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Optimal simple monetary policy rules for a resource-rich economy and the Zero Lower Bound

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Abstract

In this article, we study the optimal simple monetary policy rules under a Zero Lower Bound (ZLB) using a DSGE model. The modeled economy is open and highly dependent on the terms of trade (TOT). Economic dynamics is the result of a TOT shock and an external interest rate shock. Using impulse response functions, we show that the presence of the ZLB reduces the impact of positive external shocks. This means greater growth in real interest rates and lesser growth in consumption and production. The monetary authority minimizes the volatility of key macroeconomic indicators. The optimal parameters for the rule turn out to be such that the regulator de facto reduces the probability of being at the ZLB. At the ZLB, the regulator is less responsive to inflation changes, and the interest rate is more persistent. In the case of Russia, we have got low probability estimate of hitting the ZLB under the current monetary policy and a long-term value of the interest rate of 6%. The gap reaction parameter and interest rate persistence parameter for the current monetary policy are in the range of values for optimal monetary policy rules. The current CPI reaction parameter is much less than the optimal one. This implies a higher probability of hitting the ZLB in the optimum than under the current monetary policy. We also found that under current monetary policy, the likelihood of reaching the effective lower bound (ELB), defined by the alternative households' ability to save, is quite significant.

Keywords: DSGE models, zero lower bound, nonlinear models, optimal policy, monetary policy, terms of trade

JEL: D58, E32, E52, E58.

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1. Introduction

The zero lower bound (ZLB) of nominal interest rates is a natural constraint that monetary authority has to take into account. In developed economies, when negative shocks lead to a slowdown in price and output growth, central banks usually lower nominal interest rates in order to stimulate economic activity. In the event of major negative shocks or a prolonged sequence of minor negative shocks, the rate may reach its zero bound and remain there for a long time, thus limiting the ability of central banks to influence aggregate demand and stabilize the business cycle.

The ZLB problem was faced by Japan in the late 1990s, the US and, to a lesser extent, Europe during and after the 2008 global financial crisis. This problem has inspired numerous studies on developing economic policy measures that reduce the risks of reaching the zero lower bound of interest rates. Both options to raise the inflation target and to modify monetary policy (MP) rules are being discussed.

For example, Coibion et al. (2012), using a model calibrated with data from developed economies, estimated that the optimal inflation target under rare and costly episodes of reaching the ZLB would not exceed 2% per year. On the other hand, Williams (2009) concluded that the 2% target level for the US economy was an insufficient buffer. Eggertsson (2011) noted that when an economy is at the ZLB, a trusted regulator finds it beneficial to promise to raise the inflation target after entering the positive rate area: this raises both expected future inflation and current inflation, leading to a drop in real interest rates and an increase in current output. It turns out to be profitable to break the promise after exiting the zero-rate regime. A discretionary policy regulator with no credibility would not have access to such a policy. Schmidt (2013) showed that when the regulator is credible and able to implement a policy under commitment, the optimal policy leads to fewer losses for the economy from reaching the ZLB than a discretionary policy. At the same time fiscal policy has little impact on changes in public welfare, as monetary policy itself significantly reduces losses from zero interest rates. If the regulator is only able to pursue a discretionary policy, then the losses from being at the ZLB are higher with fiscal policy helping effectively reduce these losses.

For resource-rich countries, there may be specific features in monetary policy under the ZLB due to differences in monetary policy in resource-rich and developed countries. In non-export-oriented countries, monetary policy is countercyclical: when negative shocks lead to lower output and inflation, the regulator cuts the interest rate, supporting the economy. In export-oriented countries, when implementing MP in accordance with the Taylor Rule, the regulator lowers the interest rate when there is a positive shock to the terms of trade (TOT) in order to offset the declining inflation on the back of exchange rate appreciation. At the same time, low rates additionally accelerate output, which makes MP procyclical to output. For both developed and resource-rich countries, being at the zero lower bound of rates leads to an additional drop in output due to the limited effect of the MP. However, in developed countries, being on the ZLB increases the volatility of output, since an additional drop in output occurs during a recession, while in resource-rich countries, the ZLB reduces volatility, since an additional drop occurs during a boom. Therefore, the recipes for regulators in ZLB conditions in export-oriented countries may differ qualitatively and quantitatively from those in other countries. In this article, we do not consider the differences between the optimal monetary policy rules for resource-rich countries and developed countries. However, we further consider a highly export-dependent economy, and economic dynamics are the result of external shocks, so we can say that the results obtained are typical for resource-rich countries.

Historical episodes of the exporting economy being at the ZLB are rare. An example would be Chile 2010 (Cespedes et al. (2014)). The episode is characterized by low inflation and by a drop in rates to almost zero during the copper price rally. At the same time, commodity prices have been quite volatile in recent decades. Understanding the reasons why resource-rich economies rarely find themselves on ZLB amid high commodity price volatility is one of the motivations for this work.

In Russia, which is an export-oriented country, trend inflation has been declining over the past two decades, reaching and even falling below 4%, which is in line with the Bank of Russia’s inflation target. This has increased the likelihood of the Russian economy reaching the ZLB. Given this, it would be relevant to analyse how the increased probability of reaching the ZLB could be reflected in the Bank of Russia’s MP. The relevance of this study is also related to the fact that the Russian economy may face the ineffectiveness of monetary policy earlier than at zero interest rates (Section 7), which is known as the effective lower bound (ELB) problem.

To study the ZLB problem for resource-rich countries, we have constructed a dynamic stochastic general equilibrium (DSGE) model for a small open export-oriented economy (Section 2, Appendix). The model is calibrated in a manner typical for export-oriented economies. A number of parameters and shocks were estimated using Russian data (Section 3). The impact of the ZLB on an export-oriented economy is analyzed with the use of impulse response functions (Section 4). The monetary authority’s objective is presented in Section 5. The optimal parameters of the Taylor Rule are presented and discussed in Section 6. In section 7 we discuss the likelihood of the Russian economy to hit the ZLB and ELB.

2. DSGE model and the zero lower bound

To investigate the MP rules, we constructed a fairly standard DSGE model for export-oriented economies, including a description of non-commodity (non-traded) and commodity (traded) goods production. Manufacturers use labor and capital to produce both types of goods, while ‘land’, a special production factor, is also used to produce traded goods. Land reflects lower labor and capital costs in the exporting sector, and the cost of renting land is paid to households. The proceeds from commodity exports first go to the manufacturer of the export commodity and are spent to pay for the three production factors: the rent of the capital, labor and ‘land’ of households. Non-traded goods combined with imported products are converted into the final product spent on consumption and investment.

As is standard for DSGE models, households consume the product, offer labor to manufacturers, save money in the form of foreign bonds, and decide on the amount of productive capital to lease to manufacturers.

The model implements a New Keynesian approach: some markets are assumed to function inefficiently. Imperfections utilised in this model to reflect inefficiencies include intermediate domestic goods price rigidity, wages rigidity, household consumption habits, the cost of changing the level of investment, the cost of changing the amount of hired labor in the exporting sector, and the cost of changing the level of investment in foreign bonds.

The monetary policy authority follows Taylor's instrumental rule:

$$R_t^l - R^{l,ss} = \rho_r \left(R_{t-1}^l - R^{l,ss}\right) + \left(1-\rho_r\right)\left(\rho_{\inf}\left(\pi_t - \pi^{ss}\right) + \rho_y \left(GDP_t / GDP_t^{pot} - 1\right)\right) \quad , \qquad (1)$$

where R_t^l is the nominal interest rate, π_t is inflation, GDP_t is the total output of the exporting and domestic sectors, $R^{l,ss}, \pi^{ss}$ are the values of interest rates and inflation in the long-term equilibrium, and GDP_t^{pot} is potential output, or output in the economy with flexible prices and wages.

We provide a more detailed description of the model's equations in the Appendix. Note that this model differs from the one described in Andreev, Polbin (2019) in the absence of an entrepreneurial agent responsible for monetary relations and implementing a financial accelerator mechanism, as well as the presence of a fictitious credit market between households. The presence of an entrepreneurial agent does not bring anything important to this study and is not necessary.

In order to account for a zero lower bound of the interest rate, we assume that there is an additional constraint in the model

$$R_t^l \geq 0 \quad . \qquad (2)$$

This constraint means that at any time in any environmental state (realisation of the stochastic trajectory) either the relation (1) and the inequality (2) apply strictly, or

$$R_t^l = 0. \qquad (3)$$

In the case of (3), inflation is said not to be targeted by the monetary policy authority but determined by freely acting market forces.

A model where inequality (2) is periodically triggered is non-linear. There is no single recognised method for solving stochastic models with inequalities. However, these methods have been actively developed in recent years (e.g. see Holden (2016), and Binning, Maih (2017)). Hereinafter, we will use the OccBin toolkit developed by Guerrieri and Iacoviello (2015) to find solutions to dynamic stochastic models with such constraints as inequalities. This tool searches for piecewise smooth solutions to dynamic stochastic models with inequalities and uses Matlab with Dynare.

Many commodity exporting countries, including Russia, use fiscal rules. *Andreyev* (2020) demonstrated that the fiscal rule that smooths cyclical part of oil revenues changes the scale of economy reaction in response to the terms of trade shock. However, we do not apply the budget rule in the model concerned. Since the smoothing of the cyclical part of the oil income in Russia has been happening since 2004, and we use data from approximately the same period to calibrate the model, we implicitly take into account the presence of a budget rule. The quality of the calibration (Section 3) indirectly confirms that the key mechanisms in the modeling are not overlooked.

The DSGE model is described in more detail in the Appendix.

3. Economic shocks and calibration of the model

In the academic literature, terms of trade shocks and foreign borrowing interest rate shocks are considered the key determinants of the business cycle for export-oriented economies. In our study, we decided to focus on these two shocks and leave the other types of shocks for future research.

A terms of trade shock ε_t^{TOT} determines the trend in the terms of trade $\tilde{P}_t^{res}$ – the relationship between the price of exported (commodity) and imported goods – in line with the AR(1) process:

$$\ln\left(\tilde{P}_t^{res}\right) = \rho_{res}\ln\left(\tilde{P}_{t-1}^{res}\right) + \varepsilon_t^{TOT} \tag{4}$$

A foreign interest rate shock ε_t^{prem} sets the trend for the foreign interest rate R_t^f according to the equation:

$$R_t^f = \rho_{prem}R_{t-1}^f + \left(1-\rho_{prem}\right)R^{f,ss} + \varepsilon_t^{prem} \tag{5}$$

Estimating the standard deviations and autocorrelations of these shocks is an important stage of the study, as this procedure evaluates the probability of the economy reaching the lower bound of interest rates as well as the relative magnitude of the shocks. Therefore, we estimated the autocorrelation of the logarithm of the terms of trade ρ_{res}, the variation of the terms of trade shock

ε_t^{TOT} (see (4)), the autocorrelation of the foreign interest rate ρ_{prem}, and the variation of the foreign interest rate shock ε_t^{prem} (see (5)) using statistical data. We chose Russia as the representative country. Since oil, petroleum products, gas, and liquefied natural gas comprised an average of 60% of Russian merchandise exports from 2000 to 2019, we chose the oil price as the terms of trade. Using quarterly real Brent crude oil price data for the period from 1995 to 2019, we estimated ρ_{res} at 0.956, and the standard deviation ε_t^{TOT} at 0.144. Foreign borrowing rate R_t^f was estimated in a manner similar to Garcia-Cicco, Garcia-Schmidt (2020), Huidrom, et al. (2020) as the sum of the 3-year US Treasury bond yield and the Russian bond yield spread as calculated by JP Morgan.[1] The choice of the period used for the foreign interest rate is a separate issue in the literature. For instance, the standard deviation of the shock ε_t^{prem} is 1.1% for the period starting from 1998 and including the event of default on government bonds, and 0.17% for the period starting from 2003. Since a default is unlikely under current economic conditions, although it cannot be completely ruled out, we set the standard deviation ε_t^{prem} at 0.34%. The parameter ρ_{prem} is calibrated at 0.84, which corresponds to the LSM estimation for the period.

We calibrated the model's structural parameters and relationships in a standard manner for DSGE models and export-oriented economies.

We assumed the time preference coefficient $\beta = 0.995$. Capital depreciation rates for both producing sectors according to Motto, et al. (2010) were considered equal $\delta = 0.02$. This corresponds to an 8% annual disposal rate of productive capital. Standard parameters for the elasticity of the production function for the non-traded sector were chosen (see, e.g., Bernanke et al. (1999)): $\alpha_d = 0.35, 1-\alpha_d = 0.65$. The proportion of 'land' factor costs in the output of traded goods was set at 25%[2], and the costs of the remaining factors (capital and labor) were distributed in the ratio of $0.35/0.65$. Since the ratio of exports to GDP varies over a wide range from 0.078 to 0.64 for exporting economies (see Benkhodja (2014)), we took the average value $Y_t^{ex}/GDP_t = 0.25$ which corresponds to the Russian economy. Imports were considered equal to exports in the long-term equilibrium. The parameter for the cost of change in the volume of hired labor w^{ex} was taken approximately at the level of Dib (2003), Ambler et al. (2012). Other rigidity parameters were taken from Drobyshevsky and Polbin (2015).

We chose the model's long-run inflation value based on the inflation targets of export-oriented countries. The statistics show[3] that the annual inflation targets for many exporting countries (e.g.

[1] JP Morgan's EMBI JPSSEMRU Index
[2] If we consider the 'land' production factor cost as an additional tax levied on the exporting sector, the chosen normalisation of 25% corresponds to the difference between the tax burden of 55% on the exporting sector and 30% on the domestic sector, which is comparable to the Russian economy.
[3] http://www.centralbanknews.info/p/inflation-targets.html

Chile, Mexico, Norway, and Russia) lie in the range of 2% to 4% p.a., and that the value of 4% p.a. lies within the range of many central bank targets. In this regard, we have chosen two divergent values of long-term inflation for the study: 0% p.a. as the minimum possible target and 4% as a representative target near the upper limit for export-oriented countries. By setting the long-term real annual interest rate at 2%, we obtained two long-term nominal interest rate values for our study: 2% and 6% p.a.. We note that the nominal interest rate calibration along with the shock magnitude calibration are among the key factors behind the probability of the economy reaching the zero bound of nominal interest rates.

In order to test the adequacy of the model calibration, we tried to determine how well the model was able to describe the 2014–2015 crisis. To set up this experiment, we need to complete the model and describe the actual monetary policy. For this purpose, we estimated a simple Taylor Rule with the nominal interest rate as a function of the interest rate lag, current inflation, and the current indicator of economic activity. The model was estimated for the period from 2010 to 2019, a period with a relatively flexible exchange-rate regime compared to the period prior to the 2008–2009 crisis, when the Bank of Russia had maintained a managed ruble exchange rate regime. The MIACR rate on overnight interbank loans was chosen as the interest rate, while the growth rate of the consumer price index, seasonally adjusted using the ARIMA-X12 filter in Eviews, was chosen as the inflation rate. We used two alternative measures as the indicator of business activity: the deviation of real GDP from the potential level obtained using the Hodrick-Prescott filter and the growth rate of real GDP. In the regression under consideration, both indicators of economic activity turned out to be statistically insignificant. Therefore, we have settled on a simple specification where the interest rate depends on the interest rate lag and current inflation. The regression equation was estimated using the least squares method, which may produce shifted estimates if monetary policy shocks affect inflation at the same point in time. However, we abstract from this potential problem, as the experiment under consideration is approximate by nature, and an accurate assessment of the monetary policy parameters for the Bank of Russia is not the purpose of this paper. The parameter estimates are shown in Table 1. According to the obtained estimates, the coefficients for the dependence of the current interest rate on its lag and the degree of response to inflation were equal to $\rho_r = 0.902$ and $\rho_{\text{inf}} = 1.49$.

Table 1 – Results of LSM estimation of Taylor Rule equation for Russian data from 2010 to 2019

Variable	Multiplier	Standard deviation	t-statistic	p-value
Constant	-0.0004	0.001	-0.330	0.743
MIACR interest rate, quarterly, 1 quarter lag	0.902	0.043	20.899	0.000

Inflation, QoQ, seasonally adjusted	1.490	0.260	5.586	0.000
Observation period	2010Q2-2019Q4			
Number of observations	39			
R^2	0.951			
F-statistic	225.724			
Probability (F-statistic)	0.000			

Overall, there are many examples of a successful econometric estimation of DSGE model parameters for the Russian economy and filtering unobserved shocks (see, for example, Ivashchenko (2013), Shulgin (2014), Malakhovskaya (2016), and Kreptsev, Seleznyov (2018)). In this paper, we will follow a simpler approach: based on the calibrated model and on the observed trajectories of external variables, we run simulations of the model and analyse how well it explains the actual dynamics.

Namely, we first identified the terms of trade shocks ε_t^{TOT} and foreign interest rate shocks ε_t^{prem} based on the historic oil price and foreign interest rate series above. Using two series of shocks, we simulated the GDP series based on the model and compared it with the actual observed statistics (figure 1). It turned out that the GDP series reproduced by the model qualitatively coincides with its actual trends.

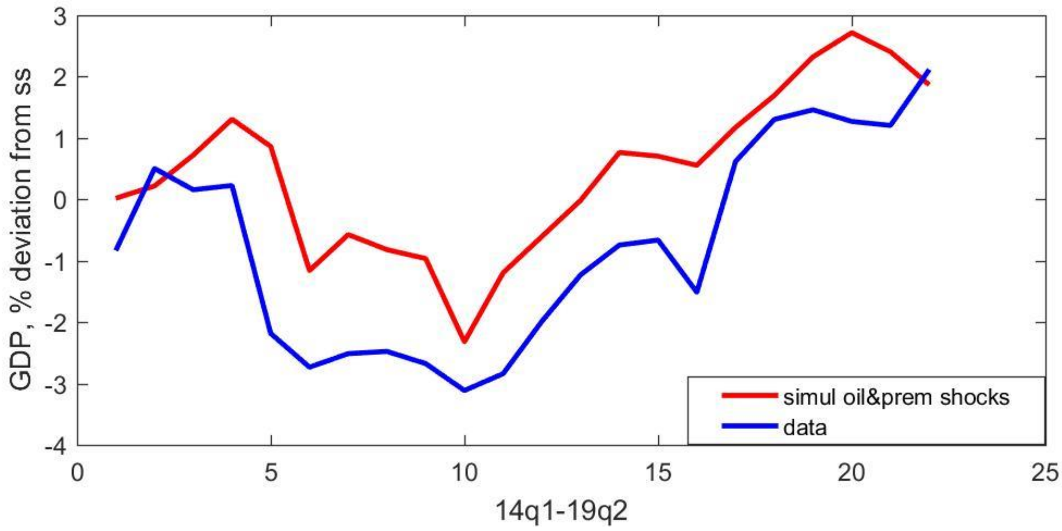

Figure 1 – The GDP data series as a % deviation from the trend (HP filter with a parameter value of 1600) and the GDP reproduced by the model as a % deviation from the long-term trend with a constant trend of 1% per year

Second, in the following test, we compared the correlation matrix of the main model variables with the correlations observed in the statistics (Table 2), as well as standard deviations (Table 3).

Almost all correlations coincide with respect to their sign (positive or negative), and most of the pairs are close in value.

Table 2 – Correlation matrix of the main model variables. Data and simulations. The long-term trend was subtracted from the data (HP filter with a parameter value of 1600). For the period from 2014 Q1 to 2019 Q3

Data/simulations	Real GDP growth rate	Households' real consumption growth rate	Oil price growth rate	MIACR rate	Inflation
Real GDP growth rate	1/1	0.94/0.96	0.81/0.69	-0.44/-0.78	0.16/-0.05
Households' real consumption growth rate	0.94/0.96	1/1	0.82/0.85	-0.38/-0.86	0.17/-0.14
Oil price growth rate	0.81/0.69	0.82/0.85	1/1	-0.61/-0.75	-0.08/-0.21
MIACR rate	-0.44/-0.78	-0.38/-0.86	-0.61/-0.75	1/1	0.62/0.44
Inflation	0.16/-0.05	0.17/-0.14	-0.08/-0.21	0.62/0.44	1/1

Table 3 – Standard deviations of the main model variables. Data and simulations.

	Data, 1999 Q2 to 2019 Q3	Data, 2010 Q1 to 2019 Q3	Data, 2014 Q1 to 2019 Q3	Simulations
Real GDP growth rate	1,4%	0,9%	0,9%	0,7%
Households' real consumption growth rate	1,9%	1,9%	2,0%	1,7%
Oil price growth rate	14,3%	12,9%	14,4%	14,5%
MIACR rate deviation from long-term trend	2,4%	1,7%	1,9%	1,1%
Inflation deviation from long-term trend	1,3%	1,1%	1,3%	0,9%

The results of both tests indicate that the model calibration is adequate.

4. The negative impact of the economy being at the ZLB

The nominal interest rate reaching the zero lower bound results in the regulator being unable to lower it further and facing limitations in the available responses to changes in the business cycle. Inflation is shaped by free market forces, which leads to its excessive volatility and harms the economy.

If the monetary authority follows the standard Taylor rule

$$R_t^l - R^{l,ss} = 1.5\left(\pi_t - \pi^{ss}\right),$$

then even with a TOT shock of less than one standard deviation, the economy meets the ZLB[4] (figures 2 and 3).

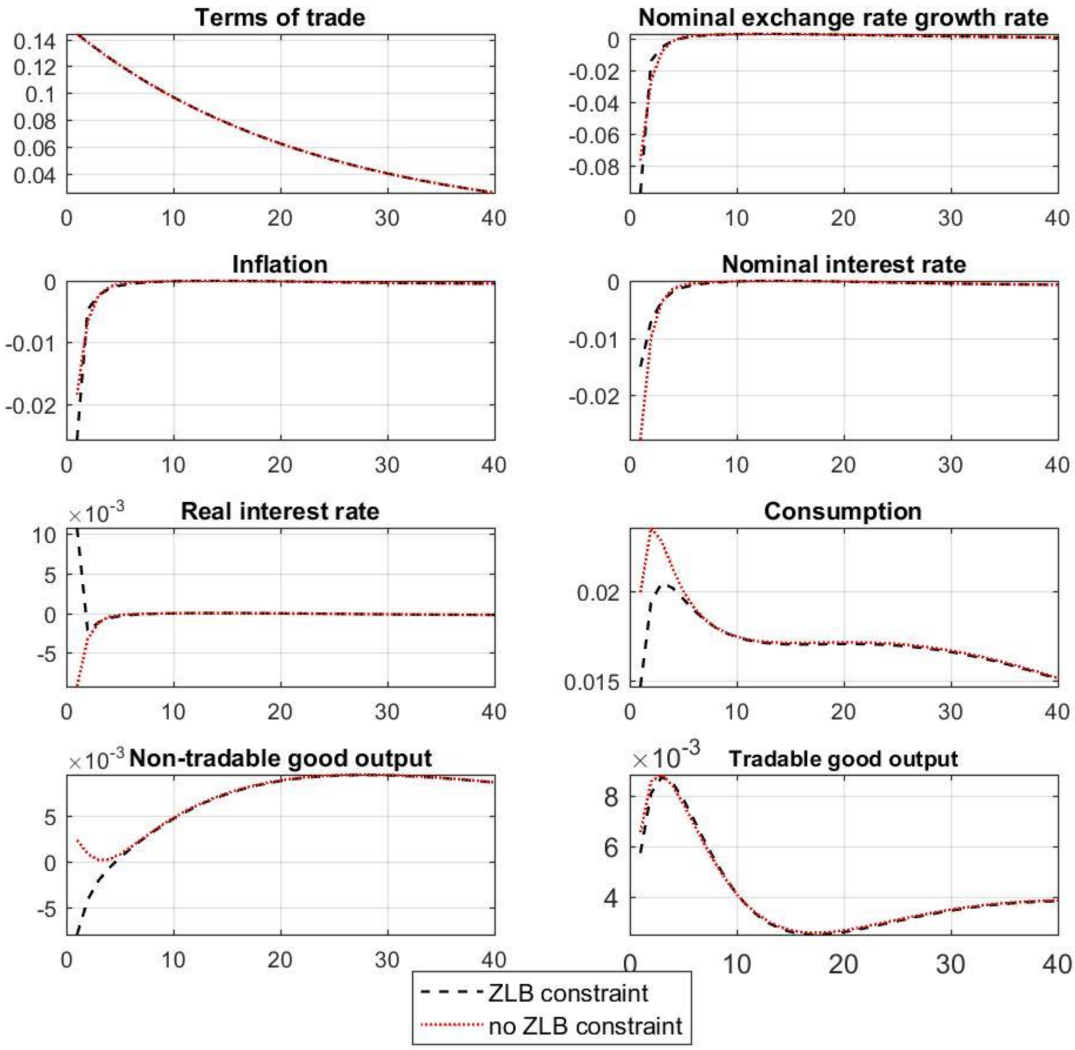

Figure 2 – the response of model variables to a positive 1-standard-deviation TOT shock under $\rho_{\text{inf}} = 1.5,\ \ \rho_y = 0,\ \ \rho_r = 0$ depending on whether the interest rate is at the bound or not. As a portion of the long-run equilibrium.

[4] The curve "nominal interest rate" for the case "ZLB constraint" (Figure 2) drops from 0 to the lower bound of -0.015. This corresponds to a fall in the rate from 6% to 0% in annual terms. Therefore, the case "ZLB constraint " corresponds to the hitting the ZLB at the same period as the shock occurred.

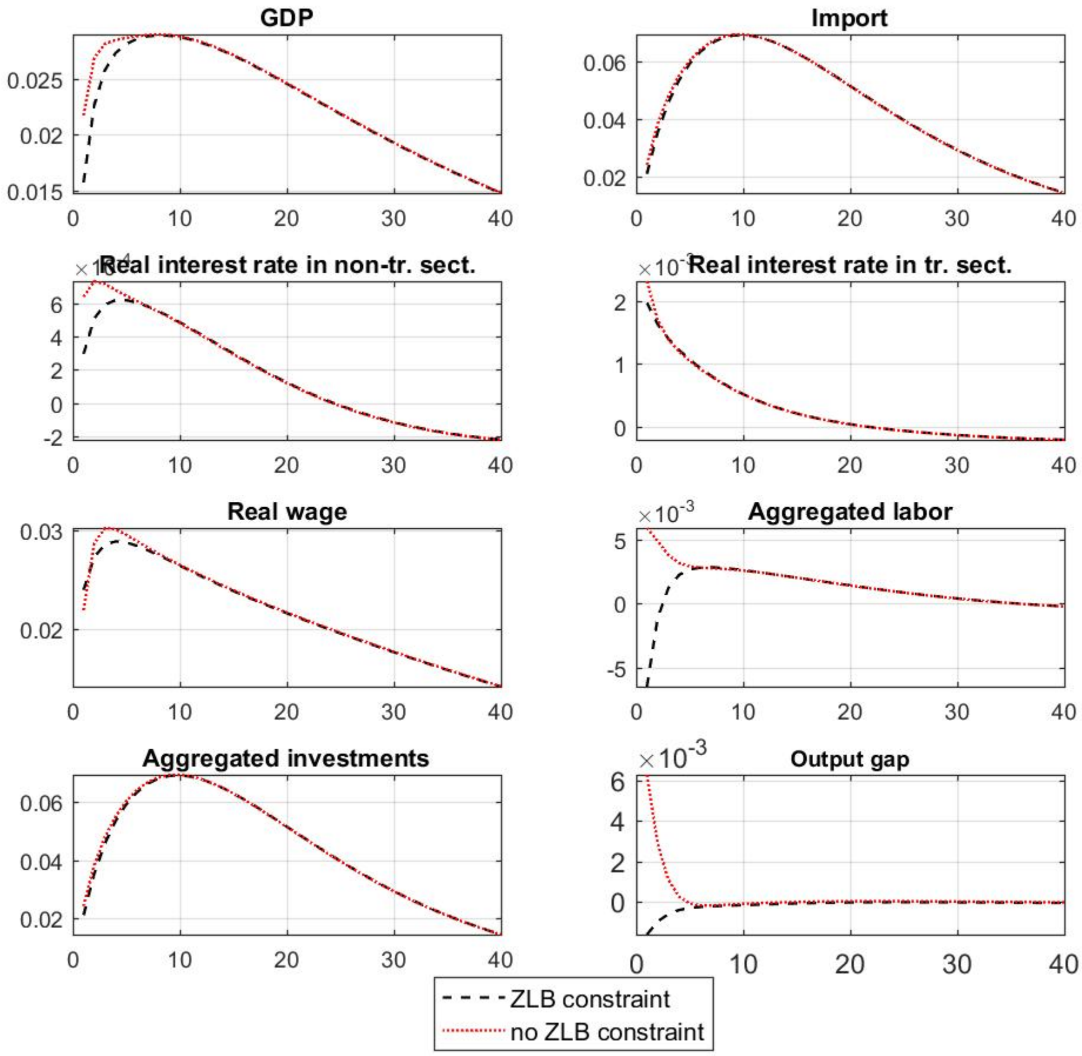

Figure 3 – the response of model variables to a positive 1-standard-deviation TOT shock under $\rho_{\inf} = 1.5,\ \ \rho_y = 0,\ \ \rho_r = 0$ depending on whether the interest rate is at the bound or not. As a portion of the long-run equilibrium. Continued.

As a result of a positive terms of trade shock, the exchange rate strengthens. Inflation with a domestic component and a foreign component determined by the exchange rate falls, forcing the regulator to lower the interest rate. In the demonstrated case, if there is a lower bound on rates, the regulator can reduce the rate only to 0% p.a. from the long-term value of 6% (Figure 2), whereas if there is no lower bound, the rate can be cut below 0%. If there is a limit, inflation falls lower than it would have had there been no such limit. The difference between the nominal rate and inflation leads to a higher real interest rate if there is a lower bound on nominal rates.

A further difference between the regimes is due to the difference in how the real interest rate responds. A higher real rate in the presence of a nominal rate limit leads households to exhibit more saving behavior and to increase consumption to a lesser extent. Lower consumer demand leads to lower output growth in the non-traded sector, while output in the traded sector changes insignificantly. Aggregate output demonstrates lower growth, which leads to lower growth of both the cost of production factors and production factors as such.

We note that the impact of the shock is asymmetric: the above differences between the regimes occur only under a positive shock to trade terms, while under a negative shock the regimes exhibit the same behavior. This means that on the paths resulting from the terms of trade shock the average inflation rate decreases and the average output rate also decreases when the nominal rate limit is present, which demonstrates the negative effect of the bound on average performance.

As for volatility, inflation volatility increases when there is an interest rate limit, whereas production volatility and nominal interest rate volatility both decrease. This means that a regulator seeking to reduce inflation volatility alone is negatively impacted by the presence of an interest rate limit. However, if a regulator has combined criteria, they may benefit from the presence of a limit. As will be seen later, the regulator can indeed improve the value of the target function to a small extent as a result of a rate limit being present.

5. Monetary authority's objective

We use the Taylor Rule as the rule on which monetary policy is based (1). The $\rho_{\inf}, \rho_y, \rho_r$ coefficients are the subject of research into the optimality of the MP rule (1). We optimised the criterion according to these parameters.

In a similar fashion to Williams (2009), Eggertsson (2011), and Adolfson, et al. (2011), we used the classical central bank loss function, which minimises the weighted sum of inflation variance, output gap, and interest rate:

$$L = Var(\pi_t) + \alpha\, Var(gap_t) + \beta\, Var(R_t^l) \qquad (6)$$

Minimising inflation variance $Var(\pi_t)$ is the primary objective of the MP authority, so this term is presented with a coefficient of 1 in the loss function. $Var(gap_t)$ corresponds to the desire of the MP authority to smooth both inflation and output gap. The last summand, the variance of the nominal interest rate $Var(R_t^l)$, is required to exclude the cases where inflation and output gap are smoothed by an overly aggressive response by the MP authority in accordance with the Taylor Rule. Aggressive interest rate changes are not usually observed in reality, as each change imposes a non-financial cost on the MP authority. Among all of the possible variants of the target function (6), we chose four: $Var(\pi_t)$, $Var(\pi_t) + Var(gap_t)$, $Var(\pi_t) + 0.35\, Var(R_t^l)$,

$Var(\pi_t)+Var(gap_t)+0.35\,Var(R_t^l)$. The $\beta=0.35$ value was chosen, firstly, by analogy with the work of Adolfson et al. (2011), and secondly, with the given value of the coefficient, interest rate variation makes an impact on the loss function comparable to inflation variation.

Next, we generated 5 sequences of terms of trade shocks ε_t^{TOT} and foreign interest rates ε_t^{prem} with 1,000 points each.[5] For each pair of 1,000-point chains, we calculated the model solution using the Occbin tool. Moving in the space of parameters $\rho_{\inf}, \rho_y, \rho_r$, we found parameter values that minimized criteria of type (6).

6. Optimal simple MP rules

The optimal parameter values are presented in Table 4.

The values of the parameters in the Table 4 are presented for specification of the Taylor rule

$$R_t^l - R^{l,ss} = \rho_r\left(R_{t-1}^l - R^{l,ss}\right) + \rho_{\inf}\left(\pi_t - \pi^{ss}\right) + \rho_y\left(GDP_t / GDP_t^{pot} - 1\right) \tag{7}$$

instead of specification (1) due to the possibility of parameter ρ_r to tend to unity and the need to limit the parameters $\rho_{\inf}$, ρ_y from above. Note that the identified parameter values for the Taylor rule (1) (Table 1) $\rho_r = 0.902$, $\rho_{\inf} = 1.49$ correspond to the value $\rho_{\inf} = 0.146$ in the specification (7).

The table contains information broken down by:

- Two long-term inflation values: 0% p.a. and 4% p.a.
- Four kinds of loss functions (6).
- The presence or absence of a zero lower bound condition (2) is indicated in the 'Avail. of ZLB constr' column. If the bound is set, the column t_1 indicates the probability of the economy being at the lower bound of interest rates. Otherwise, the probability of the rate being below the zero (unspecified) lower bound is indicated.

When looking for the $\rho_{\inf}, \rho_y, \rho_r$ policy rule parameters, we assumed that all of them are non-negative, the autocorrelation coefficient ρ_r is lower than or equal to 1, and the degree of response to inflation $\rho_{\inf}$ is lower than or equal to 3.5.

[5] The need to use several short sequences of shocks instead of a single long one arises, firstly, from a more-than-linear increase in the OccBin tool run time depending on the sequence length and, secondly, from the ability to refine the research results by generating additional iterations.

Table 4 – Parameters of optimal rules

Loss function \ parameter	Avail. of ZLB constr	$\rho_{\inf}$	ρ_y	ρ_r	t_1	Loss function value	Value $Var(\pi_t)$	Value $Var(gap_t)$	Value $Var(Rl_t)$
Long-term inflation rate – 0%									
$L = Var(\pi_t)$	no	3.5	0	0.95	(39.78%)	0.034	0.034	0.517	0.546
$L = Var(\pi_t)$	yes	3.11	0	1	33.08%	0.313	0.313	0.163	0.212
$L = Var(\pi_t) + Var(gap_t)$	no	1.29	0	0.47	(39.58%)	0.345	0.196	0.149	0.459
$L = Var(\pi_t) + Var(gap_t)$	yes	1.03	0.04	1	28.5%	0.437	0.341	0.096	0.095
$L = Var(\pi_t) + 0.35Var(Rl_t)$	no	1.44	0	1	(36.82%)	0.184	0.093	0.352	0.262
$L = Var(\pi_t) + 0.35Var(Rl_t)$	yes	0.76	0	1	26.14%	0.365	0.342	0.093	0.068
$L = Var(\pi_t) + Var(gap_t) + 0.35Var(Rl_t)$	no	0.48	0	0.86	(29.2%)	0.414	0.246	0.136	0.094
$L = Var(\pi_t) + Var(gap_t) + 0.35Var(Rl_t)$	yes	0.82	0.01	1	26.74%	0.459	0.341	0.092	0.074
Long-term inflation rate – 4%									
$L = Var(\pi_t)$	no	3.5	0	0.95	(20.96%)	0.033	0.033	0.608	0.519
$L = Var(\pi_t)$	yes	3.5	0	1	20.66%	0.156	0.156	0.296	0.304
$L = Var(\pi_t) + Var(gap_t)$	no	1.28	0.17	0.51	(20.42%)	0.366	0.224	0.142	0.443
$L = Var(\pi_t) + Var(gap_t)$	yes	1.05	0.25	1	13.28%	0.380	0.248	0.132	0.164
$L = Var(\pi_t) + 0.35Var(Rl_t)$	no	1.47	0	1	(14.58%)	0.178	0.089	0.423	0.254
$L = Var(\pi_t) + 0.35Var(Rl_t)$	yes	0.82	0	1	8.28%	0.225	0.186	0.220	0.112
$L = Var(\pi_t) + Var(gap_t) + 0.35Var(Rl_t)$	no	0.45	0.06	0.91	(4.52%)	0.431	0.267	0.135	0.086
$L = Var(\pi_t) + Var(gap_t) + 0.35Var(Rl_t)$	yes	0.56	0.11	1	5.96%	0.421	0.259	0.130	0.090
$L = Var(\pi_t)$, TR estimated parameters	no	0.146	0	0.902	0.32%	0.403	0.403	0.082	0.028

$L = Var(\pi_t)$, TR estimated parameters	yes	0.146	0	0.902	0.28%	0.404	0.404	0.081	0.028

Note that the results for the loss function $Var(\pi_t) + Var(gap_t) + 0.35\, Var(R_t^l)$, which takes into account the variations in all three variables – inflation, output gap, and interest rate – are outlying, therefore we leave the analysis of this case for the conclusion.

Otherwise, as the table shows, the differences between the regimes with and without a lower rate limit are as follows. Firstly, in the presence of an interest rate limit (2), the probability of being at the rate bound is lower than being below the bound when there is no limit. The reason for this is the increased inflation volatility in the interest rate limit regime, which leads to such optimal parameters that the probability of reaching the boundary is reduced. Secondly, for the same reason, the values of the criteria are higher when there is a rate limit. Thirdly, the parameter for policy response to inflation $\rho_{\inf}$ decreases when the limit is imposed (2). This can be explained by the undesirability of reaching the interest rate bound, the probability of which is higher in the event of more aggressive rate changes at high values of the parameter $\rho_{\inf}$. Fourthly, the policy response parameter for output gap ρ_y increases when the limit is imposed on the rate. This result turns out to be unexpected and counter-intuitive given the undesirability of reaching the interest rate bound. It can be explained by the fact that an increase in the parameter ρ_y leads to a decrease in output gap volatility, in addition to an increase in inflation volatility, which can compensate for the increase in the volatility of inflation. Fifthly, the value of the rate autocorrelation parameter ρ_r also increases when the limit is imposed. The tendency of the rate autocorrelation parameter to reach values above 1 in the optimal rules is also noted in Adolfson et al. (2014). In our case, due to economic agents making decisions that take future inflation into account, which is reflected in the equation for the variant of the Phillips curve, current inflation is lower, and thus the expected future inflation is lower. In the event of higher rate autocorrelation values ρ_r (Figure 4), corresponding, all other things being equal, to a longer return of the rate to its long-term equilibrium, the regulator promises a longer fight against the causes of inflationary changes, which reduces the reaction of inflation. This explains the optimality of high autocorrelation values for the rate ρ_r .

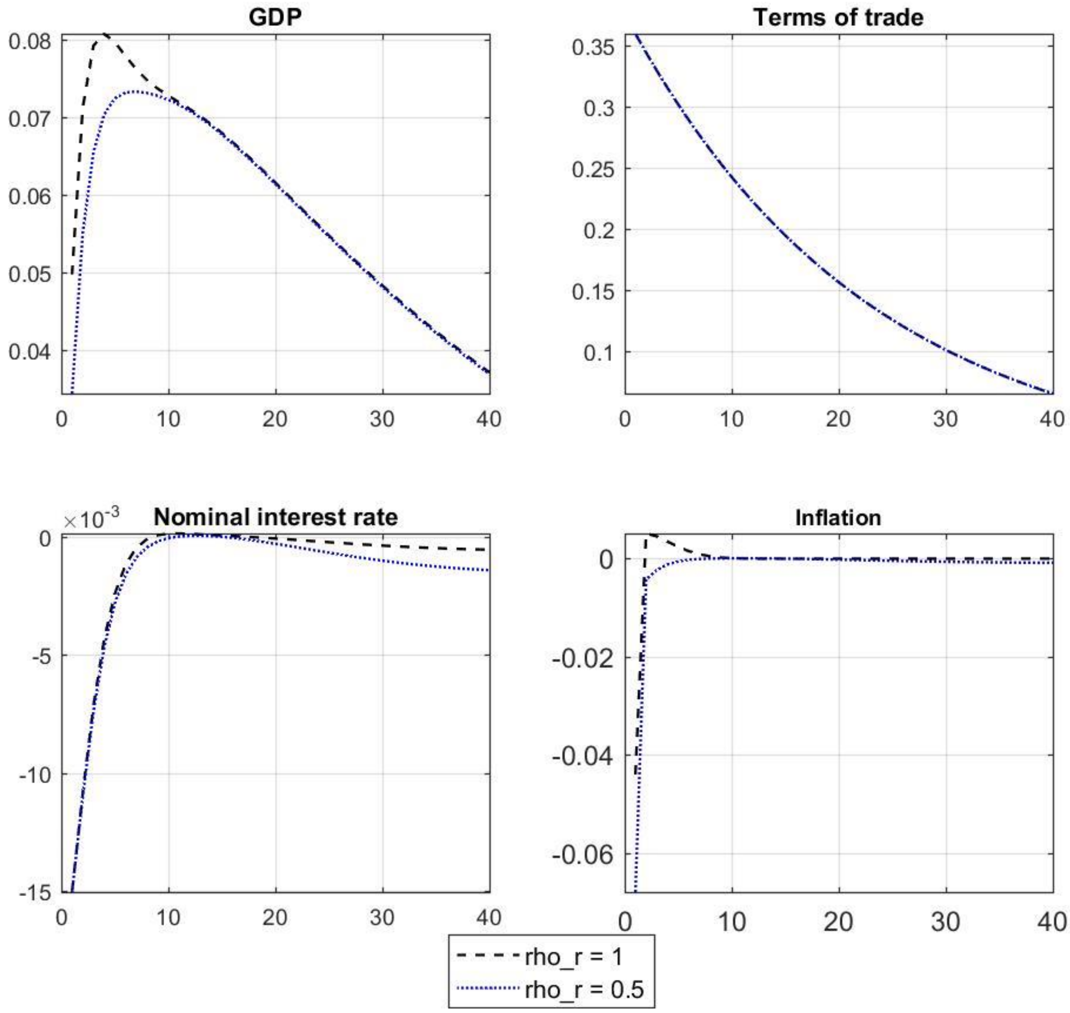

Figure 4 – the impulse response of certain model variables to a positive 2.5-standard-deviation TOT shock under the ZLB constraint under $\rho_{\inf} = 0.82, \ \rho_y = 0$. For different values of ρ_r. GDP and TOT values are given as fractions of the long-term equilibrium; nominal interest rate and inflation are quarterly.

The optimal rules corresponding to different long-term inflation values of 0% and 4% differ in terms of the probability of staying at the interest rate bound and criteria values. At 0% inflation, the probability of staying at the interest rate bound is higher because the long-term interest rate of 2% p.a. is closer to the zero lower bound than it would otherwise be at 6% p.a. Accordingly, when inflation is 0% under an interest rate limit regime, the value of the criterion is higher because the monetary authority has a decreased ability to respond to fluctuations in macro variables. The long-term value of inflation does not significantly affect the optimal values of the rule parameters.

The presence of output variation $Var(gap_t)$ in the penalty function (6) causes the interest rate to begin to respond to changes in output gap under the optimal rule. Otherwise, $\rho_y = 0$, and the rate is unresponsive to output gap. Also, the presence of output variation in the penalty function reduces the response to changes in inflation $\rho_{\inf}$ and the autocorrelation coefficient ρ_r .

Similarly, the presence of interest rate variation $Var\left(R_t^l\right)$ in the penalty function (6) causes the rate autocorrelation coefficient ρ_r to increase, while the inflation and output gap response coefficients $\rho_{\inf}, \rho_r$ decrease.

The case of the loss function $Var(\pi_t)+Var(gap_t)+0.35\,Var\left(R_t^l\right)$ with a long-term inflation rate of 4% is distinct from the above analysis. The unexpected thing is that, firstly, the value of the criterion declines when the interest rate limit regime is introduced and, secondly, the probability of reaching the interest rate bound rises when the limit regime is introduced. It would be expected that the presence of the rate limit regime should restrict monetary policy actions and lead to excessive volatility of variables and an increase in the value of the penalty function when reaching the rate bound. Indeed, in the presence of a rate limit, inflation volatility increases as the Taylor Rule ceases to apply at the rate bound. However, at the rate bound, firstly, the nominal rate itself stops changing, and secondly, output gap is less responsive to shocks, which results in a lower interest rate and output gap volatility. The reduction in interest rate and output gap volatility is greater than the increase in inflation volatility, which explains the effect[6]. Overall, Table 4 shows that adding interest rate volatility and/or output gap volatility to the loss function reduces the difference between the criterion values in the presence and absence of a rate limit.

7. The probability of being at the ZLB and at the effective lower bound (ELB)

We note that in the identified optimal rules (Table 4), the range of probabilities for reaching the ZLB at 4% inflation is quite high: from 6% in the case of a loss function that includes the volatility for all three variables to 21% in the case of a loss function for inflation variation. A high probability of being at the ZLB suggests that the desire to avoid reaching the zero bound at all costs should not be attributed to a rational monetary policy authority: a high probability of staying at the bound may be the optimal choice. A comparison of the estimated Taylor Rule parameter values (Table 1) for Russia with the optimal rules indicates that the actual policy is closer to that of regulating inflation

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[6] For the case from the Table 4, in the absence of ZLB constraint under the parameter values $\rho_{\inf}=0.45,\ \rho_y=0.06,\ \rho_r=0.91$ the loss function value is $Var(\pi_t)+Var(gap_t)+0.35\,Var\left(R_t^l\right)=0.267+0.135+0.35*0.086=0.431$. Under the ZLB constraint and under the same parameter values the loss function value is $Var(\pi_t)+Var(gap_t)+0.35\,Var\left(R_t^l\right)=0.290+0.109+0.35*0.077=0.425<0.431$. Further optimization of the parameters, as can be seen from the Table 4, reduces the value of the loss function to 0.421 and changes the value of the loss function components.

volatility with a reluctance to abruptly change the interest rate, which corresponds to a 6–8% probability of staying at the rate bound in the identified rules. Calculations using the identified parameter values (Table 4, last and second to last lines) show a 0.3% probability of being at the bound. However, a single TOT shock can push the economy to the ZLB only if it exceeds +5.4 standard deviations. The probability of such event is less than $3*10^{-8}$, while the potential for an interest rate shock is even lower. Therefore, a 0.3% probability of the economy being at the ZLB can only be realised through a prolonged series of major positive terms-of-trade shocks.

Note that the probability of being at the ZLB grows with the fall of the inflation steady state of inflation. The table 5 illustrates this.

Table 5 – The probability of being at the ZLB, depending on the inflation steady state value. For the case of identified Taylor rule parameters

Inflation steady state, % p.a.	Nominal interest rate steady state, % p.a.	The probability of being at the ZLB
4	6	0.3%
3	5	0.8%
2	4	3.2%

The zero bound of interest rates is not the only boundary near which monetary policy can be ineffective. For example, the households' saving behavior may qualitatively change if the profitability of placing deposits is much less than investments in other alternative instruments. The problem of not being able to cut the rate below a certain non-zero threshold has been called the effective lower bound (ELB) of interest rates. The trend towards a shift in focus from ZLB to ELB is aptly expressed by Governor of the Central Bank of Norway Ø. Olsen: «Before the financial crisis, most economists regarded zero as the lower bound for policy rates. At the same time, this limit was mainly of theoretical interest. Experience from the past decade has changed perspectives on the lower bound, and we now talk of "the effective lower bound" rather than "the zero lower bound"»[7].

In relation to Russia, the instrumental opportunity leading to the emergence of ELB may be an investment in foreign currency. This is due to the fact that over the past decade, keeping funds in foreign currency has turned out to be de facto highly profitable: the average annual return on holding funds in US dollars from 1999 to 2020 was 5.3%, from 2007 to 2020 – 7.6%, from 2013 to 2020 – 12.8%. At the same time, the average rate on placing deposits in national currency for a period of 1 to 3 years in the period from 2014 to 2020 was 7.2%, having decreased in 2020 to 4.5%. Thus, the profitability of keeping funds in foreign currency exceeded the profitability of deposits in the national currency. At the same time low rates on deposits in foreign currency do not create an incentive for keeping foreign currency in banks.

[7] https://www.norges-bank.no/en/news-events/news-publications/Speeches/2020/2020-10-06-cme/

We leave the question of the exact ELB level for Russia and whether this level can be determined for other studies. Here, in the Table 6, we present the probabilities of the economy hitting the potential lower bounds of rates, going over their values from 0% (ZLB) to 3% with a 0.5% step. The probabilities are given for the case of the identified parameters of the Taylor rule (Table 1).

Table 6 – The probability of being at the rate bound, depending on the value of the bound[8]. For the case of identified Taylor rule parameters

Lower bound value of the rate, % p.a.	The probability of being at the rate bound
0 (ZLB)	0.3%
0.5	0.4%
1	0.8%
1.5	1.9%
2	3.2%
2.5	>2.3%[8]
3	>3.8%[8]

As one can, the probability of hitting the rate bound grows rapidly with the growth of the bound value. This means that the Bank of Russia should not ignore the problem of the existence of ELB. At the same time, the optimal policy rules remain qualitatively the same as in Table 3, regardless of the value of the bound[9].

8. Conclusion

We have considered the DSGE model for an open export-oriented economy in which TOT shock and a foreign interest rate shock can push the domestic nominal interest rate to the zero lower bound. For an export-oriented economy, shocks which drive the economy to the interest rate bound are positive. Based on the impulse response functions, we have shown that the presence of a lower interest rate bound reduces the impact of positive shocks. This is explained by the fact that shocks that drive the economy to the interest rate bound cause inflation to decrease more sharply and the real interest rate to increase. This pushes consumers towards more saving behavior, reducing consumption and output.

In addition, inflation volatility increases under the ZLB, while output gap and interest rate volatility decreases. We have shown that if the regulator's objective is only to reduce inflation volatility, then the presence of an interest rate limit unambiguously worsens the regulator's target

[8] For the 2.5% and 3% rate bound we present only the lower estimate of the probability, since we were unable to calculate the equilibrium trajectories of macroeconomic variables with the help of Occbin for some realizations of the sequence of shocks under the estimated TR parameters. We can only say that calculation problems arise when the probability of hitting the bound turns out to be high. Therefore, the given estimate is a lower estimate.

[9] This is because the optimal rules are weakly dependent on the level of long-term inflation (Table 4). The long-term inflation value directly connected with the long-term value of the nominal interest rate. Thus, the optimal rules do not change much depending on the long-term level of the rate and so on the value of the bound.

function. If the regulator simultaneously reduces the volatility of several indicators, the negative impact of the limit becomes weaker and, in some cases, may lead to a small gain.

Generally, when the presence of an interest rate limit negatively affects the regulator's objective, the parameters of the optimal monetary policy rule are such that the probability of the economy being at the lower bound is lower than being below the bound in the event of no bound. We can say that in the optimal situation, the regulator de facto reduces the probability of being at the ZLB. The presence of the ZLB causes lower degree of a regulator's response to inflation and higher interest rate persistence. The latter leads to the economic agents' expectations that the regulator will take longer in the future to deal with the causes that affected inflation, which implies a more moderate inflationary reaction to shocks.

We used a calibration for export-oriented countries. We estimated a number of parameters using Russian data, and set long-term inflation and the interest rate at 4% and 6% p.a. respectively, which corresponds to the Bank of Russia's target. We found that the probability of reaching the interest rate bound in case of two shocks is only 0.3%. And this may be the answer to why we rarely see exporting economies on the ZLB: the long-term interest rate is too high, even for volatile commodity prices, to push the economy onto the ZLB.

In the optimum, the probability of being at the interest rate bound is estimated at 6.0–20.1%, depending on the regulator's target. This indicates that the optimal policy does not necessarily need to avoid the ZLB at all costs. Identified parameters for the interest rate response to output gap and the autocorrelation coefficient for the interest rate for Russia are within the range of values from the optimal rules, while the identified degree of inflationary response appears to be well below the optimal parameters.

Due to the fact that the profitability to save money in foreign currency in Russia in recent years is high, the Russian economy may face the ELB problem. We estimate that under the current monetary policy, the likelihood to hit the ELB is significant.

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10. Appendix: detailed description of the model

A complete description of the model used in the paper is provided below.

10.1. Households

Each household $i \in [0;1]$ optimises a target function reflecting their satisfaction from consumption $C_t(i)$ and their dissatisfaction from labour $L_t(i)$:

$$U(i) = E\sum_{t=1}^{+\infty}\beta^t\left(\ln\left(C_t(i) - H_t\right) - \frac{\sigma_L}{1+l\_el}\left(L_t(i)\right)^{1+l\_el}\right).$$

In this expression, the value $H_t = hC_{t-1}$ reflects consumption habits based on labour (Smets, Wouters, 2003).

Households are assumed to have the monopoly power in the labour market, therefore they set their labour supply based on knowledge of demand for their labour: $L_t(i) = \left(\frac{W_t(i)}{W_t}\right)^{-\eta} L_t$, where $W_t(i)$ and W_t are individual and aggregate wages, and L_t is aggregate labour. In addition, households determine the nominal volume of investments in foreign bonds $Df_t(i)$, as well as the volume of loans provided to other households $Loan_t(i)$. Similar to Smets, Wouters (2003), we assume that the interest rate on household loans is set by the monetary policy authority. This assumption eliminates the need to define financial intermediaries, such as commercial banks, which transfer to households the interest rate of the monetary policy authority with distortions that are out of the scope of this paper.

Each household in the model owns two types of productive capital: $K_t^d(i)$ for the needs of the intermediate domestic product sector and $K_t^{ex}(i)$ for the needs of the export product sector. The types of capital for the above two sectors are not considered to be completely interchangeable. Both types of capital are leased to manufacturers at nominal rates r_t^d, r_t^{ex}. Physical capital stocks $K_t^d(i)$, $K_t^{ex}(i)$ are related to new capital $DK_t^d(i)$, $DK_t^{ex}(i)$, purchased from capital producers at nominal prices Q_t^d, Q_t^{ex} as follows:

$$K_t^d(i) = (1-\delta_d)K_{t-1}^d(i) + DK_t^d(i) \quad , \quad K_t^{ex}(i) = (1-\delta_{ex})K_{t-1}^{ex}(i) + DK_t^{ex}(i) \quad , \qquad (8)$$

where δ_d, δ_{ex} are the depreciation rates of physical capital.

A household's budget limit is

$$
\begin{aligned}
&C_t(i)+\frac{Loan_t(i)}{P_t}+\frac{Df_t(i)S_t}{P_t}+\frac{Q_t^d DK_t^d(i)}{P_t}+\frac{Q_t^{ex} DK_t^{ex}(i)}{P_t}= \\
&\frac{W_t(i)L_t(i)}{P_t}+\frac{N_t}{P_t}land(i)+R_{t-1}^l\frac{Loan_{t-1}(i)}{P_t}+\frac{r_t^d K_{t-1}^d(i)}{P_t}+\frac{r_t^{ex} K_{t-1}^{ex}(i)}{P_t}+ \\
&+R_t^f\frac{Df_{t-1}(i)S_t}{P_t}+\frac{\Pi_t^{fiz}}{P_t}+\frac{\Pi_t^d}{P_t}+\frac{\Pi_t^{ex}}{P_t}-\Psi_t^L\left(W_t(i)/W_{t-1}(i)\right)-\Psi_t^{Df}\left(Df_t(i)\right)
\end{aligned} \tag{9}
$$

where P_t is the final product price index for the economy; S_t is the nominal exchange rate; $land(i)$ is the land owned by the household and rented out to the export good manufacturer; N_t is the rental value of a unit of land; R_t^l is the nominal gross rate on loans provided by households to each other as set by the monetary authorities; R_t^f is the nominal gross rate on foreign bonds denominated in foreign currency and set by the process (5); Π_t^{fiz}, Π_t^d, Π_t^{ex} are the profits transferred to households by producers of productive capital and the two types of products; and $\Psi_t^L\left(W_t(i)/W_{t-1}(i)\right)$, $\Psi_t^{Df}\left(Df_t(i)\right)$ are costs incurred by a household from changes in nominal wages and investing in non-residential bonds. In accordance with Rotemberg (1982), costs are assumed to be increasing quadratic functions of arguments and equal to zero in the long-run equilibrium: $\Psi_t^L=\frac{1}{2}w\left(W_t(i)/W_{t-1}(i)-1\right)^2\frac{W_t L_t}{P_t}$, $\Psi_t^{Df}=\frac{1}{2}d_s\left(Df_t(i)S_t/P_t Y_t\right)^2 Y_t$.

By denoting the Lagrange multiplier under the household budget limit as $\beta^t\Lambda_t$ and discarding the household index within the symmetric equilibrium analysis (households do not differ from each other), we obtain the following necessary optimal conditions for consumption, credit, bond investments, wages, and productive capital for the intermediate domestic and export product sectors:

$$\Lambda_t=\frac{1}{C_t-hC_{t-1}} \quad , \tag{10}$$

$$\Lambda_t=\beta R_t^l E_t \Lambda_{t+1}/\pi_{t+1} \quad , \tag{11}$$

$$\Lambda_t = \beta R^f E_t \Lambda_{t+1} \frac{S_{t+1}}{\pi_{t+1} S_t} - \Lambda_t \frac{P_t}{S_t} \frac{\partial \Psi_t^{Df}}{\partial Df_t} \quad , \tag{12}$$

$$\sigma_L (L_t)^{l\_el} = \frac{\eta - 1}{\eta} \Lambda_t \frac{W_t}{P_t} - \frac{W_t}{\eta L_t} \Lambda_t \frac{\partial \Psi_t^L}{\partial W_t} - \beta \frac{W_t}{\eta L_t} E_t \Lambda_{t+1} \frac{\partial \Psi_{t+1}^L}{\partial W_t} \quad , \tag{13}$$

$$\Lambda_t \frac{Q_t^d}{P_t} = \beta \Lambda_{t+1} \left(\frac{Q_{t+1}^d}{P_{t+1}} (1 - \delta_d) + \frac{r_{t+1}^d}{P_{t+1}} \right) \quad , \tag{14}$$

$$\Lambda_t \frac{Q_t^{ex}}{P_t} = \beta \Lambda_{t+1} \left(\frac{Q_{t+1}^{ex}}{P_{t+1}} (1 - \delta_{ex}) + \frac{r_{t+1}^{ex}}{P_{t+1}} \right) \quad . \tag{15}$$

In the expressions (11), (12) $\pi_t = P_t / P_{t-1}$ is the ratio of nominal prices for final products in national currency.

10.2. Manufacturers of final products

Manufacturers of the final product first buy the differentiated product $Y_t^d(j)$ in a market under perfect competition from each manufacturer j, $j \in [0;1]$, the intermediate domestic product at nominal price $P_t^d(j)$, and then aggregate the differentiated products according to the Dixit-Stiglitz method $Y_t^d = \left[\int_0^1 \left(Y_t^d(j) \right)^{(\sigma_d - 1)/\sigma_d} dj \right]^{\sigma_d/(\sigma_d - 1)}$. Second, they aggregate the domestic product Y_t^d together with the imported product bought at price P_t^f Imp_t into the final product Y_t according to the Cobb-Douglas production function

$$Y_t = \frac{(Imp_t)^{\omega} \left(Y_t^d \right)^{1-\omega}}{\omega^{\omega} (1-\omega)^{1-\omega}} \quad . \tag{16}$$

Manufacturers sell the final product in the market at price P_t, aiming to maximise their profit $P_t Y_t - S_t P_t^f Imp_t - \int_0^1 P_t^d(j) Y_t^d(j) dj$. The solution to the profit maximisation problem under technological constraints provides that the demand function for the product j of the intermediate domestic product manufacturer is as follows:

$$Y_t^d(j) = Y_t^d \left(\frac{P_t^d(j)}{P_t^d} \right)^{-\sigma_d} , \tag{17}$$

where P_t^d is the aggregate price of the domestic product. Then under the assumption of symmetry of equilibrium, the rest of the solution to the problem is

$$\frac{P_t^d}{P_t} Y_t^d = (1-\omega) Y_t , \tag{18}$$

$$\frac{S_t P_t^f}{P_t} Imp_t = \omega Y_t . \tag{19}$$

10.3. Intermediate domestic and export product manufacturers

Let us assume that manufacturers of intermediate domestic products operate in a monopolistic competition market. Manufacturers of export products operate in a perfect competition market. There is a continuum of both types of manufacturers, we will number them with the indices $j \in [0;1]$ and $\kappa \in [0;1]$ respectively.

Both types of manufacturers use labour in quantities $L_t^d(j)$, $L_t^{ex}(\kappa)$[10], which they buy in the common market at price W_t, and productive capital $K_{t-1}^d(j)$, $K_{t-1}^{ex}(\kappa)$, which they rent in separate markets from households at nominal rental rates r_t^d, r_t^{ex} as factors of production during the time period t. Export products manufacturers, unlike manufacturers of domestic products, also use a third factor of production, which is land. For the use of land to the extent of $Land_t(\kappa)$ export manufacturers pay a nominal amount $N_t Land_t(\kappa)$ to households. The specification of the production function for commodity sectors with land as a production factor was also used in the Bank of Canada's ToTEM DSGE model (Murchison, Rennison, 2006). This assumption takes into

[10] Further, index "d" corresponds to a manufacturer of domestic goods, "ex" corresponds to a manufacturer of export goods

account different availability of natural resources: the more land is available to the economy, the lower the cost of extracting a given amount of resources.

The production functions for manufacturers of intermediate domestic and export products are as follows:

$$Y_t^d(j) = a_d \left(K_{t-1}^d(j)\right)^{\alpha_d} \left(L_t^d(j)\right)^{1-\alpha_d} \quad , \tag{20}$$

$$Y_t^{ex}(\kappa) = a_{ex} \left(K_{t-1}^{ex}(\kappa)\right)^{\alpha_{ex}} \left(L_t^{ex}(\kappa)\right)^{1-\alpha_{ex}-\gamma_{ex}} \left(Land_t(\kappa)\right)^{\gamma_{ex}} \quad . \tag{21}$$

Manufacturers decide on the selection of physical capital variables $K_{t-1}^d(j)$, $K_{t-1}^{ex}(\kappa)$ during the period t. The backward time shift in the designations is due to the assumption that an entrepreneur can only lease to manufacturers the physical capital that was produced by the end of period $t-1$. This assumption allows for lagged commissioning of investments.

Domestic intermediate product manufacturers and export product manufacturers aim to maximise the expected present value of income $E\sum_{t=1}^{+\infty}\beta^t \Lambda_t \frac{\Pi_t^d(j)}{P_t}$, $E\sum_{t=1}^{+\infty}\beta^t \Lambda_t \frac{\Pi_t^{ex}(\kappa)}{P_t}$, where income is determined through the following equations:

$$\Pi_t^d(j) = P_t^d(j)\, Y_t^d(j) - W_t L_t^d(j) - r_t^d K_{t-1}^d(j) - k/2\left(P_t^d(j)/P_{t-1}^d(j) - 1\right)^2 P_t^d Y_t^d \quad . \tag{22}$$

$$\begin{aligned}\Pi_t^{ex}(\kappa) &= S_t P_t^{ex} Y_t^{ex}(\kappa) - W_t L_t^{ex}(\kappa) - r_t^{ex} K_{t-1}^{ex}(\kappa) - \\ &\quad - N_t Land_t(\kappa) - w_{ex}/2\left(L_t^{ex}(\kappa)/L_{t-1}^{ex}(\kappa) - 1\right)^2 W_t L_t^{ex}\end{aligned} \tag{23}$$

The quadratic terms in the expressions reflect the costs of changing the amount of hired labour $L_t^{ex}(\kappa)$ and nominal price $P_t^d(j)$ in accordance with Rotemberg (1982) and implement mechanisms for rigidity in these indicators.

The first-order conditions on capital, labour, and land for the export manufacturer problem after switching to aggregate variables are as follows:

$$r_t^{ex} K_{t-1}^{ex} = \alpha_{ex} S_t P_t^{ex} Y_t^{ex} \quad , \tag{24}$$

$$W_t L_t^{ex} + w_{ex}\left(\frac{L_t^{ex}}{L_{t-1}^{ex}} - 1\right)\frac{L_t^{ex}}{L_{t-1}^{ex}} W_t L_t^{ex} - \beta w_{ex} E_t \frac{\Lambda_{t+1}}{\Lambda_t}\frac{P_t}{P_{t+1}}\left(\frac{L_{t+1}^{ex}}{L_t^{ex}} - 1\right)\frac{L_{t+1}^{ex}}{L_t^{ex}} W_{t+1} L_{t+1}^{ex} = \\ = \left(1 - \alpha_{ex} - \gamma_{ex}\right) S_t P_t^{ex} Y_t^{ex}, \tag{25}$$

$$Land_t\, N_t = \gamma_{ex} S_t P_t^{ex} Y_t^{ex} \qquad , \tag{26}$$

where P_t^{ex} is the external price of the exported product. It is assumed that the trends in the terms of trade

$$\tilde{P}_t^{res} = P_t^{ex} / P_t^f$$

are described by process (4). The external price P_t^f is assumed to be constant.

By maximising the objective function for an individual domestic intermediate product manufacturer with production factors and price $P_t^d(j)$ under information limit (17), production limit (20) and ratio (22) after transforming aggregate variables we obtain the following ratios:

$$r_t^d K_{t-1}^d = \alpha_d\, Mc_t\, P_t^d Y_t^d \qquad , \tag{27}$$

$$W_t L_t^d = \left(1 - \alpha_d\right) Mc_t\, P_t^d Y_t^d \qquad , \tag{28}$$

$$Mc_t = \frac{\sigma_d - 1}{\sigma_d} + \frac{k}{\sigma_d}\left(\frac{P_t^d}{P_{t-1}^d} - 1\right)\frac{P_t^d}{P_{t-1}^d} - \beta \frac{k}{\sigma_d} E_t \frac{\Lambda_{t+1}}{\Lambda_t}\frac{Y_{t+1}^d}{Y_t^d}\frac{P_t}{P_{t+1}}\left(\frac{P_{t+1}^d}{P_t^d} - 1\right)\left(\frac{P_{t+1}^d}{P_t^d}\right)^2 \qquad . \tag{29}$$

The value of Mc_t is interpreted as the marginal cost of the manufacturer.

10.4. Manufacturer of productive capital

A manufacturer of productive capital buys investments in the final product market Inv_t^d, Inv_t^{ex} to create productive capital DK_t^d, DK_t^{ex} for subsequent sale to households at nominal prices Q_t^d, Q_t^{ex}. The technology for transforming investment into capital takes into account the non-linearity of capital production costs and is described by the following equations

$$DK_t^d = Inv_t^d - \Psi_t^d\left(Inv_t^d / Inv_{t-1}^d\right) Inv_t^d \quad , \quad DK_t^{ex} = Inv_t^{ex} - \Psi_t^{ex}\left(Inv_t^{ex} / Inv_{t-1}^{ex}\right) Inv_t^{ex} \quad , \qquad (30)$$

where $\Psi_t^d\left(Inv_t^d / Inv_{t-1}^d\right) = 1/2\ k_d\left(Inv_t^d / Inv_{t-1}^d - 1\right)^2$, $\Psi_t^{ex}\left(Inv_t^{ex} / Inv_{t-1}^{ex}\right) = 1/2\ k_{ex}\left(Inv_t^{ex} / Inv_{t-1}^{ex} - 1\right)^2$.

The goal of a capital manufacturer is to maximise expected adjusted profits $E_t \sum_{t=1}^{+\infty} \beta^t \Lambda_t \frac{\Pi_t^{fiz}}{P_t}$. The expression for current period profit Π_t^{fiz} is

$$\Pi_t^{fiz} = Q_t^d DK_t^d + Q_t^{ex} DK_t^{ex} - P_t Inv_t^d - P_t Inv_t^{ex} \quad . \qquad (31)$$

As a result of maximising the expected adjusted profit, prices of productive capital are determined from the following ratios:

$$\frac{Q_t^d}{P_t}\left(1 - \Psi_t^d - Inv_t^d \frac{\partial \Psi_t^d}{\partial Inv_t^d}\right) = 1 + \beta E_t Inv_{t+1}^d \frac{\Lambda_{t+1}}{\Lambda_t} \frac{Q_{t+1}^d}{P_{t+1}} \frac{\partial \Psi_{t+1}^d}{\partial Inv_t^d} \quad , \qquad (32)$$

$$\frac{Q_t^{ex}}{P_t}\left(1 - \Psi_t^{ex} - Inv_t^{ex} \frac{\partial \Psi_t^{ex}}{\partial Inv_t^{ex}}\right) = 1 + \beta E_t Inv_{t+1}^{ex} \frac{\Lambda_{t+1}}{\Lambda_t} \frac{Q_{t+1}^{ex}}{P_{t+1}} \frac{\partial \Psi_{t+1}^{ex}}{\partial Inv_t^{ex}} \quad . \qquad (33)$$

If $k_d = k_{ex} = 0$, then the investments are identified with the new capital, and the prices for the capital Q_t^d, Q_t^{ex} are equal to the price of the final product P_t .

10.5. Market equilibrium

The equilibrium conditions for the labor market, land market, credit market, final product market, and foreign exchange market are as follows:

$$L_t = L_t^d + L_t^{ex} \quad , \qquad (34)$$

$$\int_0^1 Land_t(\kappa)\, d\kappa = \int_0^1 land(i)\, di = land \quad , \qquad (35)$$

$$\int_0^1 Loan_t(i)\, di = 0 \qquad (36)$$

$$C_t + Inv_t^d + Inv_t^{ex} + \Psi_t^L\left(W_t/W_{t-1}\right) + k/2\left(P_t^d / P_{t-1}^d - 1\right)^2 P_t^d Y_t^d / P_t + \\ + w_{ex}/2\left(L_t^{ex} / L_{t-1}^{ex} - 1\right)^2 W_t L_t^{ex} / P_t = Y_t \quad , \qquad (37)$$

$$P_t^{ex} Y_t^{ex} + R_{t-1}^f Df_{t-1} = P_t^f Imp_t + Df_t + P_t^f \Psi_t^{Df} / S_t \quad . \qquad (38)$$

Ratio (35) means that the supply of land is fixed because the amount of land owned by individual households is assumed to be constant. The last 3 terms in the left part of (37) are the costs associated with changes in nominal wages, manufacturer prices, and hours worked in the export production sector.

10.6. Monetary policy

The monetary authority is assumed to follow the Taylor Rule (1) as long as the nominal interest rate is strictly above zero (2). The MP authority ceases to follow the Taylor Rule (1), if the nominal interest rate rests at the zero lower bound (3).

10.7. Model equations

The final model is represented by ratios (4), (5), (8) – (16), (18) – (38), and – depending on whether we are at the lower bound of the nominal interest rate – expressions (1) or (3).