' welfare, the volatility of output, and volatility of inflation. As we can see, the communication policy will operate in the model only if the central bank is trying to prick the bubble. I will come back to the discussion of the central bank's actions related to the pricking of bubbles in Section 2.3.

The initial portfolio of the trader i in week zero consists of cash, $cash_{i,0}$, and some amount of futures, $futures_{i,0}$. It should be noted that $cash_{i,0}$ and $futures_{i,0}$ for each trader are chosen randomly from the uniform distributions over the intervals $[0, \overline{cash}]$ and $[0, \overline{futures}]$ respectively, where $\overline{cash}$ and $\overline{futures}$ are some parameters. Following Harras and Sornette (2011), to introduce the differences in risk aversion between traders, I assume that each week trader i decides on her participation in trading based on the parameter $\underline{\omega_i}$. This parameter is set randomly for each trader over the interval $[0, \underline{\Omega}]$, where $\underline{\Omega}$ is the parameter of the differences in risk aversion. The value of $\underline{\omega_i}$ is compared with the value of $\omega_{i,w}$, and the trader i makes the decision following these rules:

$$\begin{aligned} &if \quad \omega_{i,w} > \underline{\omega_i} : order_{i,w}^d = +1\ (buy),\ size_{i,w}^d = fraction * \frac{cash_{i,w-1}}{price_{w-1}} \\ &if \quad -\underline{\omega_i} \le \omega_{i,w} \le \underline{\omega_i} : order_{i,w}^d = 0\ (hold),\ size_{i,w}^d = 0 \\ &if \quad \omega_{i,w} < -\underline{\omega_i}\ order_{i,w}^d = -1\ (sell), size_{i,w}^d = fraction * futures_{i,w-1} \end{aligned} \tag{29}$$

where $size_{i,w}^d$ is the number of futures the trader i wants to buy or sell, and $order_{i,w}^d$ denotes the indicator of the trading operation. $fraction$ is the model parameter that means what fraction of futures contracts or of cash the trader i wants to use in one trading operation. Following Harras and Sornette (2011), I use the value for $fraction$ that is much smaller than 1 to ensure time diversification.

2.2.2. Liquidity Flows

Traders also additionally buy (sell) futures in the case of positive (negative) liquidity flow from the real sector to the financial market. The variable $liquidity_w$ shows by how much the value of the trader's portfolio should be changed due to liquidity flow. If $liquidity_w > 0$, then the liquidity flow is positive, and if $liquidity_w < 0$ then the liquidity flow is negative. The trader i additionally buys and sells futures according to the following rules:

$$\begin{aligned} &if \quad liquidity_w > 0 : order_{i,w}^l = +1, size_{i,w}^l = liquidity_w * futures_{i,w-1} \\ &if \quad liquidity_w = 0 : order_{i,w}^l = 0\ ,\ size_{i,w}^l = 0 \\ &if \quad liquidity_w < 0 : order_{i,w}^l = -1,\ size_{i,w}^l = liquidity_w * futures_{i,w-1} \end{aligned} \tag{30}$$

where $size_{i,w}^l$ is the number of futures the trader i wants to buy or sell due to the liquidity flow, and $order_{i,w}^l$ is the indicator of the trading operation.

2.2.3. Price Clearing Condition

Once all traders have made their trading decisions on the basis of their opinions and the liquidity flow, they send trading orders without any transaction costs to a market maker, who has an unlimited amount of cash and stocks. The market maker sets the price in week w according to the following market clearing rules:

$$r_w = \frac{1}{\lambda * H} \sum_{i=1}^{S} \left(order_{i,w}^{d} * size_{i,w}^{d} + order_{i,w}^{l} * size_{i,w}^{l} \right) \tag{31}$$

$$log[price_w] = log[price_{w-1}] + r_w \tag{32}$$

where r_w is the market return, while λ represents the market depth, i.e. the relative impact of the excess demand upon the price.

2.2.4. Cash and Futures Positions

The cash and futures positions held by the trader i are updated according to:

$$cash_{i,w} = liquidity_w * cash_{i,w-1} - \left(s_{i,w}^{d} * v_{i,w}^{d} + s_{i,w}^{l} * v_{i,w}^{l} \right) * p_{m,w} \tag{33}$$

$$futures_{i,w} = futures_{i,w-1} + s_{i,w}^{d} * v_{i,w}^{d} + s_{i,w}^{l} * v_{i,w}^{l} \tag{34}$$

2.3. The Interaction of the Real Sector and the Financial Market

As mentioned earlier, one period in the real sector of the model corresponds to one quarter, while the agent-based financial market operates weekly. To combine the two parts of the model into the joint one, I assume that one quarter always consists of 13 weeks, so one year, which is four quarters, always includes 52 weeks.

2.3.1. Interactive Channels

The interaction between the real sector and the financial market is based on four channels. The first channel is related to the market price formation. The market price of capital in the financial market determines the market price of capital in the real sector through the market change impulse τ_t^F in equation (9), which has the following log-linearised form:

$$f_t - q_t = f_{t-1} - q_{t-1} + \tau_t^F, \tag{35}$$

where $f_t = \frac{F_t - \bar{F}}{\bar{F}}$ and $q_t = \frac{Q_t - \bar{Q}}{\bar{Q}}$ are the deviations of the market price of capital, F_t, and the fundamental price of capital, Q_t, from their steady state values $\bar{F} = 1$ and $\bar{Q} = 1$ respectively. τ_t^F is calculated according to the following equation that includes the average market price in the financial market over 13 weeks $\frac{\sum_{w=1}^{13} price_w}{13}$ in the quarter t:

$$\tau_t^F = sen_1 * \left(\frac{\sum_{w=1}^{13} price_w}{13} - Q_{t-1}\right) - (F_{t-1} - Q_{t-1}), \quad (36)$$

where sen_1 is the model parameter that represents the sensitivity of changes in the real sector to changes in the financial market. All calculations in the real sector take place at the end of each quarter, when the dynamics of the agent-based model in this quarter is already known.

The second channel is the liquidity flow from the real sector to the financial market. I assume that the liquidity flow is proportional to changes in the net worth of entrepreneurs and is set as follows:

$$liquidity_w = sen_2 * (n_{t-1} - n_{t-2})^{\frac{1}{13}}, \quad (37)$$

where $n_{t-1} = \frac{N_{t-1} - \bar{N}}{\bar{N}}$ is the deviation of the entrepreneurs' net worth from its steady state value in the last quarter. sen_2 is the model parameter that shows the sensitivity of the liquidity flow to changes in the entrepreneurs' net worth. This relationship between the entrepreneurs' net worth and the liquidity flow corresponds to reality, because the growth of the firms' net worth in the economy means an increasing amount of available collateral for loans. This subsequently leads to the growth of available liquidity in the economy, as well as to the increase in liquidity flows to financial markets. Moreover, the higher the value of the net worth, the more firms or institutional investors can invest in, for example, different funds, like mutual and hedge funds that operate in financial markets.

The third and the fourth channels pertain to the transmission of information from the real sector to the financial market through traders' opinions in equation (27) that demonstrates the formation of the fundamentalist strategy's signal. Specifically, they are the impact of the fundamental price of capital from the real sector on traders' opinions and, respectively, the influence of the central bank's communication policy on traders' opinions.

2.3.2. Central Bank's Actions

I investigate the consequences of pricking bubbles by the central bank, assuming that it starts pricking a bubble only when the latter has already grown to a significant size. This assumption corresponds to reality, because even if possible it is very difficult to identify the bubble at the initial stage when it is small. Moreover, if the central bank can identify the bubble at the initial stage, a small deviation in the market price from the fundamental price may not grow in the future. Thus, at this stage, there may not be any significant threat to economic growth and financial stability, and it is unreasonable to raise the interest rate at the expense of economic growth.

The central bank sets the interest rate according to the interest rate rule (22):

$$\ln\left(\frac{R_t}{\bar{R}}\right) = \rho_r \ln\left(\frac{R_{t-1}}{\bar{R}}\right) + (1-\rho_r)\left(\rho_\pi \ln\left(\frac{\pi_t}{\bar{\pi}}\right) + \rho_y \ln\left(\frac{Y_t}{\bar{Y}}\right)\right) + \Delta r_t^{Bubble}$$

where the additional increase of the interest rate Δr_t^{Bubble} is a piecewise function based on the deviation of the market price of capital from the fundamental one (D_w from equation (28)):

$$If\ D_w \geq levCB:\ \Delta r_t^{Bubble} = \Delta$$
$$Else: \quad \Delta r_t^{Bubble} = 0, \tag{38}$$

where $levCB$ is a model parameter that means a threshold size. If the deviation of the market price of capital from the fundamental price, D_w, is less than this threshold size in the last week of the current quarter, t, the interest rate rule (22) corresponds to the standard Taylor rule based on the deviations of output and inflation from their steady state values. However, if D_w is greater than $levCB$, the central bank raises the interest rate by the fixed value Δ beyond the necessary reaction determined by the standard Taylor rule in each quarter – until the bubble again becomes lower than the threshold size.[3] In Section 5, I study the effect of different values for the threshold size on the consequences of pricking bubbles for social welfare and financial stability.

[3] The considered specification of the interest rate rule for pricking asset price bubbles may not be optimal in terms of the maximisation of social welfare or minimisation of the central bank's loss function. However, whether it is possible to find a better specification is still unknown. Moreover, if such a specification exists, it may not be robust to changes in the model parameters (for example, parameters related to the agent-based financial market). Therefore, I use a simple specification of the interest rate rule for pricking bubbles to make the results more robust and realistic.

An additional increase of the interest rate for pricking a bubble in the financial market has an immediate effect on the net worth of entrepreneurs and the fundamental price of capital. A growing interest rate slows down the economy, as well as reduces the net worth of entrepreneurs and the fundamental price of capital. This subsequently affects the financial market through channels two and three discussed in Section 2.3.1 earlier. In addition to these indirect effects on the financial market, pricking the bubble has a direct effect on the traders' opinion through the channel four, if the central bank implements the communication policy, or, in other words, if the parameter $CPolicy$ in equation (27) is greater than 0.

2.3.3. Model Solution

To solve the joint model, I first log-linearise the equations related to the real sector and obtain transition matrixes, following typical steps when solving DSGE models. The log-linearised version of the real sector is presented in Appendix A. Thereafter, I iterate the following algorithm:

1. I simulate the dynamics of the agent-based financial marker during 13 weeks in the current quarter, to calculate the market price change impulse τ_t^F from the real sector.
2. At the end of the current quarter, I compute the values of the variables from the real sector using the transition matrixes.
3. I simulate once again the agent-based model during 13 weeks in the next quarter and so on.

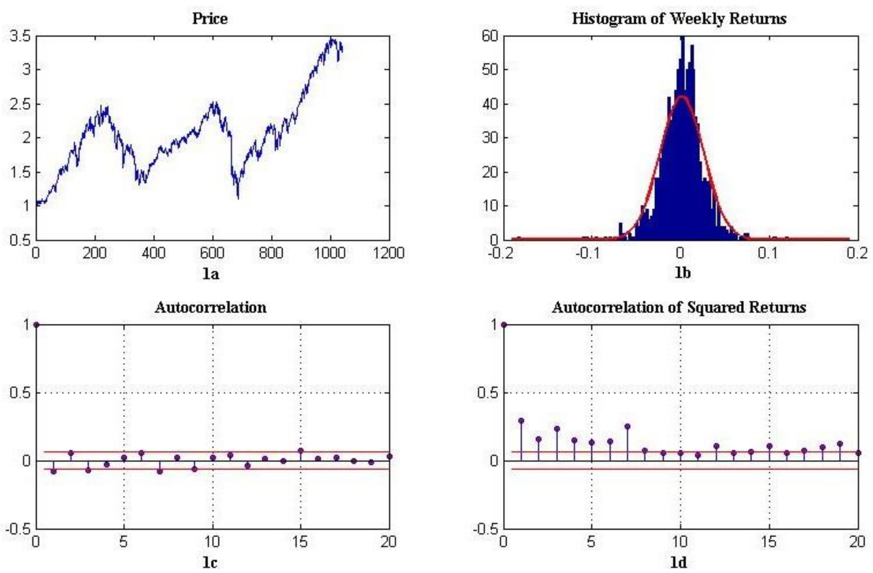
3. CALIBRATION

The goal of the model calibration is to find the value of parameters under which the model will be able to generate realistic dynamics of the economy for a period of 20 years accompanied by bubbles in the financial market. For this purpose, I calibrate the agent-based financial market to reproduce well-known stylised facts regarding weekly statistical characteristics of the S&P 500 index for the period of 1996-2016. Figure 1a presents historical prices of the S&P 500 over this period, during which in the US stock market there were two large crashes: the dot-com bubble and the financial crisis in the years 2007-2009. Thus, for each realisation of random shocks the model should generate approximately from 1 to 4 crises. A greater number of crises seems unrealistic because it is difficult to find a 20 year period in US or other countries' history with such a large number of stock market bubbles.

In addition to the number of stock market crashes, the statistical characteristics of the market price of capital set by the agent-based financial market should comply with the following well-known stylised facts:

- Weekly returns have small autocorrelation. Figure 1c shows that autocorrelation in weekly returns over the period 1996-2016 is insignificant for any lag.
- The distribution of weekly returns does not follow the normal distribution. Figure 1b illustrates that the real distribution has a higher kurtosis (fatter tails, more peaked around zero) and is negatively skewed. Moreover, it is not possible to reject the hypothesis of the zero mean return.
- The dynamics of the market price can be divided into volatility clusters; in some periods volatility will be high, while in others it will be low. The positive autocorrelation in squared returns on Figure 1d represents this phenomenon.
- In the periods of high volatility, the market price is more likely to fall, while in the periods of low volatility, it is more likely to grow. Thus, there is a negative correlation between volatility and stock returns.

Figure 1. Statistical Characteristics of the S&P 500 index



Notes: The figure presents the following data for the period of 1996-2016: the weekly adjusted price of the S&P 500 index (Figure 1a), the histogram of weekly S&P 500 returns (Figure 1b), the autocorrelation of weekly S&P 500 returns (Figure 1c), and the autocorrelation of squared weekly S&P 500 returns (Figure

1d). The red line on Figure 1b shows the probability density function of a normal distribution with the mean and standard deviation of weekly S&P 500 returns over the sample period.

The agent-based financial market has many possible combinations of parameters that correspond to the mentioned stylised facts (as usual for agent-based models). For this reason, in the description of the parameters calibration I focus primarily on the explanation of the parameters' effects on the statistical characteristics of the market price. The summary of the calibration results can be found in Appendix B.

The number of traders in the model is set at the relatively large value of $H = 10000$, which guarantees that in any week for any type of trading decision (buy, sell or do not participate in trading) there will be many traders who choose this type. Values for the amount of cash and futures in the initial week, $\overline{cash} = 1$ and $\overline{futures} = 1$, are taken from Harras and Sornette (2011), as well as the share of traders' cash or stocks they trade each time, $fraction = 0.02$. In reality, the world economy has had positive average long-term growth since World War II, so I assume that the fixed long-run component in the chartist strategy's signal, LR, is positive and equal to 0.6. To create the growing dynamics of the financial market with $LR = 0.6$, I find that the parameter of a variable medium-run component in equation (25) for the chartist strategy's signal, $trend$, the distribution parameter related the fundamentalist strategy, X_3, and the distribution parameter related the chartist strategy, X_2, should approximately have the following values: $trend = 1.2$, $X_3 = 1$, $X_2 = 20$. At the same time, to allow the bursting of bubbles, the parameters of the fundamentalist strategy's signal $\xi 1$, $\xi 2$, m, and σ^{FU} should be approximately equal: $\xi 1 = 3, \xi 2 = 750$, $m = 12$, and $\sigma^{FU} = 2$. The parameters of the differences in risk aversion, $\underline{\Omega}$, and the market depth, λ, specify, respectively, the form and the scale of the distribution of returns,. To match the form and the scale of the distribution of returns from the agent-based financial market with the same distribution in Figure 1b, I calibrate these parameters as $\underline{\Omega} = 40$ and $\lambda = 0.05$. The distribution parameter of individual strategy, X_1, enables me to simultaneously adjust the autocorrelation of returns and the autocorrelation of squared returns. I find that with $X_1 = 15$, the market price in the agent-based financial market has realistic levels for the autocorrelation of returns and the autocorrelation of squared returns that are similar to the levels in Figures 1c and 1d. A smaller value of X_1 leads to a higher value of autocorrelations, and vice versa.

For the sensitivity parameters, sen_1 and sen_2, I take the values that lead to realistic fluctuations of output over the 20 years:. $sen_1 = 0.06$ and $sen_2 = 0.075$. In the real sector of the model, for all but two parameters, I use the values estimated and frequently used in the literature. These values can be found in Appendix B. The value of the additional increase in the interest rate for pricking asset price bubbles, Δ , is set to $\Delta = 0.25\%$ because it is the minimum value that is typically used by the Federal Reserve System. For the parameter of the financial accelerator mechanism I take the value $\psi = 0.02$. This value decreases the financial accelerator effect in comparison to Bernanke and Gertler (2000) (they use $\psi = 0.05$), but it enables obtaining more realistic dynamics of the joint model.

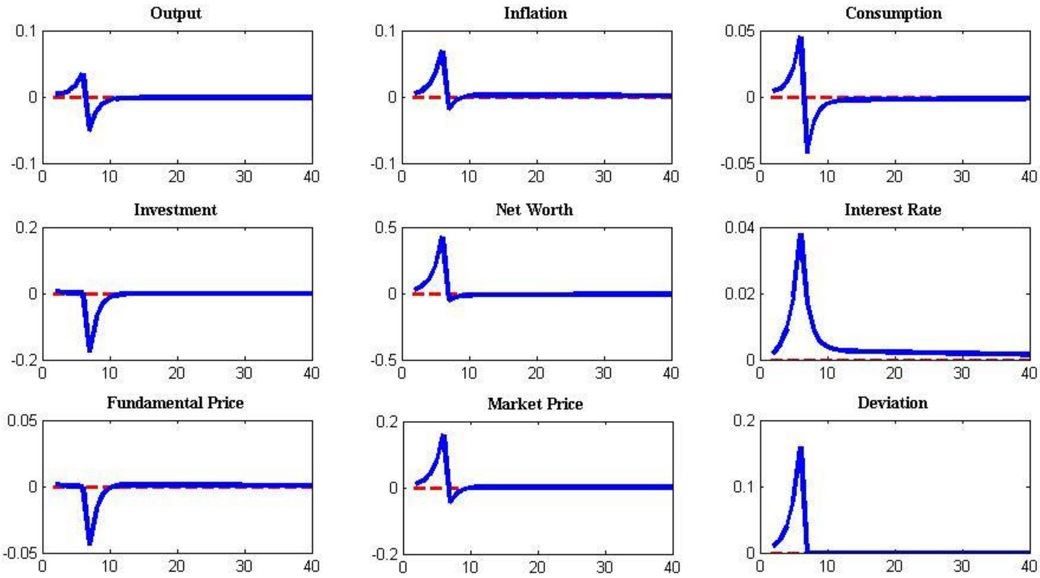
4. MODEL SIMULATIONS

In this section, I analyse the model dynamics over the period of 1040 weeks, when the central bank does not prick bubbles, so $\Delta r_t^{Bubble} = 0$ in equation (22). I assume that each quarter consists of 13 weeks, so the analysed period is equivalent to 20 years or 80 quarters. In Section 4.1, I show the dynamics of the real sector in response to an exogenous bubble as in Bernanke and Gertler (2000). Then in Section 4.2, I consider the dynamics of the agent-based financial market when it operates without any connections to the real sector. Finally, in Section 4.3, I analyse the dynamics of the joint model. It is worth noting that I present the dynamics of the agent-based model in Section 4.2 and the dynamics of the joint model in Section 4.3 for a random realisation of shocks. For other realisations, these dynamics may differ, but they will still correspond to stylised facts and will be accompanied by bubbles in the financial market, because of the considered calibration.

4.1. Real Sector Response to an Exogenous Bubble

As in Bernanke and Gertler (2000), I consider an exogenous bubble – a 1% market price change shock, which grows twice in each quarter and bursts when the market price is 16% percent higher than the fundamental price. The impulse responses to the bubble are presented in Figure 2. The bubble growth increases the net worth of entrepreneurs and the inflation acceleration. The net worth increase leads to a lower rate of external financing for entrepreneurs; therefore, they start borrowing more funds from households, purchase more capital, hire more household abour, and produce more intermediate goods. The inflation acceleration and output growth force the central bank to raise the interest rate. Under the considered model calibration, the negative effect from the increasing interest rate on capital and investment during the boom phase of the bubble approximately matches the positive effect from the net worth increase. The output growth is also accompanied by consumption growth. However, after the bubble bursts, almost all key variables in the economy including consumption sharply fall, so the welfare of households significantly decreases.

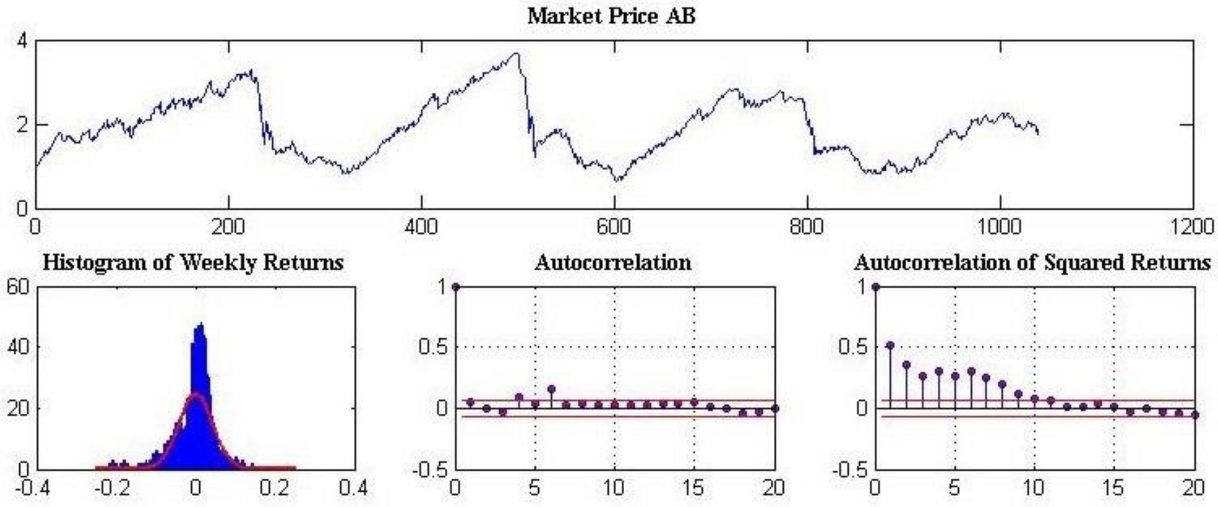
Figure 2. Impulse Responses of the Real Sector to an Exogenous Bubble



4.2. Agent-Based Financial Market Dynamics

As already mentioned, the market price dynamics in the agent-based financial market depends on the random realisation of shocks, and there exist an infinite number of possible shocks' realisations. Typical agent-based financial market dynamics is presented in Figure 3. The market price dynamics, the distribution and autocorrelation of market returns, and the autocorrelation of squared market returns match the real data in Figure 1. In over 1,040 weeks, the agent-based financial market experienced two large crashes and one smaller correction. The first two episodes are very similar to bubbles, where the boom phase of a bubble takes approximately four years. As in the real data, weekly market returns in the agent-based financial market have small autocorrelation, but a significant autocorrelation in squared market returns exists. The distribution of weekly market returns in the agent-based financial market also has fat tails and is more peaked around zero than normal distribution and negatively skewed.

Figure 3. Agent-Based Financial Market Dynamics



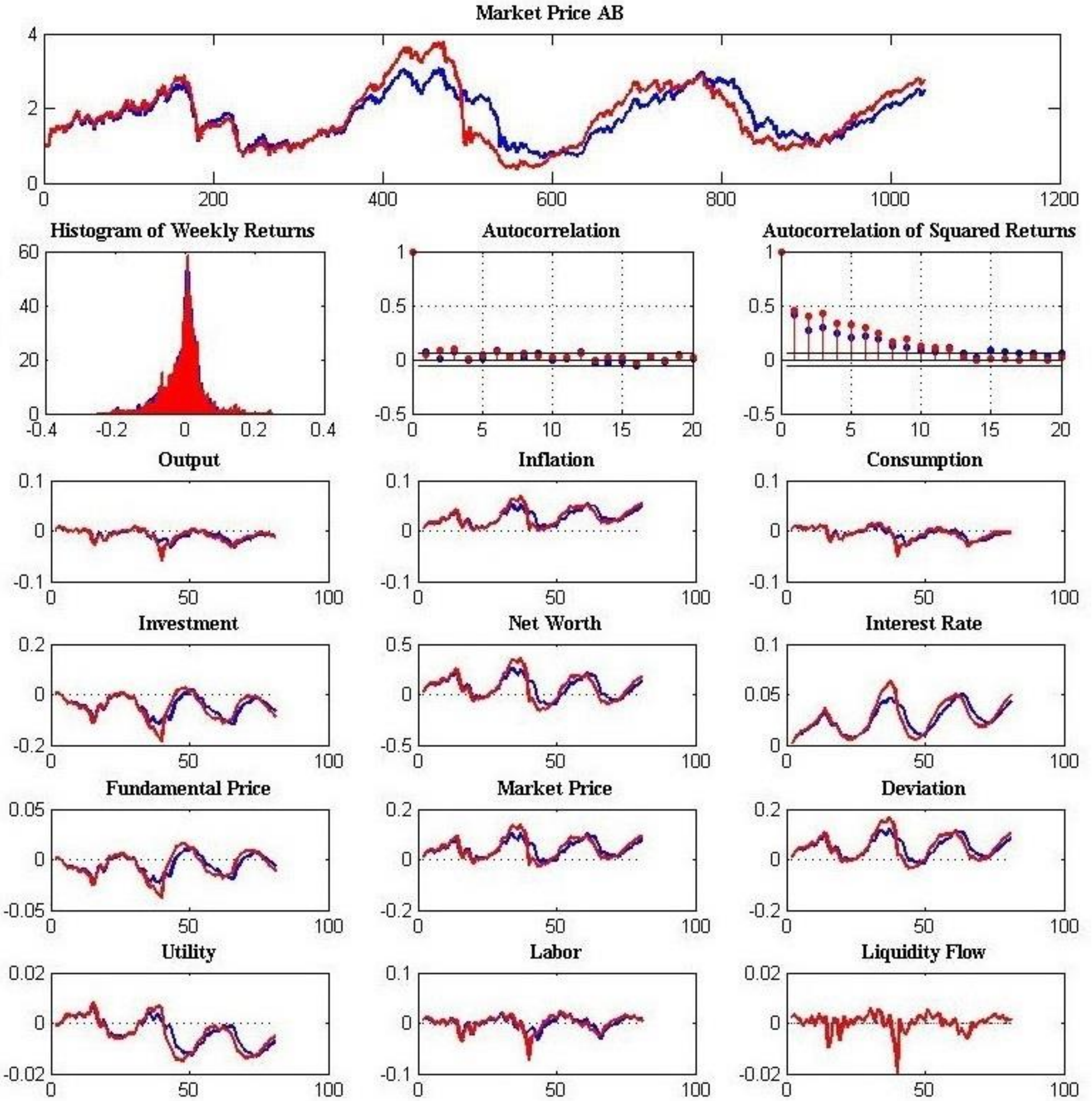
4.3. The Joint Model Dynamics.

Figure 4 presents the dynamics of the real sector and of the agent-based financial market when both parts operate simultaneously and endogenously connected with each other. The red and blue lines present the joint model dynamics with and without the liquidity flow from the real sector to the financial market respectively. The market price growth in the agent-based financial market leads to the increase in output, consumption, entrepreneurs' net worth, and households' utility. The output growth and inflation acceleration cause the interest rate increase. We also observe a positive liquidity flow from the real sector to the financial market (in the case with liquidity flows – red lines) because of the entrepreneurs' net worth increase. In the case of the sharp market price fall, similar to the bubble burst or a market crash, the dynamics become the opposite. A market crash in the model typically occurs quickly, whereas an economic recovery takes longer; these dynamics correspond to the real data. Figure 4 illustrates that the inclusion of the liquidity flow in the model increases the absolute values of macroeconomic variables' fluctuations. For example, the liquidity flow inclusion leads to larger bubbles and deeper recessions in the real sector after the market crashes.

4.4. Dynamics Robustness

To check the robustness of the model dynamics, I simulate the joint model changing values of each parameter related to the agent-based financial market that can affect statistical characteristics of the market price of capital by 10%. The statistical characteristics of the market price of capital remain the same for each 10% change of a parameter when other parameters are fixed.

Figure 4. Joint Model Dynamics



Notes: The red and blue lines show the dynamics of variables in the joint model with and without, respectively, the liquidity flow from the real sector to the financial market,

5. SHOULD THE CENTRAL BANK PRICK BUBBLES?

The effect of pricking bubbles on the economy depends on two key parameters: the threshold size of the bubble at which the central bank starts raising the interest rate beyond the necessary reaction determined by the standard Taylor rule in each quarter, $levCB$, and the effectiveness of the central bank's communication policy, $CPolicy$. Figure 5 shows a possible effect of pricking bubbles on the economy for $levCB = 1$ and

$CPolicy = 500$ over 1040 weeks for the same realisation of random shocks used for model simulations in Section 4. The red and blue lines show the dynamics of variables in the joint model with and without, respectively, pricking bubbles,. In Figure 5, we can see that when the central bank pricks bubbles, the dynamics of the market price is substantially different from the case in which it does not. The highest market price values are lower in the case of pricking bubbles, and the deviations of the market price from the fundamental price are smaller. This leads to the difference in the dynamics of the variables in the real sector; the central bank's actions related to pricking bubbles reduce the decrease in output, consumption, and households' utility caused by the bubbles bursting.

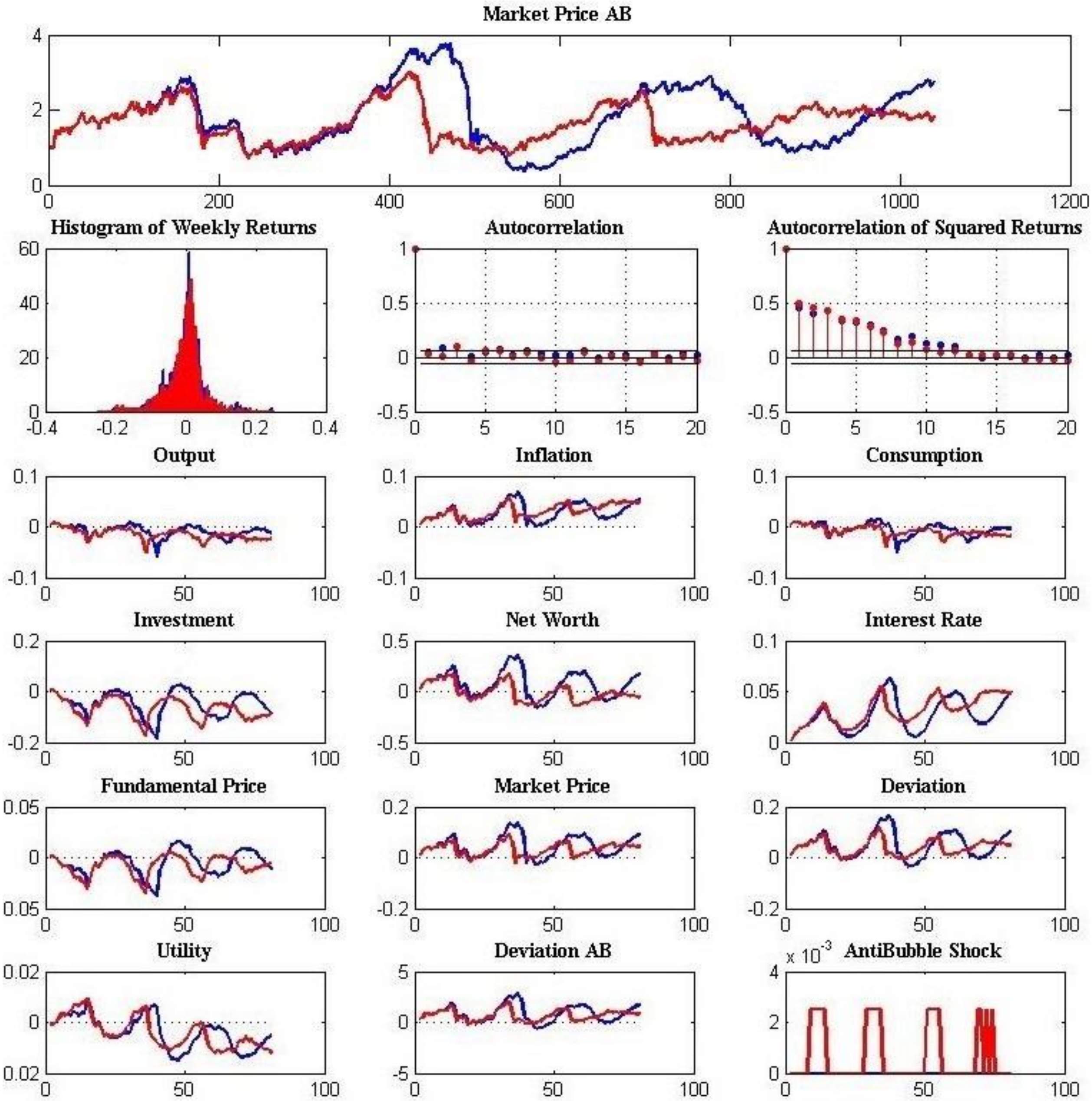
To explore whether and how the central bank should prick bubbles, I calculate the effect of pricking bubbles on social welfare and financial stability under several combinations of $levCB$ and $CPolicy$. I use the following levels for the threshold size of the bubble: $levCB \in [1; 1.2; 1.4; 1.6; 1.8; 2]$. I do not consider larger values for $levCB$ because bubbles of $levCB > 2$ occur quite rarely, only in some infrequent realisations of random shocks. A lower level of $levCB$ does not seem realistic, as in this case the central bank will prick bubbles more often than will not. I consider several non-negative values for the effectiveness of the central bank's communication policy: $CPolicy \in [0; 200; 400;\ 600; 800; 1000]$. For $CPolicy$ values that are significantly larger than 1000, the speed of decrease in the market price after the bursting of the bubbles is too fast and does not correspond to the real data.

For each combination of the parameters $levCB$ and $CPolicy$, I calculate changes in social welfare caused by volatile dynamics in the financial market during the considered period. Specifically, I compute changes in households' welfare as discounted differences between the utility of households in each period and the utility of households in the steady state, divided by the consumption of households in the steady state:

$$W = \sum_{t=0}^{T} \beta^t \left(\frac{U_t - \bar{U}}{\bar{C}}\right), \quad (39)$$

where $\mathrm{U_t}$ is the utility of households at time t; $\bar{U}$ denotes the value of the steady-state utility of households.

Figure 5. Model Dynamics with and without Pricking Bubbles



Notes: The blue lines show the dynamics of variables in the case without pricking bubbles while the red lines show the same information in the case when the central bank pricks bubbles with the values of the threshold size $levCB = 1$ *and the effectiveness of the central bank's communication policy* $CPpolicy = 500$.

Schmitt-Grohe and Uribe (2004) show that for the welfare analysis, it is necessary to use the second-order approximation of the welfare function:

$$U_t = \bar{U} + \frac{1}{\bar{C}}(C_t - \bar{C}) - \bar{L}^{\sigma_l}(L_t - \bar{L}) - \frac{1}{\bar{C}^2}\frac{(C_t - \bar{C})^2}{2} - \sigma_l \bar{L}^{\sigma_l - 1}\frac{(L_t - \bar{L})^2}{2}$$

$$= \bar{U} + c_t - \bar{L}^{\sigma_l + 1} l_t - \frac{{c_t}^2}{2} - \sigma_l \bar{L}^{\sigma_l + 1}\frac{{l_t}^2}{2}, \quad (40)$$

where $c_t = \frac{C_t - \bar{C}}{\bar{C}}$ and $l_t = \frac{L_t - \bar{L}}{\bar{L}}$ are the deviations of consumption and labour from their steady-state values $\bar{C}$ and $\bar{L}$ at time t, respectively. Using (39) and (40), we can get:

$$W = \sum_{t=0}^{T} \beta^t \left(\frac{1}{\bar{C}} c_t - \frac{\bar{L}^{\sigma_l+1}}{\bar{C}} l_t - \frac{1}{2\bar{C}} {c_t}^2 - \frac{\sigma_l \bar{L}^{\sigma_l+1}}{2\bar{C}} {l_t}^2 \right) \quad (41)$$

In addition to changes in households' welfare, I also calculate relative changes in the volatility of output, $\Delta Var_{levCB,CPolicy}(y)$, and in the volatility of inflation, $\Delta Var_{levCB,CPolicy}(\pi)$, for given values of $levCB$ and $CPolicy$ over the considered period compared to the case without pricking bubbles:

$$\Delta Var_{levCB,CPolicy}(y) = \frac{Var_{levCB,CPolicy}(y) - Var_{wp}(y)}{Var_{wp}(y)} \quad (42)$$

$$\Delta Var_{levCB,CPolicy}(\pi) = \frac{Var_{levCB,CPolicy}(\pi) - Var_{wp}(\pi)}{Var_{wp}(\pi)}, \quad (43)$$

where the subscript wp denotes the case without pricking bubbles while the subscript $levCB, CPolicy$ represents the case with pricking bubbles for given values of $levCB$ and $CPolicy$ and for the same realisation of random shock as in the case without pricking bubbles. The volatility of output and the volatility of inflation are frequently used in the literature as variables in the central bank's loss function and as indicators of financial stability.

As already mentioned, the dynamics of the model depends on the realisation of random shocks for the assigned period of 1040 weeks, and it is different for different realisations of random shocks, although each realisation corresponds to the stylised facts discussed in Section 3. To compare the values of changes in households' welfare, the volatility of output, and the volatility of inflation for the different values of the parameters $levCB$ and $CPolicy$, I calculate the average changes in these indicators for 200 realisations. Table 1 reports the results for different combinations of the parameters $levCB$ and $CPolicy$ for the joint model without the liquidity flow from the real sector to the financial market (in this configuration of the joint model, $sen_2 = 0$). Meanwhile, Table 2 reports the same information for the joint model with the liquidity flow (in this configuration of the joint model, $sen_2 = 0.075$).

Table 1. Average Changes in Households' Welfare, the Volatility of Output and Inflation for the Case without the Liquidity Flow from the Real Sector to the Financial Market

$sen_2 = 0$						
	$\Delta W_{average}$,%					
$levCB$	1	1.2	1.4	1.6	1.8	2
$CPolicy = 0$	-6.09***	-5.07***	-4.10***	-3.10***	-2.22***	-1.62***
$CPolicy = 200$	-3.68***	-2.79***	-2.08***	-1.46***	-1.07***	-0.69***
$CPolicy = 400$	-1.96***	-1.15***	-0.62***	-0.27***	-0.11	0.06
$CPolicy = 600$	-0.73***	-0.01	0.42***	0.62***	0.61***	0.66***
$CPolicy = 800$	0.27	0.86***	1.14***	1.30***	1.11***	1.15***
$CPolicy = 1000$	1.21***	1.63***	1.78***	1.76***	1.61***	1.59***
	$\Delta Var_{average}(y)$, %					
$levCB$	1	1.2	1.4	1.6	1.8	2
$CPolicy = 0$	96.67***	74.68***	53.01***	35.86***	23.05***	15.24***
$CPolicy = 200$	64.99***	45.87***	28.93***	18.50***	11.13***	6.89***
$CPolicy = 400$	42.44***	25.17***	13.51***	6.66***	2.89***	1.68***
$CPolicy = 600$	24.11***	10.30***	1.83	-1.65	-2.99***	-2.27***
$CPolicy = 800$	9.27***	-1.33	-6.29***	-7.69***	-6.94***	-4.76***
$CPolicy = 1000$	-2.47	-9.99***	-12.82***	-12.44***	-9.36***	-6.33***
	$\Delta Var_{average}(\pi)$, %					
$levCB$	1	1.2	1.4	1.6	1.8	2
$CPolicy = 0$	80.24***	53.91***	31.55***	16.64***	8.03***	3.44***
$CPolicy = 200$	46.16***	24.47***	8.68***	0.72	-1.96***	-2.88***
$CPolicy = 400$	23.36***	4.80***	-5.69***	-9.91***	-9.51***	-7.41***
$CPolicy = 600$	5.39***	-9.12***	-15.83***	-17.24***	-13.64***	-10.45***
$CPolicy = 800$	-8.41***	-19.57***	-23.25***	-20.96***	-16.68***	-12.46***
$CPolicy = 1000$	-18.67***	-27.21***	-28.91***	-25.30***	-18.97***	-13.94***

Notes: ***, **, and * indicate significance at the 1%, 5%, and 10% levels, respectively.

In Table 1, we can see that the central bank's actions related to pricking bubbles in the joint model with zero effectiveness of the communication policy ($CPolicy = 0$) lead to welfare losses and to the growth of average output and inflation volatilities. In this case, the central bank only raises the interest rate by $\Delta = 0.25\%$ beyond the Taylor rule in each quarter until the bubble bursts without affecting traders' opinions. The gains from pricking bubbles are typically reflected in smaller decreases, for example, in output and consumption after the bursting of bubbles, but if $CPolicy = 0$, these gains are lower than the losses from raising the interest rate, which slows down the economy. Moreover, without an effective communication policy, an increasing interest rate in some realisations of random shocks is unable to prick bubbles at all. However, the growth of the effectiveness of communication policy causes average welfare losses to decrease for any value of $levCB$. When $CPolicy = 1000$, the central bank's actions related to pricking bubbles lead to the highest average welfare gains for all values of $levCB$ with the maximum at 1.78% of the steady-state consumption level in the case where $levCB = 1.4$. Similar positive results are observed for the average volatility of output and inflation; the maximum reductions of average volatilities occur in the case where $levCB = 1.4$ and have the following values: -12.82% for the average volatility of output and -28.91% for the average volatility of inflation.

According to Table 2, the results in the case where the joint model includes the endogenous liquidity flow from the real sector to the financial market are approximately the same. Compared to Table 1, the maximum value of average welfare gains caused by pricking bubbles is 4.04% of the steady-state consumption, this value is achieved where $levCB = 1.6$ and $CPolicy = 1000$. In addition, the maximum values of the decrease in the average volatilities of output and inflation are equal -38.93% and -54.04% respectively. These values are obtained in the case where $levCB = 1.2$. It is worth noting that the maximum average welfare gains from pricking bubbles in the case with the endogenous liquidity flow are approximately 2.3 times larger than in the case without it. The corresponding maximum decreases in the average volatility of output and inflation are approximately 3 and 1.9 times larger respectively.

In sum, the results of the analysis demonstrate that pricking asset price bubbles can enhance social welfare as well as reduce the volatility of output and inflation. This positive effect is greater when asset price bubbles are partially caused by the liquidity flow from the real sector to the financial market, or when the central bank implements an effective communication policy to prick the bubble. However, pricking asset price bubbles only by raising the interest rate without an effective communication policy leads to negative consequences for social welfare and financial stability. Thus, the effect of pricking asset price bubbles through the interest rate policy is significantly dependent on the effectiveness of the central bank's communication policy.

Table 2. Average Changes in Households' Welfare, the Volatility of Output and Inflation for the Case with the Liquidity Flow from the Real Sector to the Financial Market

$sen_2 = 0.075$						
	$\Delta W_{average}$, %					
$levCB$	1	1.2	1.4	1.6	1.8	2
$CPolicy = 0$	-5.51***	-4.80***	-4.18***	-3.53***	-2.98***	-2.42***
$CPolicy = 200$	-2.48***	-1.84***	-1.22***	-0.80***	-0.51***	-0.36**
$CPolicy = 400$	-0.31	0.43***	0.90***	1.27***	1.39***	1.31***
$CPolicy = 600$	1.14***	1.79***	2.22***	2.43***	2.49***	2.34***
$CPolicy = 800$	2.30***	2.83***	3.13***	3.26***	3.42***	3.08***
$CPolicy = 1000$	3.23***	3.63***	3.74***	4.04***	3.89***	3.74***
	$\Delta Var_{average}(y)$, %					
$levCB$	1	1.2	1.4	1.6	1.8	2
$CPolicy = 0$	33.74***	27.31***	22.58***	18.77***	15.05***	12.15***
$CPolicy = 200$	10.40***	5.53***	1.29	-0.56	-1.73	-1.16
$CPolicy = 400$	-6.39***	-10.53***	-12.23***	-12.90***	-12.44***	-11.71***
$CPolicy = 600$	-19.48***	-23.09***	-22.95***	-22.23***	-19.42***	-17.75***
$CPolicy = 800$	-29.84***	-32.01***	-30.60***	-28.45***	-25.87***	-21.74***
$CPolicy = 1000$	-37.08***	-38.93***	-37.37***	-33.89***	-30.39***	-25.47***
	$\Delta Var_{average}(\pi)$, %					
$levCB$	1	1.2	1.4	1.6	1.8	2
$CPolicy = 0$	13.54***	5.79***	0.98	-2.17***	-4.05***	-4.88***
$CPolicy = 200$	-10.32***	-15.86***	-19.27***	-19.73***	-19.03***	-17.00***
$CPolicy = 400$	-25.92***	-30.73***	-31.94***	-30.16***	-27.98***	-25.08***
$CPolicy = 600$	-37.08***	-41.09***	-40.65***	-37.99***	-34.12***	-30.10***
$CPolicy = 800$	-45.08***	-47.92***	-47.01***	-43.42***	-38.83***	-33.69***
$CPolicy = 1000$	-51.55***	-54.04***	-52.39***	-47.71***	-42.45***	-36.79***

Notes: ***, **, and * indicate significance at the 1%, 5%, and 10% levels, respectively.

6. CONCLUSION

This paper contributes to the existing literature by studying whether and how the central bank should prick asset price bubbles, if the effect of interest rate policy on these bubbles greatly varies across periods, and also if the central bank should start pricking a bubble when it has already grown to a significant size. For this purpose, I employ a novel theoretical framework based on the integration of an agent-based financial market in a financial accelerator model with a bubble in the spirit of Bernanke and Gertler (2000). The proposed framework can endogenously generate bubbles in the financial market. A bubble burst may lead to negative consequences in the real sector of the economy, such as a decline in consumption, investment, and output. The central bank may try to prick bubbles by two policies: the interest rate policy and the communication policy – for example, by verbal interventions.

To explore whether and how the central bank should prick asset price bubbles, I calculate the effect of several interest rate rules for pricking bubbles that have a piecewise form on social welfare and financial stability. The results demonstrate that pricking asset price bubbles can enhance social welfare as well as reduce the volatility of output and of inflation. This positive effect is greater, when asset price bubbles are caused by credit expansion, or when the central bank implements an effective communication policy to prick the bubble. However, pricking asset price bubbles only by raising the interest rate without an effective communication policy leads to negative consequences for social welfare and financial stability because an increasing interest rate, in this case, may not burst asset price bubbles but slow down the economy.

Future research may add macroprudential policy to the analysis of pricking asset price bubbles. For example, it is worth investigating the interaction of monetary and macroprudential policies aimed at pricking bubbles and accompanied by regulators' communications with market players. Besides, researchers could calibrate the model to other advanced and emerging market economies. Such work could reveal the extent to which the results obtained in this paper are applicable in other countries and could be used by policymakers.

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APPENDIX A. LOG-LINEARISED REAL SECTOR

$$\lambda_t = -c_t \quad \text{(A1)}$$

$$\lambda_t + \pi_{t+1} = \lambda_{t+1} + r_t \quad \text{(A2)}$$

$$y_t = \alpha * k_{t-1} + (1-\alpha) * \Omega * l_t \quad \text{(A3)}$$

$$s_t = y_t + mc_t - l_t \quad \text{(A4)}$$

$$r_t^k = y_t + mc_t - k_{t-1} \quad \text{(A5)}$$

$$q_{t+1} = \chi(i_t - k_{t-1}) \quad \text{(A6)}$$

$$\beta\pi_{t+1} = \pi_t - \frac{(1-\beta\theta_p)(1-\theta_p)}{\theta_p} mc_t \quad \text{(A7)}$$

$$c_t^e = f_t + k_t \quad \text{(A8)}$$

$$\frac{n_t}{\upsilon * \overline{R^F}} = \frac{\bar{K}}{\bar{N}} * r_t^F - \left(\frac{\bar{K}}{\bar{N}} - 1\right)(r_t - \pi_t) - \psi\left(\frac{\bar{K}}{\bar{N}} - 1\right)(k_{t-1} + f_{t-1}) + \left(\psi\left(\frac{\bar{K}}{\bar{N}} - 1\right) + 1\right) n_{t-1} \quad \text{(A9)}$$

$$b_t = \frac{\bar{K}}{\bar{B}}(q_t + k_t) - \frac{\bar{N}}{\bar{B}} n_t \quad \text{(A10)}$$

$$y_t = c_t^e \frac{\overline{C_e}}{\bar{Y}} + c_t \frac{\bar{C}}{\bar{Y}} + i_t \frac{\bar{I}}{\bar{Y}} + g_t \frac{\bar{G}}{\bar{Y}} \quad \text{(A11)}$$

$$r_t^F = r_t + \psi * (f_t + k_t - n_t) - \pi_{t+1} \quad \text{(A12)}$$

$$r_t = \rho_r r_{t-1} + (1-\rho_r)(\rho_\pi \pi_t + \rho_y y_t) + \Delta r_t^{Bubble} \quad \text{(A13)}$$

$$f_t - q_t = f_{t-1} - q_{t-1} + \tau_t^F \quad \text{(A14)}$$

APPENDIX B. MODEL PARAMETERS

Table B1. Calibrated Model Parameters.

Real sector		Agent-based financial market	
Parameters	Values	Parameters	Values
β	0.99	H	10000
σ_l	1	$\overline{cash}$	1
α	0.35	$\overline{futures}$	1
Ω	0.99	$\underline{\Omega}$	40
A	1	X_1	15
υ	0.9728	X_2	20
lev	2	X_3	1
δ	0.025	LR	0.6
χ	0.25	$trend$	1.2
θ_p	0.75	σ^{FU}	2
ϵ_y	6	$\xi 1$	3
$\overline{R^F} - \bar{R}$	0.02	$\xi 2$	750
ρ_r	0.7	m	12
ρ_π	1.1	$fraction$	0.02
ρ_y	0.2	λ	0.05
Δ	0.0025	sen_1	0.06
ψ	0.02	sen_2	0.075