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DEMUR, a regional semi-structural model of the Ural Macroregion

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Oleg Kryzhanovsky, Alexander Zykov

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Oleg Kryzhanovsky, Alexander Zykov
Bank of Russia, Economic Division, Ural Main Branch

E-mail: 65svc_seu1@cbr.ru

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12 Neglinnaya Street Moscow 107016
Tel.: +7 499 300 30 00, +7 495 621 64 65 (fax)
Bank of Russia website: www.cbr.ru

Abstract

This paper offers an introduction into a new macroeconomic model of the Ural Macroregion named DEMUR (the Dynamic Equilibrium Model of the Ural Region). DEMUR is a regional semi-structural model that includes some key characteristics of the Ural economy for analysing the implications of monetary policy measures and forecasting. DEMUR is built in the logic of neo-Keynesian models with real and nominal rigidity. It also takes into account the structure of a small open economy, external (relative to the region) monetary conditions and other factors that drive changes of the Ural economy.

The model is estimated by Bayesian methods based on international OECD, EAI, FRED and FAO statistical data, federal and regional statistical data by Rosstat and the Bank of Russia for 2009 Q1–2020 Q4. While describing DEMUR's properties, we demonstrate the model's capabilities by decomposing historical and forecast data. The model enables the analysis of changes in economic indicators on both Russian and macroregional levels in response to domestic or external macroeconomic shocks, and quantifies the macroregion's contribution to changes in countrywide indicators, making it a valuable tool for macroeconomic analysis.

Keywords: gross regional product, forecasting models, QPM, quarterly projection models, semi-structural models, monetary policy models, inflation targeting.

JEL classification: C11, C13, E30

1. Introduction

To make effective monetary policy decisions, monetary authorities need to not only consider their implications for the overall national economy, but also for individual regions. Regions may vary in terms of the business cycle stage, the rate of inflation, budget expenditures, and involvement in global production chains. In Russia, the heterogeneity of macroeconomic development can be clearly seen at regional level (as we discuss in this paper), which may call into question price stability in individual regions. The analysis of disaggregated information is needed to identify major trends and structural changes that shape the dynamics of regional economic growth.

This approach is widely covered in academic literature. The issue has been drawing the most attention in the euro area where monetary policy is within the remit of the supranational European Central Bank (ECB) – unlike fiscal policy. Some researchers consider monetary policy implications for the welfare of the entire euro area in the context of regional heterogeneity (Benigno, 2004). According to their findings, accounting for the specifics of national economic development may under certain conditions drive improvements in the entire euro area's welfare. Having said that, some researchers (Cristadoro, et al., 2013), (Bouvet, et al., 2013) make the case for the sufficiency of overall euro area information to make informed decisions on the policy rate, whereas an indicator constructed on the basis of national economic data does not add any value.

Some other researchers (Meade, et al., 2005), (Brauning, et al., 2018) investigate the decision-making process at a central bank (the US Fed and the ECB), aiming to highlight potential geographical shifts related to regional/national specifics of economic development. To this end, they have checked on the relevance of specific regional/national variables in the central bank's response policy function. Their findings suggest that US Fed governors give the highest priority to their region's unemployment data. The ECB takes into account inflation divergence within the euro area; its fight against the general inflationary gap is a second priority. This is probably explained by the ECB's fears that tight monetary policy could easily cause deflation in low-inflation economies should the regulator take a tough stance against Europe-wide inflation.

There are a number of papers focused on the implications of the ECB's single policy rate for euro area economies. For example, the paper (Quint, 2016) introduces the *monetary policy stress* concept, defined as the difference between actual ECB interest rates and Taylor-rule implied optimal rates at the member state level for 11 countries, which explicitly takes into account the natural rate of interest to capture changes in trend growth. Monetary policy stress began to intensify when the euro crisis emerged in 2009, after a period of sustained decline between the euro introduction and before the crisis. Importantly, current levels of monetary policy stress have been lower compared to the late 1990s.

This problem is not irrelevant to the Russian economy. The work (Новак, и др., 2020) provides a dual treatment of the need to incorporate regional heterogeneity in the workings of monetary policy: ignoring regional heterogeneity is opposed to reliance on regional representatives of monetary authorities in the formulation of monetary policy. With deviations in monetary policy decisions showing a high correlation with models incorporating complete information, the authors were unable to argue that the regional approach substantially enhances monetary policy analysis enabled by reliance on aggregate data.

Regions' structural specifics, relative changes in effective exchange rates, movements in incomes and stocks (Жемков, 2019) were analysed; the analysis showed that these factors can drive persistent deviations of regional inflation from the national average. It was shown that when the overall nationwide inflation target is reached, price growth rates may be higher in a number of regions (the Central Federal District (Central FD) and the North-Western FD) and lower in others (the Volga-Vyatka FD, the Ural FD, and the Far Eastern FD).

This paper aims to develop a methodology for assessing the implications of monetary policy decisions (i.e. changes in the key rate) for the economy of the Ural Macroregion (hereinafter, the Ural MR) over a medium-term horizon. To solve this problem, we analyse movements in a number of key interrelated indicators at the Ural MR level (the price level, output and the regional interest rate) as well as their changes in response to various domestic and external macroeconomic shocks. A monetary policy shock in this paper is understood to mean solely a change in the key rate level. Shocks of global commodity prices (especially oil price shocks), country risk premium shocks (that accompany changes in international capital movements), monetary policy shocks, and government spending shocks are among the most important shocks driving the economic cycle in the case of the Russian economy (Khotulev, 2020). Such shocks as oil price shocks, global economic shocks, and the Russian economy's shocks (excluding those of the Ural MR) are external to the Ural MR. This approach makes it possible to assess the price level as part of the Bank of Russia's efforts to maintain macroregional price stability via monetary policy, in an environment of domestic and external economic changes, as well as to take into account the state of prices and other indicators (output and the interest rate) with a view to making further monetary policy decisions.

To solve problems like this, a special type of models has been developed and is used – quarterly projection models (QPMs). They include the dynamics of domestic and external variables of the model, as well as their shocks, in forecasting – by quantifying their contribution. This type of model is used by the IMF, the US Fed, the ECB, the Bank of England and many other central banks. The Bank of Russia has developed several modifications of this type of model. Major quarterly projection models are provided in the following papers: (Орлов, 2021), (Крепцев, и др., 2016), (Kreptsev, и др., 2017), (Seleznev, 2016), as well as (Andreev, 2020) and (Новак, и др., 2020).

This study presents our proprietary approach enabling a focus on the specifics of macroregion economies. This work offers an introduction into a new macroeconomic model of the Ural MR named DEMUR (the Dynamic Equilibrium Model of the Ural Region). DEMUR is a regional semi-structural model aimed at describing the main characteristics of the Ural economy for the purposes of monetary policy analysis and forecasting. DEMUR is built in the logic of neo-Keynesian models with real and nominal rigidity – (Christiano, et al., 2005), (Smets, et al., 2003), (Woodford, 2003), (Galí, 2008). Neo-Keynesian models are flexible structures that can include multiple economic mechanisms that are of interest to researchers; they are adapted for monetary policy analysis and produce appropriate forecasting results.

DEMUR is a model of a small open economy operating in a single foreign currency area, in this case, in the Russian ruble space. Monetary authorities set short-term nominal interest rates under the Taylor rule based on countrywide economic indicators. An open economy involves the presence of exporting and importing firms, as well as the ability of agents to save or borrow foreign financial assets.

DEMUR features many properties of existing DSGE-based macromodels: those by the US Fed, the ECB, the Bank of Canada, the Bank of Spain and others. Thus, DEMUR is comparable to its counterpart models and can rely on many years' experience.

DEMUR is estimated by Bayesian methods, which supports the view that they afford a powerful, consistent, flexible and promising tool for estimating economic dynamic models – (An, et al., 2006), (Burriel, et al., 2010):

1. Bayesian analysis is built on a clear set of axioms and is directly linked to decision theory. This relation is especially relevant to DEMUR, considering that the model was developed for applied analysis of monetary policy. Many decisions may have to recognise uncertainty and consider estimates for asymmetric losses.
2. Bayesian analysis offers a clear solution to the problems of incorrect specification and identification that are quite common in assessing DSGE models (Canova, et al., 2006), (Iskrev, 2008).
3. Bayesian estimates are less demanding of a sample size and asymptotic properties, even if assessed against the classic criteria (Fernández-Villaverde, et al., 2004).
4. Ex ante values allow us to input pre-selection information and reduce the dimensionality problem associated with the number of parameters.

5. A probabilistic method such as the Bayesian estimation allows us to restore all the parameter values necessary for monetary policy analysis.
6. Bayesian methods have important computational advantages over the maximum likelihood method in large models such as DEMUR. Simulating the posterior distribution of parameters is a simpler task compared to maximising the likelihood of high dimensionality.

DEMUR can be used to solve three main problems: understanding the dynamics of fluctuations, analysing monetary policy including alternative key rate-setting experiments, and forecasting. Although DSGE models were not specifically developed for this purpose, the history of forecasting with their use was recognised as satisfactory for this class of models – used by the Fed and the ECB – (Edge, et al., 2009), (Christoffel, et al., 2007).

The key advantage of the proposed approach over existing models is the recognition of the interaction and mutual influence of economic indicators at three different levels: the global economy, Russia and the Ural MR, making it possible to forecast and measure the contribution of factors on each level. Among its drawbacks is the fact that the authors identify a limited number of regional factors (metal prices, oil output, expenditures of regional consolidated budgets), which needs expansion in the future in order to enhance the explanatory capability of the model in decomposing historical and forecast data.

Therefore, the main objective of this research is to develop a semi-structural model to investigate the dynamics of core macroindicators of the region (based on the example of the Ural MR) across various scenarios of key rate decisions, taking into account both internal and external factors such as changes in the exchange rate, prices for core commodities (oil, metals, food) and the global interest rate among others.

The paper is structured as follows. First, we show Russia's regional heterogeneity in output dynamics; thereafter we present the structure of the model in diagrams with detailed descriptions of DEMUR equations; further on, the model is solved and estimated based on quarterly data (from 2009 Q1 to 2020 Q4); subsequently, the study presents the results of a number of exercises made with the model. The Conclusions section presents findings of the study.

2. Russia's regional heterogeneity in macroregions' output

The section that follows shows differences in the output variable, which, when aggregated for all regions, loses part of the information necessary for setting the policy rate. The differences in macroeconomic variables suggesting heterogeneity at regional level may bring extra inflationary pressure beyond the pressure reflected in nationwide inflation expectations, thereby posing a threat to overall price stability.

Regional shifts are explained by a number of reasons. When the central bank rate is uniform, rates on actually issued loans vary considerably across regions. The lowest regional premium is recorded in the Central FD, a region that concentrates most credit institutions.

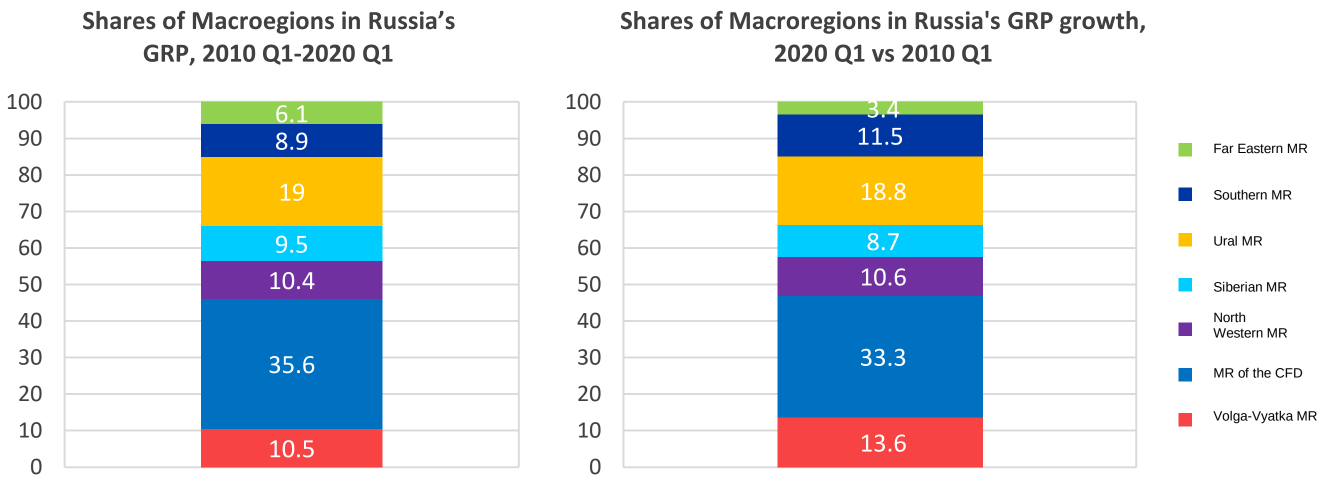
The second reason for shifts is the regional variance in real interest rates depending on the level of expected inflation. If expected inflation in a specific region is high as compared to others, the real interest rate is relatively low then, and economic agents in such regions will show a stronger propensity to borrow and invest, which stimulates economic activity.

Therefore, if economic activity is expected to grow above a countrywide level, the period of stronger economic growth may push up prices stronger, resulting in another spiral of rising inflation. Global experience shows that the most common outcome is an adverse economic situation such as a property market bubble. The opposite is also true: regional inflation holding persistently below the countrywide level is likely to become an additional recession factor.

Estimates of GRP movements by Russian macroregions[1] between 2010 Q1–2020 Q1 show a mismatch between the structure of Russia's total GRP and macroregions' contributions to its growth. This paper relies on calculations of the leading GRP indicator made by the temporal disaggregation method (Boyko, 2020).

Importantly, the contributions of the Far Eastern MR and the Siberian MR to growth are far lower than their share in the structure of the national economy. Conversely, the contributions of the Southern MR and the Volga-Vyatka MR are much higher. This suggests the regional structure of the national economy has been changing rapidly over the past decade.

Chart 1. Shares of Macroregions in Russia's GRP and GRP growth in 2010–2020, %



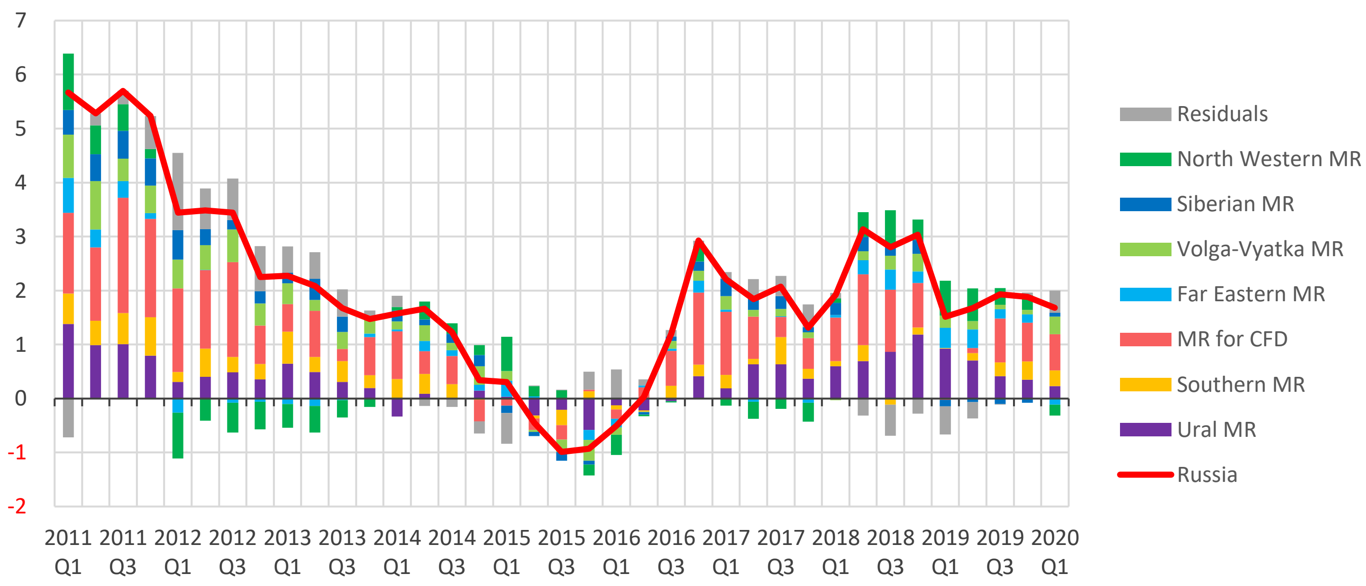
[1] In accordance with the Bank of Russia's regional structure, there are seven macroregions: The Main Branch (MB) for the Central FD, the North-Western MB, the Volga-Vyatka MB, the Southern MB, the Ural MB, the Siberian MB, and the Far Eastern MB. From here on, we shall name as macroregions (MR) the corresponding Main Branches.

In 2015, the structure of growth in GRP saw a drastic change. The strongest decline in growth was recorded in the Far Eastern MR, Siberian MR, Volga-Vyatka MR and Southern MR (roughly threefold in each); a more moderate decline was reported in the MR of the Central Federal District and the North Western MR (between 1.6 and 1.7 times), and the lowest decline – in the Ural MR (1.35 times).

Table 1. Average GRP growth, YoY

Region	2010 Q1–2020 Q1	2010 Q1–2014 Q4	2015 Q1–2020 Q1
Russia	2.06	3.00	1.27
North-Western MR	2.19	2.23	1.46
Far Eastern MR	1.21	1.74	0.56
MR of the Central Federal District	2.01	2.25	1.33
Siberian MR	1.95	3.08	1.03
Volga-Vyatka MR	2.78	4.54	1.17
Ural MR	2.11	2.53	1.87
Southern MR	2.76	3.59	1.23

Chart 2. Russia's GRP breakdown by macroregion, YoY



The breakdown of Russia's GRP by macroregion (Chart 2) shows that the macroregions' contributions vary in different periods, both in absolute numbers and in the plus/minus sign. For instance, since 2012 Q1 the North-Western MR had made a negative contribution to Russia's GRP for eight consecutive quarters.

Comparisons between Russia's and macroregions' output gap movements, obtained with the help of the HP Filter, suggest that the business cycle duration from region to region varies in a broad range.

Table 2. Changes in Russia's and macroregions' business cycles

Region	Changeover to the positive output gap	Changeover to the negative output gap	Changeover to the positive output gap
Russia	2011 Q1	2015 Q2	2018 Q2
North-Western MR	2011 Q1	2013 Q3	2018 Q1
Volga-Vyatka MR	2011 Q2	2015 Q1	2017 Q4
MR for the Central Federal District	2011 Q2	2014 Q4	2017 Q4
Southern MR	2010 Q4	2015 Q1	2017 Q4
Ural MR	2010 Q3	2014 Q4	2017 Q4
Siberian MR	2011 Q2	2014 Q4	2017 Q4
Far Eastern MR	2010 Q2	2012 Q4	2018 Q1

For example, the Far Eastern MR recorded the negative output gap faster than others – in 2012 Q4, whereas the North-Western MR, in 2013 Q3. Other regions and the overall country recorded the negative output gap much later, in late 2014 or early 2015. Consequently, the decline in output growth in the Far Eastern MR and North Western MR is caused by other structural reasons and cannot be explained by countrywide factors.

3. Constructing a regional semi-structural model

3.1. Literature review

DSGE models are frequently used in forecasting, and when estimating the contribution of specific factors and their shocks to changes in key variables. A standard DSGE model may be described as a combination of various theoretical ideas that may be grouped into five components:

1. The model is intended to examine macroeconomic fluctuations, that is co-movements in aggregate time series around a stochastic trend (Lucas, 1977), (Nelson, et al., 1982). The relevant theoretical toolkit is a neo-Walrasian general equilibrium and, specifically, an optimal growth model (Kydland, et al., 1982).

2. The model economy is populated by representative (homogeneous) agents (households and firms). People's behaviour is rational, which suggests that:

1) each agent solves the optimisation problem in a constrained environment (utility/profit maximisation, cost minimisation) within an unlimited number of periods;

2) each agent forms rational expectations as regards the future state of its environment (Muth, 1961);

3) individual optimal plans are interdependent and compatible. Consequently, all markets are perfectly competitive (concurrently and interdependently), and the equilibrium is unique, stable and intertemporal;

4) a total of characteristics of the economy is the sum of agents' individual behaviours, consistent with the microfoundations concept brought forward by Lucas.

3. The model's dynamic results depend on stochastic shifts (shocks). Such shifts are the impulse for fluctuations, while shifts in optimal behaviour of specific agents are responses to or mechanisms for the propagation of oscillations, following the *rocking horse* model (Frisch, 1933). Shocks can be real (related to technology, consumer preferences and producer margins) or nominal (related to interest rates and prices).

4. At the individual level, changes in prices and wages do not occur immediately, which suggests the rigidity of prices at the aggregate level. Nominal rigidity at the macroeconomic level depends on an imperfect competitive environment (monopolistic competition) (Dixit, et al., 1977)), as well as the degree and speed at which prices/wages adjust to changing conditions (Calvo, 1983).

5. Monetary policy plays an active role in the way aggregate equilibrium is determined through the nominal interest rate (Woodford, 2003). The central bank's behaviour follows the Taylor rule (Taylor, 1993).

A classic forward-looking form of the DSGE model (Clarida, et al., 1999) is a system of three equations:

$$\hat{y}_t = E_t(\hat{y}_{t+1}) - \frac{1}{\sigma^c}[i_t - E_t(\pi_{t+1})] + \epsilon_t^c \quad \text{(I)}$$

$$\pi_t = \rho_{\pi+1} E_t(\pi_{t+1}) + \varphi \hat{y}_t + \epsilon_t^a \quad \text{(II)}$$

$$i^*{}_t = (\bar{r} + \bar{\pi}) + \rho_{\bar{\pi}}(E_t \pi_{t+1} - \bar{\pi}) + \rho_{\hat{y}} \hat{y}_t + \epsilon_t^i \quad \text{(III)}$$

Equation (I) (the curve IS) describes a goods market equilibrium (the output gap $\hat{y}_t$) as a function of the expected output gap $E_t(\hat{y}_{t+1})$ of intertemporal elasticity of consumption σ^c and the expected real interest rate; the presence of the negative correlation between the output gap and the real interest rate is assumed.

Equation (II) (NKPC, the neo-Keynesian Phillips curve) determines inflation dynamics π_t as the function of expected inflation, the price rigidity degree φ and output gaps; a positive correlation is assumed between inflation and the output gap.

Equation (III) (a forward-looking Taylor rule) is a central bank tool to set the nominal interest rate $i^*{}_t$ subject to the real long-term interest rate $\bar{r}$, the inflation target $\bar{\pi}$, and deviations in inflation expectations from the target and the output gap. $\rho_{\pi+1}, \rho_{\overline{\pi}}, \rho_{\hat{y}}$ are sensitivity parameters. Deviations of the economy from its steady state depend on the consumer preference shocks ϵ_t^c, the technology shocks ϵ_t^a and the monetary policy shocks ϵ_t^i.

The equation data are the core of the DEMUR model. It would not be enough to just short-list the variables: high-quality macroeconomic analysis of a regional economy requires a much more diverse list of variables. Before adding a new variable to the model – which is capable of enhancing the model's explanatory power (that is, above all, of reducing the model's residuals), this variable needs to be presented in a structural form, that is adjusted for seasonality, with the trend, gaps and residuals identified. The latter are interpreted as shocks to the economy, that is an unexpected change in the variable that any previous historical observations fail to explain.

To investigate the contribution of specific factors and their shocks and to forecast macroeconomic variables, the Bank of Russia uses quarterly forecast models build on a reduced form of a log-linear simple neo-Keynesian DSGE model for a small open economy.

The basic quarterly projection model (QPM) by the Bank of Russia Monetary Policy Department (Орлов, 2021) suggests overall treatment of the Russian economy and the other world as an external factor. This model is based on the principles of a neo-Keynesian DSGE model with the Euler equation (the aggregate demand curve and the Phillips curve), the aggregate supply curve, and the Taylor rule (the monetary policy rule), as well as the absence of arbitration in financial markets. This model is the result of optimising the objectives of consumers, firms and the state; these objectives are however not explicitly formulated. This QPM is more flexible and empirically more aligned with the data; however, it deviates from the theoretical validity of a standard DSGE. Another feature of such models is the need to select one inflation targeting regime, the exchange rate (fixed or floating) and a fiscal rule applicable to the entire period.

Structural models

The present-day methodology of modelling and forecasting time series is known in academic literature under several names:

− Bayesian structural time series models;

− state-space models;

− dynamic linear models;

− Kalman filter models.

Suppose y_t is the value of a quantitative variable recorded at a moment at a certain point t. The structural model of the time series is set by two equations:

$$y_t = Z_t^T \alpha_t + \epsilon_t, \qquad \epsilon_t \sim N(0,\ H_T), \qquad \text{(IV)}$$

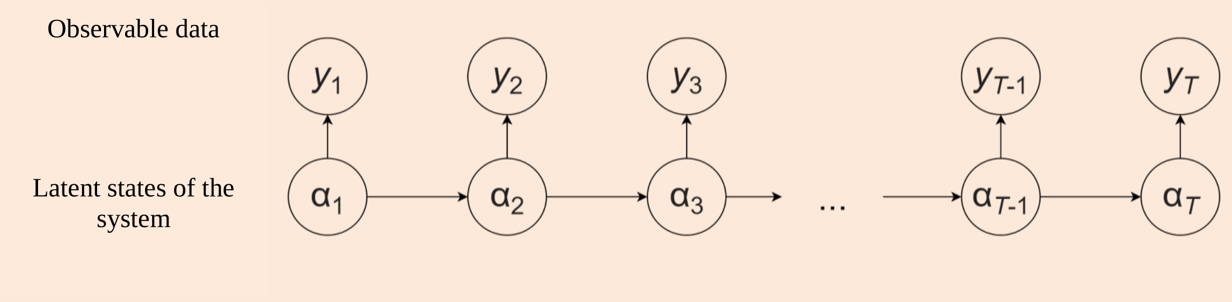
$$\alpha_{t+1} = T_t \alpha_t + R_t \eta_t, \qquad \eta_t \sim N(0,\ Q_T). \qquad \text{(V)}$$

It is assumed that at each point of time t the dynamic system may be in an explicitly unobservable state α_t. Equation (IV) links observed data with the latent state vector and is known as the observation equation. Equation (V) sets the process for a transition from one latent state to another; it is known as the transition equation.

The state of the system at each point of time is conditional only on its state at a previous point, which is commonly termed as the Markovian nature of the system. The residuals ϵ_t and η_t are independent of each other and have a normal distribution with the average value equalling to zero. The matrices Z_t, T_t and R_t are termed as structural parameters.

Models that can be described with the help of equations (IV) and (V) are known as state-space models.

Chart 3. Transition of latent states of the system



Structural time series models are at the same time state-space models. In structural models, the time series is represented as the sum of unobservable components that can be interpreted as a trend, seasonality, predictor effects, and residuals:

$$y_t = \mu_t + \gamma_t + \beta^T x_t + \epsilon_t, \quad \text{(VI)}$$

$$\mu_t = \mu_{t-1} + \delta_{t-1} + u_t, \quad \text{(VII)}$$

$$\delta_t = \delta_{t-1} + v_t, \quad \text{(VIII)}$$

$$\gamma_t = \sum_{s=1}^{S-1} \gamma_{t-s} + w_t, \quad \text{(IX)}$$

where μ_t is the current value of the trend and δ_t is the trend growth factor.

The seasonal component γ_t is presented via S−1 variables with factors that are different for each period.

As defined by equations (IV)–(V),

η_t=(u_t, v_t, w_t) joins together normally distributed random fluctuations.

Q_T is the diagonal matrix whose diagonal contains $\sigma_u^2, \sigma_v^2, \sigma_w^2$.

H_T is the scalar of σ_ϵ^2.

Therefore, based on the data, we need to estimate the variances $\sigma_u^2, \sigma_v^2, \sigma_w^2, \sigma_\epsilon^2$ and the coefficients β of predictors.

In the simplest terms, present-day semi-structural macroeconomic models may be described as follows:

1) key macroindicators have interrelations that can be described theoretically and presented in the form of equations;

2) the time series of each macroindicator may be divided into the following components: the trend, seasonality, the gap (shock) (a change in the period that is absent from the preceding three components);

3) there is a steady state for each macroindicator towards which it moves:

4) each steady state is described by the AR (1) process.

The Bank of Russia’s set of operating models also includes a small DSGE model (Крепцев, и др., 2016). The model is intentionally based on a small number of equations and includes households, companies, the central bank and the external sector. The model features the absence of capital production (nothing else than oil exports). Such a model may serve as a good core for additional variables to be included as add-ons; it may be a good approximation for the Russian economy in certain circumstances but its augmented version is preferred. The authors consider two modifications of such a model for the 2003–2015 period, one with a fixed and one with a floating exchange rate, and conclude that the model with a fixed exchange rate is preferred for the given period of time. The model also outperforms the BVAR model in terms of forecasting capacity as regards the CPI and the exchange rate, although the same does not hold for output forecasts.

A large DSGE model is also in use (Kreptsev, и др., 2017); it is an extended modification of a standard DSGE model and includes the banking sector. Also, the model includes a series of standard agent types with their own conditions optimised, that is households, companies (manufacturers of consumer and investment goods (capital)), retailers and packagers of imported and exported goods, the central bank and the public centre, as well as the external world. The authors conclude that such models have sufficient forecast capabilities with well interpretable impulse responses, enabling analysis of macroeconomic effects of monetary and macroprudential policy.

In addition to these models, we highlight the work (Seleznev, 2016) that is focused on the approximation of a DSGE model around stochastic trends for solving a model without a balanced growth path – which may be called a time-varying DSGE. In a certain way, this model version permits some flexibility to simulate the dynamics of variables following a random walk that does not assume a return to their average levels (e.g. oil prices).

The quarterly projection model in the paper (Andreev, 2020) includes additional equations governing the operating principle of the Russian Finance Ministry’s fiscal rule. This is also a DSGE model that includes standard sectors (households, companies and the central bank) and additionally includes the fiscal rule providing for oil and gas revenue to top up budget accounts and budget spending to be limited to a sum of certain amounts of oil and gas revenues, non-oil and gas revenues and of interest income, as well as by the cap at 0.5% of GDP. The introduction of this fiscal rule did not result in a substantial improvement in the forecasts, but had a stabilising effect on a number of model indicators by reducing output and exchange rate volatility, with a disinflationary effect on prices with oil price shocks.

Importantly, for all the strengths of the above papers in terms of forecasting countrywide indicators, their forecast fail to take regional indicators into account. The regional heterogeneity factor is indeed considered in the model (Новак, и др., 2020); yet, under the approach outlined in the paper, some region variables originate from regional level data, and some are aggregated at the countrywide level. Key results of the work were obtained by the simulation method, for which three models were used. The multi-regional (global) version of the model supports fully informed decisions on monetary policy. The regional approach to monetary policy is based on the elaborated regional model featuring an independent monetary policy instrument. The third approach relies on a model similar to that used in modelling a region, but its estimates are based on information aggregated for the whole country. The approach of weighing aggregate and regional information enables potential reduction in monetary policy decision errors related to regional heterogeneity.

DEMUR's key feature (provided that all the above-mentioned models have the same construction methodology – QPM) is including a third block in the analysis (in addition to the *country* and the *rest of the world* blocks). Unlike the large DSGE model, DEMUR does not incorporate the banking sector and, as opposed to the quarterly projection model in the work (Andreev, 2020), does not apply the Russian Ministry of Finance's fiscal rule. Unlike the Bank of Russia's basic QPM (Орлов, 2021), DEMUR governs the ruble/dollar exchange rate by equation (29) that does not contain oil price and/or output indicators. As opposed to all the models studied, the public sector at regional level in DEMUR is represented by expenditure of the Ural MR's consolidated budget in constant prices.

Further avenues of research may include calculations of a Russian equivalent to monetary policy stress (Quint, 2016), determined as the difference between actual Bank of Russia rates and rates calculated under the Taylor rule for the seven macroregions of Russia. In the original paper (Quint, 2016), the author views 'the stress from the common monetary policy as the difference between the actual ECB interest rates and Taylor-rule implied optimal rates at the member state level, which are estimated on the basis of actual data assuming the presence of common targets for inflation and output gap. The analysis concludes that the 'stress' for euro area countries dropped ahead of the 2008–2009 crisis and went up after the crisis, driven by diverse movements in member country indicators. The paper allows us to suggest that a single monetary policy covering a range of heterogeneous economies may incur certain economic losses and costs for at least some countries since in each specific case it may be suboptimal. The method of the paper, as well as that of the QPM, makes it possible to estimate the effects of monetary policy decisions; however, it is another type of estimation that is fundamentally different from the QPM models and goes beyond the scope of this research.

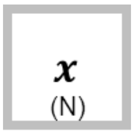
The work (Burriel, et al., 2010) also provides insights into the effect of key rate decisions on a national economy. With focus on the implications of ECB decisions on the Spanish economy, the authors introduce MEDEA (Modelo de Equilibrio Dinámico de la Economía Española), a dynamic equilibrium model for the Spanish economy, a DSGE-type QPM. It is used for analysis of monetary policy decisions and forecasting macroeconomic indicators. MEDEA is built on the neo-Keynesian growth model with households optimising their welfare, and firms growing as the population grows and technology advances. The model also assumes the nominal and real rigidity of prices and wages, the presence of consumption patterns and a range of adjustment costs – in investing, importing and exporting, as well as the presence of fiscal and monetary authorities that set tax rates and interest rates respectively. Separately, there are stochastic shocks in population growth and technological advances, as well as in consumer preferences and economic policy, which highlight the specific nature of the Spanish economy such as slow technological growth and a sizeable share of migrants in the labour market. One of the particular qualities of the model – relevant to the concept of our work – is the inclusion of variables of different levels. MEDEA uses variables of the Spanish economy level, the euro (ECB policy) area, and the entire world. Similar to the terms of this paper, Spain can be viewed as a kind of *macroregion* in the 'ECB state'. The authors' analysis relies on two approaches: under the first, Spain is artificially estimated for the entire period from 1986 to 2007 as a country with its own independent monetary policy conducted by the national body, as had been the case before the euro area emerged. Under the second approach, the Spanish economy is estimated in the presence of the ECB interest rate and does not make a substantial impact on ECB indicators. The researchers note that the introduction of the exogenous common ECB rate significantly changes a number of indicators, including an increase in the mark-ups and the deviation of the Taylor rule shock, similar to the monetary policy shock described above, which is natural if the deviation from the optimal interest rate is rising both at the level of a certain country (Spain) and in aggregate for ECB countries.

However, the case of the Russian economy and its regions has a number of differences from the euro area/ECB case. First of all, these are different characteristics of the two economies that introduce additional variables for analysis. For example, the influence of oil and metal prices stands out in the case of Russia, and specifically the Ural MR given its focus of exports. Furthermore, Russian macroregions operate as components of a single state, which involves not only a single monetary policy, as in the case of the ECB, but also fiscal policy, as well as a range of shocks typical of the entire state – unlike the euro area where shocks are rarely shared by all member countries. All these considerations represent additional rationale for a separate model to be created to analyse and forecast Russian macroregions' indicators within the national economy and the outside world.

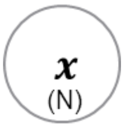
3.2. DEMUR description

Our work presents DEMUR, a regional semi-structural model of the Ural MR economy. All DEMUR indicators have a one-quarter frequency. The gaps are denoted by $(\widehat{x})$, the trend components are represented by $(\bar{x})$, the growth rates on a sustained growth path are denoted by $(\bar{x}^{ss})$, and the shocks are written as (ε_x). The seasonal component was removed using X13-ARIMA-SEATS. Large symbols (X) denote variables in original shape, small symbols denote (x) in natural logarithms. Annualised seasonally adjusted growth rates on a quarter-on-quarter basis are denoted as $(X_{i,\ t}^{QoQ\ SAAR})$, annual growth rates, as $(X_{i,\ t}^{YoY})$. The initial values of estimated coefficients of the indicators are taken from academic literature on the subject (Benes, 2017), (Новак, et al., 2020); alternatively, coefficients of lag (1) from the process AR (1) were taken, with their subsequent Bayesian assessment conducted in IRIS Toolbox.

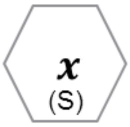
To understand the general structure of the model, below are diagrams showing interaction between core DEMUR components, divided into three blocks: the Ural MR, Russia, and the Rest of the World (ROW). Diagram components are grouped into the following categories:



– variables of IS-PC-MR equations;



– synthetic variables representative of real (in constant prices) indicators or conditions in a specific area;



– the rest of variables, defined within structural models, with each of them governed by the set of equations S:

$$x_{i,\ t}^{QoQ\ SAAR} = 4 * (x_{i,\ t} - x_{i,\ t-1}); \tag{S1}$$

$$x_{i,\ t} = \bar{x}_{i,\ t} + \hat{x}_{i,\ t}; \tag{S2}$$

$$\bar{x}_{i,\ t}^{QoQ\ SAAR} = \rho_{\bar{x}_{i,\ t}^{QoQ\ SAAR}} * \bar{x}_{i,\ t-1}^{QoQ\ SAAR} + \left(1 - \rho_{\bar{x}_{i,\ t}^{QoQ\ SAAR}}\right) * \bar{x}_{i,\ t-1}^{ss\ QoQ\ SAAR} + \varepsilon_{\bar{x}_{i,\ t}^{QoQ\ SAAR}}; \tag{S3}$$

$$\bar{x}_{i,\ t}^{QoQ\ SAAR} = 4 * (\bar{x}_{i,\ t} - \bar{x}_{i,\ t-1}); \tag{S4}$$

$$\hat{x}_{i,\ t} = \rho_{\hat{x}_{i,\ t}} * \hat{x}_{i,\ t-1} + \varepsilon_{\hat{x}_{i,\ t}} \tag{S5}$$

Figures 4 and 5 are interaction diagrams for core DEMUR components – for the blocks the *Ural MR* and *Russia* between each other and the *Rest of the World* respectively. The number in brackets under the factor name is the counting number of the equation in the model. The arrows show the direction of influence between the factors, the number next to the arrow shows which model equation governs the relationship. The signature (S) under the factor name suggests that its dynamics are governed by the set of equations S.

These diagrams represent the key distinguishing feature of DEMUR vs the QPM used by the Bank of Russia. This model demonstrates dynamically interacting three (rather than two) sectors: the regional sector, based on the example of the *Ural MR* sector (absent from the QPM used by the Bank of Russia), the *Russia* sector and the *rest of the world* sector. Each of the sectors is described by both mandatory general and specific factors: for the Ural MR sector, additional factors have been introduced to describe regional specifics, which enables more accurate forecasts and the decomposition of the macroregion's historical statistics. This approach can be applied to any of Russia's seven macroregions (consistent with the structure of the Bank of Russia's main branches), if the variables influencing the macroregion under study are included in the model.

Figure 4. Interaction between core components of DEMUR: the Ural MR block

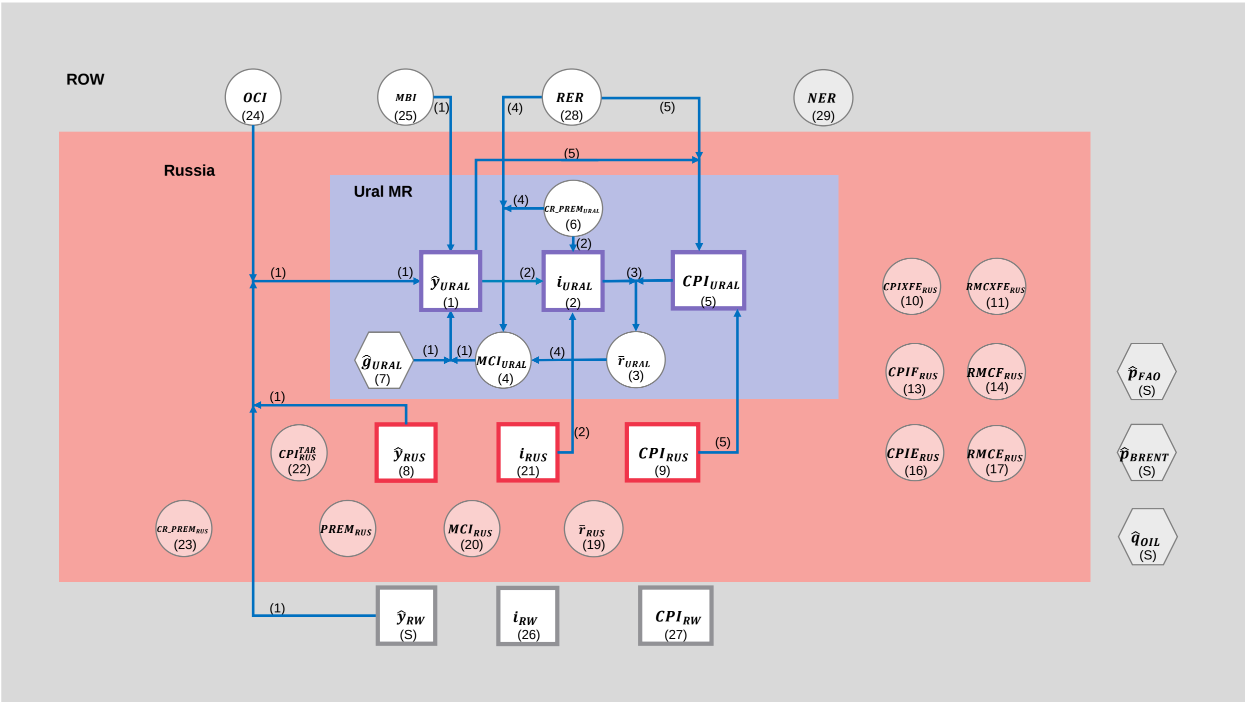
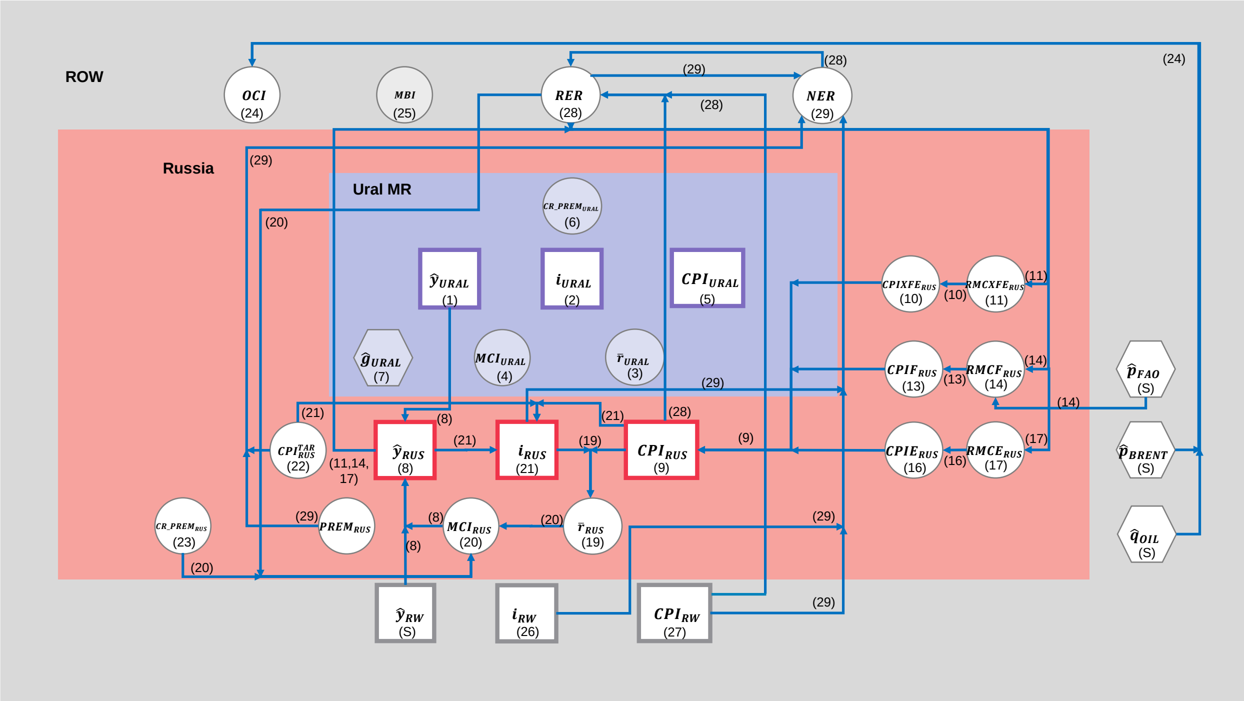


Figure 5. Interaction between core components of DEMUR: the Russia block



Ural MR sector

In addition to the classic QPM factors, the aggregate demand equation (IS) of the Ural MR (1) accounts for the output gap of the Russian economy (excluding the Ural MR), the oil conditions index (OCI_t), and the core metal exports index (MBI_t), where both OCI_t and MBI_t represent deviations from long-run annual average value:

$$\hat{y}_{URAL,\,t} = b1_{URAL} * \hat{y}_{URAL,\,t-1} - b2_{URAL} * MCI_{URAL,\,t} + b3_{URAL} * \hat{y}_{RW,\,t} + \\ +b_{GDP_{RUSwoURAL}} * \left(\hat{y}_{RUS,\,t} - w_{GDP_{URAL}} * \hat{y}_{URAL,\,t}\right) + b9_{URAL} * OCI_t + b7 * MBI_t + \\ +b12 * \hat{g}_{URAL,\,t} + \varepsilon_{\hat{y}_{URAL},\,t}, \quad (1)$$

where $\hat{y}_{URAL,\,t}$ is the output gap for the Ural MR,

$MCI_{URAL,\,t}$ is the monetary conditions index for the Ural MR,

$\hat{y}_{RW,\,t}$ is the output gap for the rest of the world,

$\hat{y}_{RUS,\,t}$ is the output gap for Russia,

$w_{GDP_{URAL}}$ is the weight of the Ural MR in Russia's output,

OCI_t is the oil conditions index, based on current oil output and global oil prices,

MBI_t is the core metal exports index for the Ural MR,

$\hat{g}_{URAL,\,t}$ is spending gap of consolidated budgets for the Ural MR, and

$\varepsilon_{\hat{y}_{URAL},\,t}$ is the output gap shock for the Ural MR.

The nominal interest rate equation for the Ural MR (2) defines the value of the regional nominal interest rate via its previous value, the expected value of the Bank of Russia key rate, the output gap and the Ural MR's credit premium:

$$i_{URAL,\ t} = g1_{URAL} * i_{URAL,\,t-1} + (1 - g1_{URAL}) * i_{RUS,\,t+1} + \\ + g3_{URAL} *\ \hat{y}_{URAL,\,t} + CR_{PREM_{URAL,\,t}} + \varepsilon_{i_{URAL},\,t}\ , \tag{2}$$

where $i_{URAL,\ t}$ is the nominal interest rate in the Ural MR,

$i_{RUS,\,t+1}$ is the Bank of Russia key rate,

$CR_{PREM_{URAL,\,t}}$ is the Ural MR's credit premium, and

$\varepsilon_{i_{URAL},\,t}$ is the nominal interest rate shock for the Ural MR.

$i_{URAL,\ t}$ is the actual observed weighted-average credit interest rate in the Ural MR. The rate's impact on the Ural MR's output cannot be directly compared with the influence of the Bank of Russia key rate on the Russian output (it is far weaker).

The real interest rate for the Ural MR (3) is calculated as the difference between the nominal rate in period *t* and the expected inflation in the Ural MR in the next period:

$$r_{URAL,\ t} = i_{URAL,\,t} - CPI^{YoY}_{URAL,\,t+1}, \tag{3}$$

where $r_{URAL,\ t}$ is the real interest rate for the Ural MR,

$CPI^{YoY}_{URAL,\,t+1}$ is the consumer price index in the Ural MR (year-on-year).

On the one hand, the Ural MR nominal credit interest rate is significantly higher than the Bank of Russia key rate and, on the other hand, the consumer price index for the Ural MR is lower than the countrywide level.

The Ural MR's regional monetary conditions (4) are calculated as the sum of gaps between the real interest rate and the credit premium (for the Ural MR) with the weight $b4_{URAL}$, and the real exchange rate gap with the weight $(1 - b4_{URAL})$:

$$MCI_{URAL} = b4_{URAL} * (\hat{r}_{URAL} + CR_{PREM_{URAL,\ t}}) + (1 - b4_{URAL}) * (-\widehat{RER}_t), \tag{4}$$

where $\hat{r}_{URAL}$ is the real interest rate gap for the Ural MR, and

$\widehat{RER}_t$ is the real ruble/dollar exchange rate gap.

The consumer price index of the Ural MR (5) is computed by a standard forward-looking equation with rational expectations (based on the neo-Keynesian Phillips curve principle), including regional inflation, Russian inflation (without the Ural MR) and output variables.

$$CPI_{URAL,t}^{QoQ\,SAAR} = a1_{URAL} * CPI_{URAL,t-1}^{QoQ\,SAAR} + (1 - a1_{URAL}) * CPI_{URAL,t+1}^{QoQ\,SAAR} + \\ + a55 * ((CPI_{RUS,t}^{QoQ\,SAAR} - w_{CPI_{URAL}} * CPI_{URAL,t}^{QoQ\,SAAR}) - CPI_{URAL,t}^{QoQ\,SAAR}) + a4_{URAL} * \hat{y}_{URAL,t} \\ + \varepsilon_{CPI_{URAL},t}, \tag{5}$$

where $CPI_{URAL,\ t}^{QoQ\,SAAR}$ is the consumer price index for the Ural MR,

$CPI_{RUS,\ t}^{QoQ\,SAAR}$ is the consumer price index for Russia, and

$\varepsilon_{CPI_{URAL},t}$ is the consumer price shock for the Ural MR.

The credit premium for the Ural MR (6) is defined by the process AR (1):

$$CR_{PREM\,URAL,t} = b5_{URAL} * CR_{PREM\,URAL,t-1} + \varepsilon_{CR_{PREM\,URAL,t}}, \tag{6}$$

where $\varepsilon_{CR_{PREM\,URAL,t}}$ is the credit premium shock for Ural MR.

The Ural MR’s consolidated budget expenditures (7) are also defined by the AR (1) process:

$$\hat{g}_{URAL,t} = \rho_{\hat{g}_{URAL}} * \hat{g}_{URAL,\ t-1} + \varepsilon_{\hat{g}_{URAL},t}, \tag{7}$$

where $\varepsilon_{g_{URAL},t}$ is the public spending shock, and

$\rho_{g_{URAL}}$ is the autoregressive coefficient of the Ural MR’s consolidated budget spending gap.

Russian sector

The output gap of Russia (8) is defined by its previous values (inertia), Russian monetary conditions and the global output gap:

$$\hat{y}_{RUS,t} = b1_{RUS} * \hat{y}_{RUS,t-1} - b2_{RUS} * MCI_{RUS,\ t} + b3_{RUS} * \hat{y}_{RW,\ t} + \\ + (\varepsilon_{\hat{y}_{RUS},t} - w_{GDP_{URAL}} * \varepsilon_{\hat{y}_{URAL},t}) + w_{GDP_{URAL}} * \varepsilon_{\hat{y}_{URAL},t}, \tag{8}$$

where $MCI_{RUS,\ t}$ is the monetary condition index for Russia, and

$\varepsilon_{\hat{y}_{RUS},\,t}$ is the output gap shock for Russia.

The consumer price index of Russia (9) consists of three components: fuel price indices, food price indices and indices for other consumer goods and services, weighted by consumption in respective categories:

$$cpi_{RUS,\,t} = w_{cpie} * cpie_{RUS,\,t} + w_{cpif} * cpif_{RUS,\,t} + \\ +(1 - w_{cpie} - w_{cpif}) * cpixfe_{RUS,\,t} + \varepsilon_{cpi_{RUS},\,t}, \tag{9}$$

where $cpie_{RUS,\,t}$ is the fuel price index for Russia,

$cpif_{RUS,\,t}$ is the consumer food price index for Russia,

$cpixfe_{RUS,\,t}$ is the consumer price index excluding fuel and food, Russia,

w_{cpie} the weight of fuel in the consumer basket,

w_{cpif} the weight of food in the consumer basket, and

$\varepsilon_{cpi_{RUS},\,t}$ is the consumer price index shock for Russia.

The Russian consumer price index excluding fuel and food (10) depends on its own previous and expected values, as well as real marginal costs of relevant goods producers:

$$CPIXFE_{RUS,\,t}^{QoQ\ SAAR} = a1_{RUS} * CPIXFE_{RUS,\,t-1}^{QoQ\ SAAR} + (1 - a1_{RUS}) * CPIXFE_{RUS,\,t+1}^{QoQ\ SAAR} + \\ +a2_{RUS} * RMCXFE_{RUS,t} + \varepsilon_{CPIXFE_{RUS,\,t}^{QoQ\ SAAR}}, \tag{10}$$

where $RMCXFE_{RUS,t}$ is the real marginal costs of goods producers excluding fuel and food in Russia, and

$\varepsilon_{CPIXFE_{RUS,\ t}^{QoQ\ SAAR}}$ is the Russian price shock excluding fuel and food.

The real marginal costs of goods producers excluding fuel and food in Russia (11) are determined by the Russian output gap, the deviation of the real ruble/dollar exchange rate from its equilibrium level and the deviation of relative prices of respective goods from its long-run average:

$$rmcxfe_{RUS,\ t} = a3_{RUS} * \hat{y}_{RUS,\ t} + (1 - a3_{RUS}) * (\widehat{rer}_t - \widehat{rpxfe}_{RUS,\ t}), \tag{11}$$

where $\widehat{rpxfe}_{RUS,\ t}$ is the relative price gap excluding fuel and food for Russia.

The gap of Russian relative prices excluding fuel and food (12.1) is equal to the deviation of these relative prices from their long-run average values. The relative prices of goods excluding fuel and food (12.2–12.3) are equal to the deviation of their price index from the overall consumer price index of the country:

$$\begin{cases} \widehat{rpxfe}_{RUS,\ t} = rpxfe_{RUS,\ t} - \overline{rpxfe}_{RUS,\ t}\ ; & (12.1) \\ rpxfe_{RUS,\ t} = cpixfe_{RUS,\ t} - cpi_{RUS,\ t}\ ; & (12.2) \\ \overline{RPXFE}^{QoQ\ SAAR}_{RUS,\ t} = 4 * \left(\overline{rpxfe}_{RUS,\ t} - \overline{rpxfe}_{RUS,\ t-1}\right). & (12.3) \end{cases}$$

Several comments are needed on the above equations. The Russian consumer price index excluding fuel and food (10) depends on its expected one-quarter ahead value, as well as on its previous value. This inflation is also in positive dependence from the domestic production gap and the real exchange rate gap (since real devaluation increases the domestic cost of imported intermediate and finished goods and creates upward pressure on prices). In addition to these standard mechanisms, this modified Phillips curve includes the gap of relative domestic prices excluding fuel and food (11).

The food consumer price index in Russia (13) is determined in the same manner as the consumer price index excluding fuel and food:

$$CPIF^{QoQ\ SAAR}_{RUS,t} = a21_{RUS} * CPIF^{QoQ\ SAAR}_{RUS,t-1} + (1 - a21_{RUS}) * CPIF^{QoQ\ SAAR}_{RUS,t+1} + \\ + a22_{RUS} * RMCF_{RUS,t} + \varepsilon_{CPIF^{QoQ\ SAAR}_{RUS,t}}, \tag{13}$$

where $RMCF_{RUS,\ t}$ is the real marginal costs of food producers for Russia and

$\varepsilon_{CPIF^{QoQ\ SAAR}_{RUS,\ t}}$ is the food price shock for Russia.

The real marginal costs of food producers (14) depend on the Russian output gap, the global food price gap, the real exchange rate gap and the relative food price gap:

$$rmcf_{RUS,\ t} = a23_{RUS} * \hat{y}_{RUS,\ t} + (1 - a23_{RUS}) * (\hat{p}_{FAO_t} + \widehat{rer}_t - \widehat{rpf}_{RUS,\ t}), \tag{14}$$

where $\hat{p}_{FAO_t}$ is the global food price gap;

$\widehat{rpf}_{RUS,\ t}$ is the relative food price gap for Russia.

The gap of Russian relative food prices as well as relative prices themselves are determined in the same manner as relative prices excluding fuel and food (15.1–15.3):

$$\begin{cases} \widehat{rpf}_{RUS,\ t} = rpf_{RUS,\ t} - \overline{rpf}_{RUS,\ t}\ ; & (15.1) \\ rpf_{RUS,\ t} = cpif_{RUS,\ t} - cpi_{RUS,\ t}\ ; & (15.2) \\ \overline{RPF}^{QoQ\ SAAR}_{RUS,\ t} = 4 * \left(\overline{rpf}_{RUS,\ t} - \overline{rpf}_{RUS,\ t-1}\right). & (15.3) \end{cases}$$

The consumer price index for Russia (16) is determined in the same manner as the consumer price index excluding fuel and food:

$$CPIE^{QoQ\ SAAR}_{RUS,\ t} = a31_{RUS} * CPIE^{QoQ\ SAAR}_{RUS,\ t-1} + (1 - a31_{RUS}) * CPIE^{QoQ\ SAAR}_{RUS,\ t+1} + \\ + RMCE_{RUS,\ t} + \varepsilon_{CPIE^{QoQ\ SAAR}_{RUS,\ t}}, \tag{16}$$

where $\varepsilon_{CPIE^{QoQ\ SAAR}_{RUS,\ t}}$ is the Russian fuel price shock.

The real marginal costs of fuel producers (17) depend on the Russian output gap, the global oil price gap, the real exchange rate gap and the relative oil price gap:

$$RMCE_{RUS,\ t} = a33_{RUS} * \hat{y}_{RUS,\ t} + (1 - a33_{RUS}) * (\hat{p}_{BRENT_t} + \widehat{rer}_t - \widehat{rpe}_{RUS,\ t}), \tag{17}$$

where $\widehat{rpe}_{RUS,\ t}$ is the Russian relative fuel price gap (18.1).

$$\begin{cases} \widehat{rpe}_{RUS,\ t} = rpe_{RUS,\ t} - \overline{rpe}_{RUS,\ t}\ ; & (18.1) \\ rpe_{RUS,\ t} = cpie_{RUS,\ t} - cpi_{RUS,\ t}\ ; & (18.2) \\ \overline{RPE}^{QoQ\ SAAR}_{RUS,\ t} = 4 * \left(\overline{rpe}_{RUS,\ t} - \overline{rpe}_{RUS,\ t-1}\right). & (18.3) \end{cases}$$

The real interest rate in Russia (19) is determined based on the current value of the nominal interest rate, adjusted for expected inflation:

$$r_{RUS,t} = i_{RUS,t} - CPI^{YoY}_{RUS,t+1}, \tag{19}$$

where $r_{RUS,t}$ is the real interest rate for Russia.

Monetary conditions in Russia (20) are determined by the gap of the Russian real interest rate and by the Russian credit premium, as well as the deviation of the ruble to US dollar real exchange rate from its equilibrium value:

$$MCI_{RUS,\ t} = b4_{RUS} * (\hat{r}_{RUS,\,t} + CR_{PREM_{RUS,\,t}}) + (1 - b4_{RUS}) * (-\widehat{rer}_t), \tag{20}$$

where $\hat{r}_{RUS,\,t}$ is the real interest rate gap for Russia and

$CR_{PREM_{RUS,\,t}}$ is the Russian credit premium.

The nominal interest rate (Bank of Russia key rate) (21) includes its previous value as well as its neutral level, the deviation of inflation from target, and the Russian output gap:

$$\begin{gathered} i_{RUS,t} \ = g1_{RUS} * i_{RUS,\ t-1} + \\ (1 - g1_{RUS}) * \left(i^{*}_{RUS,\,t} + g2_{RUS} * \left(CPI^{YoY}_{RUS,\,t+4} - CPI^{TAR\ YoY}_{RUS,\,t+4}\right) + g3_{RUS} * \hat{y}_{RUS,\,t}\right) + \\ + \varepsilon_{i_{RUS,\,t}}\,, \end{gathered} \tag{21}$$

where $i^{*}_{RUS,\,t}$ is the neutral interest rate for Russia,

$CPI^{TAR\ YoY}_{RUS,\,t}$ is the inflation target for Russia (YoY), and

$\varepsilon_{i_{RUS,\,t}}$ is the Bank of Russia key rate shock.

The Russian inflation target of 4% is described in the model by the equation (22):

$$CPI^{TAR\ YoY}_{RUS,\,t} = \rho_{CPI^{TAR\ YoY}_{RUS}} * CPI^{TAR\ YoY}_{RUS,\,t-1} + \left(1 - \rho_{CPI^{TAR\ YoY}_{RUS}}\right) * CPI^{ss\ TAR\ YoY}_{RUS} + \varepsilon_{CPI^{TAR\ YoY}_{RUS,\,t}}\ ; \tag{22}$$

where $\varepsilon_{CPI^{TAR\ YoY}_{RUS,\,t}}$ is the inflation target shock.

The Russian credit premium (23) is determined by the AR(1) standard process similar to the Ural MR’s credit premium :

$$CR_{PREM_{RUS,\,t}} = b5_{RUS} * CR_{PREM_{RUS,\,t-1}} + \varepsilon_{CR_{PREM_{RUS,\,t}}}, \tag{23}$$

where $\varepsilon_{CR_{PREM_{RUS,t}}}$ is the Russian credit premium shock.

The country risk premium of Russia $PREM_{RUS,t}$ is determined by the actual 5-year CDS values for Russia.

External sector

Oil conditions OCI_t (24) are determined by deviations of Brent prices and Russian oil output from their equilibrium values:

$$OCI_t = b6 * \hat{p}_{BRENT,t} + (1 - b6) * \hat{q}_{OIL,t}, \quad (24)$$

where $\hat{q}_{OIL,t}$ is the oil out put gap for Russia.

The Ural MR core metal exports index MBI_t (25) is determined by the deviation of global price indices from their own equilibrium (long-term average) values – for iron, copper, titanium, aluminum and zinc (World Bank Commodity Price Data (The Pink Sheet), FRED):

$$MBI_t = b_{IRON} * \hat{p}_{IRON,t} + b_{COPPER} * \hat{p}_{COPPER,t} + b_{TITANIUM} * \hat{p}_{TITANIUM,t} + \\ + b_{ALUMINUM} * \hat{p}_{ALUMINUM,t} + b_{ZINC} * \hat{p}_{ZINC,t}, \quad (25)$$

where $\hat{p}_{IRON,t}$, $\hat{p}_{COPPER,t}$, $\hat{p}_{TITANIUM,t}$, $\hat{p}_{ALUMINUM,t}$, $\hat{p}_{ZINC,t}$ are gaps of global price indexes for iron, copper, titanium, aluminum and zinc respectively;
b_{IRON}, b_{COPPER}, $b_{TITANIUM}$, $b_{ALUMINUM}$, b_{ZINC} are coefficients of respective metals in MBI_t.

The global interest rate (26) depends from its own historical values with the rigidity degree $\rho_{i_{RW}}$, and depends from the sum of the global real long-term interest rate $\bar{r}_{RW}$ and global annualised quarterly inflation with the degree $(1-\rho_{i_{RW}})$:

$$i_{RW,t} = \rho_{i_{RW}} * i_{RW,t-1} + \left(1 - \rho_{i_{RW}}\right) * (\bar{r}_{RW} + CPI_{RW,t}^{QoQ\ SAAR}) + \varepsilon_{i_{RW}}, \quad (26)$$

where $\varepsilon_{i_{RW}}$ is the global interest rate shock and

$CPI_{RW,t}$ is the global consumer price index.

The global consumer price index $CPI_{RW,t}^{QoQ\ SAAR}$ (27) is formed as the sum of the previous consumer price index for OECD countries and its long-run equilibrium level, adjusted for the global output gap:

$$CPI_{RW,t}^{QoQ\ SAAR} = \rho_{CPI_{RW}^{QoQ\ SAAR}} * CPI_{RW,t-1}^{QoQ\ SAAR} + \left(1 - \rho_{CPI_{RW}^{QoQ\ SAAR}}\right) * \\ * \left(CPI_{RW,t+1}^{ss\ SAAR} + a_{RW} * \hat{y}_{RW,t}\right) + \varepsilon_{CPI_{RW}^{QoQ\ SAAR}}. \tag{27}$$

The ruble/dollar real exchange rate rer_t (28) accounts for the corresponding nominal exchange rate, global and domestic inflation:

$$rer_t = ner_t + cpi_{RW,t} - cpi_{RUS,t}\ , \tag{28}$$

where rer_t is the logarithm of the ruble/dollar real exchange rate and

ner_t is the logarithm of the logarithm of the ruble/dollar nominal exchange rate.

The ruble/dollar nominal exchange rate (29) takes into account its own previous and future values, the equilibrium ruble/dollar real effective exchange rate, the global inflation rate consistent with the balanced growth path, the Russian inflation target, the Bank of Russia key rate, the global nominal interest rate and the Russian country premium:

$$ner_t = [(1 - e1) * ner_{t+1} + e1 * (ner_{t-1} + \\ + \frac{2}{4} * \left(CPI_{RUS,\,t}^{TAR\ YoY} - CPI_{RW}^{ss\ QoQ\ SAAR} + \overline{RER}_t^{QoQ\ SAAR}\right) + \frac{-i_{RUS,t} + i_{RW,t} + PREM_{RUS,t}}{4}] \\ + \varepsilon_{ner_t}, \tag{29}$$

where $\overline{RER}_t^{QoQ\ SAAR}$ is the trend of changes in the real exchange rate of the ruble,

$PREM_{RUS,t}$ is the country premium for Russia,

$i_{RW,t}$ is the global nominal interest rate, and

ε_{ner_t} is the ruble nominal exchange rate shock.

The calibrated values of all coefficients used are shown in Appendix 2. The model framework affords a wide range of capabilities for forecasting future values of macroindicators:

1) if it is not known how the indicators will change in the future: all indicators change according to their path of motion to a stationary state;

2) if future values of one (or more) indicators are exactly known, for example, the key interest rate for several periods ahead: the implications for other macroindicators are then analysed;

3) if it is assumed that shocks (a combination of shocks) are possible, for example, a sharp downturn in oil prices, a decline in global output, or a weakening of the national currency: the values of shocks are established by experts, and their consequences are analysed within the model.

The basic forecasting algorithm within the model:

1) set the key rate for 4–12 periods ahead (e.g. keep the key rate at 4.25% for all the periods ahead);

2) set domestic output shock values (e.g. the expected shock for 2020 Q3 is minus 3%);

3) set global output shock values (e.g. the expected shock for 2020 Q3 is minus 3%);

4) set the level of global oil prices (e.g. Brent is assumed to average $45 a barrel in 2020 Q3).

As a result of the simulation, we can obtain the impulse response functions (examples are shown in Appendix 3) and the values of indicators under review in tabular and graphic forms. The key advantage of models of this type is the possibility to decompose indicators, based on actual historical data, and the forecast, based on the assumptions.

Figure 6. The Ural MR’s output gap decomposition: example

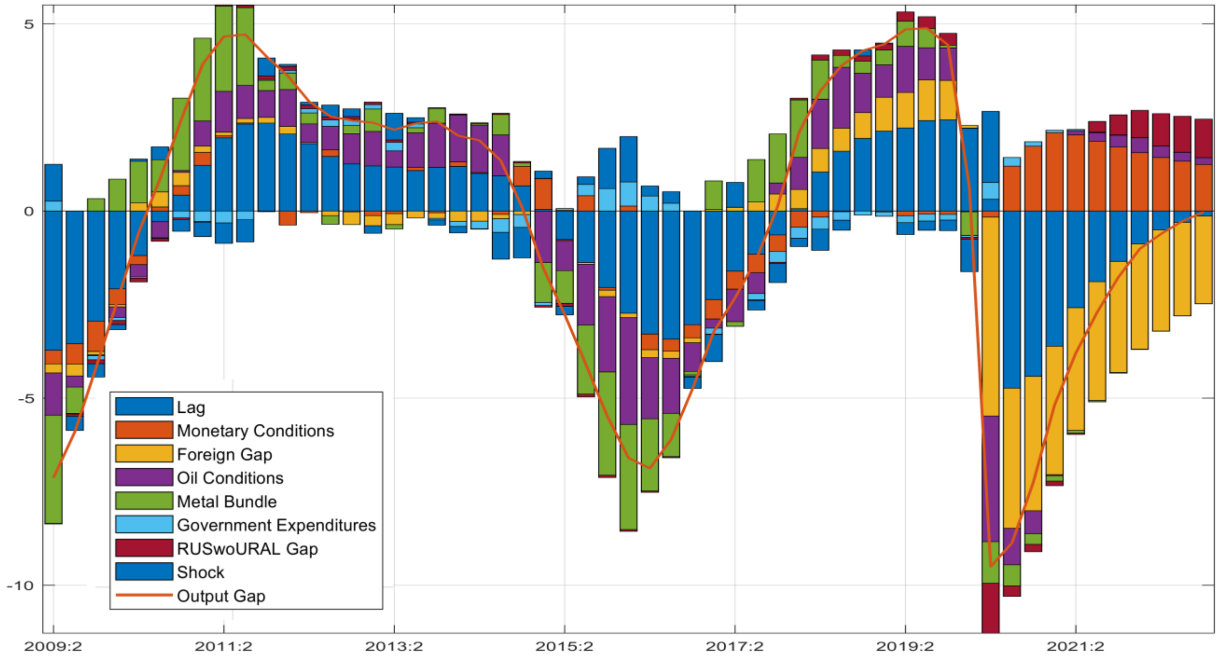


Figure 6 shows the decomposition of the Ural MR's output gap beginning from 2009 Q2. The model suggests that core components of the positive output gap in 2010–2015 were a high (higher than the long-run average) metals basket price, a benign oil conditions index (thanks to prices and output deviating from long-run averages) and the inertia of the output gap. These components are chiefly behind the negative output gap in 2015–2017. In addition, we observe a positive contribution of expenditures of consolidated budgets, which helped stave off a deeper decline in the specified period. The positive output gap in 2017–2019 (besides metals and oil output) was driven by positive output gaps for Russia (excluding the Ural MR) and the global economy (OECD countries).

Bayesian estimation

Estimating the variances $\sigma_u^2, \sigma_v^2, \sigma_w^2, \sigma_\epsilon^2$ and coefficients β of predictors is based on the Kalman filter and Markov Chain Monte Carlo (MCMC).

Let us assume that we have a certain probabilistic distribution. Can we simulate it? It is not always possible to approximate the distribution (as in Laplace's method) with any known distribution so as to calculate it explicitly. The most common problem is a multi-modal distribution with a large set of parameters.

The following two problems for a certain $p(x)$ distribution are normally solved:

1) it is necessary to be able to generate $\{x^{(r)}\}_{r=1}^{R}$ samples according to $p(x)$;

2) it is necessary to be able to estimate the functions' expectations according to the distribution $p(x)$, that is to be able to estimate integrals of the following type:

$$E_p[f] = \int p(x)f(x)dx. \quad \text{(X)}$$

Suppose that x is a vector from R^n with the components x_n, f functions are the moments of stochastic variable dependent on x. Also, suppose that the $p(x)$ function is too complicated and cannot be explicitly calculated.

The problem of estimating the expected function is solved when the sampling problem is successfully solved: the samples $\{x^{(r)}\}_{r=1}^{R}$ need to be taken to make the following calculations:

$$\hat{f} = \frac{1}{R}\sum_r f(x^{(r)}). \quad \text{(XI)}$$

The expected $\hat{f}$ is equal to $E_p[f]$, the variation decreases in inverse proportion R.

Suppose that the function $p^*(x)$ is given, different from $p\ (x)$ only in the normalisation constant:

$$Z = \int p^*(x)dx\,, \quad p(x) = \frac{p^*(x)}{Z}. \quad \text{(XII)}$$

Problems: we do not know Z (most often); the right samples p^* often find their way where p^* is great, which is why we need to determine where p^* is great without calculating it all over.

The Markov chain is given by the initial probability distribution $p^0(x)$ and the probability of the transition $T(x'; x)$, where $T(x'; x)$ is the distribution of the next element of the chain depending on the element it follows; the distribution at the step $t+1$ is equal to:

$$p^{t+1}(x') = \int T(x'; x) p^t(x) dx. \quad \text{(XIII)}$$

The chain should converge to the distribution we are after.

The property of the equilibrium in Markov chains for p and T:

$$\forall x, x' \quad T(x, x')p(x') = T(x', x)p(x). \quad \text{(XIV)}$$

That is, the probability of us choosing x and reaching x' is equal to the probability of choosing x' and reaching x. Such chains are called reversible.

General idea of the Metropolis-Hastings algorithm

Given: the family $q(x', x^{(i)})$, where $x^{(i)}$ is the current state.

q is not an approximation of p, but should be a sampled distribution (e.g. Gaussian). The candidate for the new state x' is sampled from $q(x', x^{(i)})$:

1. It is calculated as follows:

$$a = \frac{p^*(x') * q(x^{(i)}; x')}{p^*(x^{(i)}) * q(x'; x^{(i)})}. \quad \text{(XV)}$$

2. With the probability a: $x^{(i+1)} = x'$, or else $x^{(i+1)} = x^{(i)}$.

The point is that we reach a new distribution centre if we accept the next step. We come to a random walk dependent on the $p^*(x)$ distribution.

Clearly, $x^{(i)}$ are not independent since independent samples are obtained only at large intervals. Equilibrium criterion:

$$\begin{aligned} p(x)q(x; x')a(x'; x) &= min\big(p(x)q(x; x'), p(x')q(x'; x)\big) = \\ &= min\big(p(x')q(x'; x), p(x)q(x; x')\big) = p(x')q(x'; x)a(x; x'). \end{aligned} \quad \text{(XVI)}$$

An important parameter is the distribution variance q. It sets the balance between frequent acceptance and fast movement across the state space.

The decline in the output gap in 2020 Q2 occurred on the back of all components except for inertia and consolidated budget spending. The output gap will close by 2022 Q4 thanks to soft monetary conditions, recovering oil prices and oil output, as well as a positive output gap in Russia (excluding the Ural MR) and a shrinking negative output gap in the global economy (OECD countries). This analysis suggests, for example, that a rise in the price of the metals basket may push up the volume of non-ferrous and ferrous metals output in excess of potential growth rates, and consequently, increase the output gap for the Ural MR. By including this extra factor as an explanatory variable in equation (1), we obtain more accurate estimates of coefficients in the presence of the main factors: inertia, monetary conditions, and the output gap of the rest of the world. Recognition of the relationship between the metals basket and the output gap enables a more accurate assessment of the impact of a common monetary policy.

The model is a tool using the quantitative relationships of economic variables estimated on actual data. In response to the introduction of possible scenarios (sets of a number of indicators, e.g. oil output consistent with the OPEC+ agreement) it provides forecast values for a number of variables for a certain period, in the form of charts and tables with their values. The causal relations between monetary policy measures, inflation and output movements in the macroregion are consistent with regular transmission channels. For instance, the interest rate channel operates as follows. An increase in the Bank of Russia key rate (21) sequentially causes an increase in the interest rate in the Ural MR (2), tighter monetary conditions (4), a lower output gap for Ural MR (1) and, as a result, a lower consumer price index (5). In addition, we observe the effect of the foreign currency channel: the increase in the Bank of Russia key rate (21) leads to a strengthening in the nominal exchange rate (29), which worsens monetary conditions (4) (by increasing the absolute value of the index), reduces the output gap for the Ural MR (1) and as a result reduces the consumer price index (5). The presented model also displays the convergence channel for the MR Ural and countrywide prices (as shown in equation (5)).

The authors do not set out to demonstrate how modelling results can help solve the task of formulating common monetary policy for Russia. The presented work aims to present an estimate of effects of key rate decisions for the economy of a particular macroregion (the Ural MR).

3.3. Quality assessment of DEMUR compared to alternative models

The assessment of the model's capabilities involves a study of its properties compared to alternative models. It is impossible to compare the model's analytical properties with other models used at the Bank of Russia and discussed in the scientific literature that studies the same monetary policy measures, since there had been no task before this study to assess the implications of key rate decisions for the economy of a specific macroregion (the Ural MR).

In terms of theoretical assumptions, the presented model relies on the neo-Keynesian approach briefly discussed in Subsection 3.1. The model is constructed as an add-on of additional variables to the core of IS-PC-MR equations for three blocks: the Ural MR, Russia, and the Rest of the World (ROW). This approach makes it possible to build multiple alternative models, which enable us to identify several alternatives and make a comparative study of each.

We built two additional truncated DEMUR models: DEMURcore (with equation (1) excluding prices of the metals basket, the oil conditions index and regional consolidated budget spending), and DEMURcoreOCI (with equation (1) stripping out metals basket prices and regional consolidated budget expenditure). Also, a VECM model (two co-integrating equations) was built with five lags of three endogenous variables: the output gap for MR Ural $\hat{y}_{URAL,\,t}$, the consumer price index for MR Ural $CPI_{URAL,\,t}^{QoQ\,SAAR}$, the nominal interest rate for MR Ural $i_{URAL,\,t}$; with exogenous variables: $cpi_{RUS,\,t}$, $i_{RUS,t-1}$, $i_{RW,t-1}$, $PREM_{RUS,t-1}$, ner_{t-1}, $CPI_{RW,t-1}^{QoQ\,SAAR}$, $\hat{y}_{RW,\,t-1}$, $\hat{q}_{OIL,\,t-1}$, $\hat{y}_{RUS,\,t-1}$, $\hat{p}_{BRENT,\,t-1}$, $\hat{p}_{FAO\,t-1}$, $\hat{p}_{IRON,\,t-1}$, $\hat{p}_{COPPER,\,t-1}$, $\hat{p}_{TITANIUM,\,t-1}$, $\hat{p}_{ALUMINUM,\,t-1}$, $\hat{p}_{ZINC,\,t-1}$, $\hat{g}_{URAL,\,t-1}$.

To build out-of-sample forecasts of the VECM model, the function predict_rolling of the package tsDyn for R was used.

This function allows checking the accuracy of out-of-sample forecasts by estimating the model on the basis of the sub-sample of the original, each time thereafter making an *nroll* of forecasts for the horizon *n.ahead* while updating the sample. In other words, with this model estimated on the basis of 100 observations, the function will estimate it, for example, on the basis of the 90 first observations (*nroll* = 10) and generate, for example, a one-step ahead forecast (*n.ahead* = 1) of 90 observations; thereafter it will use the true value of 91, 92 observations... until the sample is complete. Unlike the conventional pred () methods, the instruction n.ahead = 2 will not generate one and two step forward forecasts but only an *nroll* of two one-step ahead forecasts (Fabio Di Narzo, et al., 2020).

Also, the random walk (RW) method and ARIMA were used to compare the quality of the forecast estimate. Out-of-sample forecasts were made based on the sample from 2017 Q1 to 2020 Q4 (16 quarterly observations). This time lag was chosen because the VECM model in the sample for a period shorter than 2010 Q1–2016 4 (28 quarterly observations) had an extremely high forecast error for endogenous variables.

Table 3. RMSE of the nominal interest rate for the Ural MR

$i_{URAL,\ t}$	ARIMA	RW	VECM	DEMUR	DEMUR core	DEMUR coreOCI
t1	0.82	0.53	2.62	*0.30*	**0.23**	0.36
t2	1.59	0.92	2.02	*0.68*	**0.54**	0.81
t3	2.12	1.18	2.05	*1.09*	**0.90**	1.26
t4	2.67	*1.42*	2.16	1.45	**1.25**	1.69
t5	2.69	*1.57*	2.10	1.71	**1.52**	2.19
t6	2.74	**1.58**	1.74	1.84	*1.67*	2.72
t7	2.79	**1.52**	1.91	1.86	*1.72*	3.08
t8	2.79	**1.45**	*1.70*	1.82	*1.70*	3.16
t9	2.78	**1.48**	2.96	1.74	*1.63*	2.93
t10	2.71	1.61	**1.49**	1.65	*1.56*	2.44
t11	2.61	1.79	**1.51**	*1.56*	**1.50**	1.76
t12	2.49	1.95	1.62	*1.43*	*1.43*	**1.05**
t13	2.35	1.99	*1.13*	1.27	1.32	**0.68**
t14	2.15	1.91	**0.48**	1.01	1.14	*0.83*
t15	1.94	1.80	2.20	**0.73**	*0.90*	0.95
t16	1.49	1.43	0.91	**0.40**	*0.57*	0.76

Estimated RMSEs of the nominal interest rate for the Ural MR are shown in Table 3. Let us consider the obtained values for the line t1: beginning from 2016 Q4, forecasts were made for each of the 16 subsequent quarters for 16 observations ahead by sequentially adding actual data. Thus, 16 forecast values were obtained based on data available before the forecast period: the forecast for 2017 Q1 was obtained from actual data, as were subsequent forecasts, until the one for 2020 Q4, which was obtained from actual data available as of 2020 Q3. For each obtained forecast value, a forecast error was calculated and the quality of RMSE forecasts assessed to compare the various models with each other. Each line in Table 3 shows values of the model with the lowest RMSE (in bold); the second-best model is put in italics. We can conclude that DEMURcore has the lowest error in forecasting the nominal interest rate for the Ural MR.

Table 4. RMSE of the consumer price index for the Ural MR

$CPI_{URAL,\,t}^{QoQ\ SAAR}$	ARIMA	RW	VECM	DEMUR	DEMUR core	DEMUR coreOCI
t1	2.16	**1.47**	9.08	1.61	*1.59*	2.44
t2	2.86	**1.66**	7.43	1.94	*1.87*	5.05
t3	3.23	**2.16**	8.11	2.22	*2.17*	7.11

t4	3.36	2.16	8.71	*2.12*	**2.08**	8.63
t5	3.47	**1.88**	7.05	*1.92*	**1.88**	9.32
t6	3.49	**1.67**	8.15	1.77	*1.71*	8.91
t7	3.74	**1.11**	7.58	1.71	*1.67*	7.70
t8	3.93	**1.15**	7.00	1.70	*1.68*	6.42
t9	4.04	**1.02**	11.42	*1.70*	*1.70*	5.73
t10	3.95	**1.45**	3.07	*1.68*	1.69	5.26
t11	3.50	**1.60**	5.02	**1.60**	1.64	4.62
t12	3.01	**1.46**	12.42	*1.56*	1.64	3.93
t13	2.23	**1.34**	8.53	*1.43*	1.58	3.30
t14	1.51	**0.86**	14.78	*1.40*	1.58	2.68
t15	**0.97**	*1.11*	20.02	1.36	1.50	2.31
t16	**0.33**	*0.61*	2.08	1.19	1.27	2.10

RMSE estimates of the consumer price index for the Ural MR (Table 4) are quite high for all the models compared to RW. The high error of ARIMA and VECM is explained by the insufficient number of observations. DEMUR and DEMURcore estimates came very close; overall, DEMURcore forecasts the Ural MR's consumer price index for 1–8 periods ahead a little more accurately, while DEMUR slightly better forecasts the Ural MR's consumer price index for 9–16 periods ahead.

RMSE estimates of the output gap for the Ural MR (Table 5) are quite high for all the models compared to DEMUR. A special feature of this indicator is that it is an unobservable variable calculated within the semi-structural model itself. Therefore, while Tables 3 and 4 show comparisons of estimated accuracy of forecasts against the actual observed nominal interest rate for the MR Ural and the consumer price index for the MR Ural, here for the models ARIMA, RW, VECM and DEMUR we use the identical range of values of this indicator, while DEMURcore and DEMURcoreOCI use slightly different data series obtained with the help of the Kalman filter.

The high error of ARIMA and VECM is also explained by the insufficient number of observations. Output gap estimates for the Ural MR by DEMURcore are generally slightly worse than those by DEMUR.

Table 5. RMSE of the output gap for the Ural MR

$\hat{y}_{URAL,t}$	ARIMA	RW	VECM	DEMUR	DEMUR core	DEMUR coreOCI
t1	3.76	1.56	3.08	**0.54**	*0.60*	1.00
t2	4.14	2.49	2.64	**0.83**	*0.89*	1.79
t3	4.44	2.98	2.64	**0.90**	*1.00*	2.46
t4	4.48	3.17	2.51	**0.90**	*1.08*	3.13
t5	4.35	3.05	2.94	**0.87**	*1.13*	3.84
t6	4.20	2.87	3.03	**0.84**	*1.14*	4.27
t7	4.13	2.77	2.73	**0.81**	*1.11*	4.25
t8	4.27	2.79	2.63	**0.78**	*1.07*	3.84
t9	4.60	2.73	9.38	**0.77**	*1.05*	3.19
t10	4.39	2.49	2.83	**0.76**	*1.04*	2.38
t11	4.13	2.15	5.23	**0.70**	*1.00*	1.51
t12	2.83	1.81	4.19	**0.67**	*0.97*	1.14
t13	2.72	1.49	2.04	**0.64**	*0.94*	1.40
t14	2.78	1.36	4.30	**0.56**	*0.86*	1.61
t15	2.35	0.94	11.35	**0.27**	*0.49*	1.35
t16	1.57	0.45	6.16	*0.19*	**0.12**	0.84

We can therefore conclude that DEMUR is suitable for forecasting the Ural MR's key macroeconomic indicators and analysing their contribution to their own changes and the shocks that have occurred, that is, an unexpected change (in excess of the potential (trend) growth rate and consistent approximation to it) in the values of external factors such as the ruble exchange rate, oil prices, OPEC+ oil output curbs, the Bank of Russia key rate, prices for metals mined and/or processed in the Ural MR and the rest of factors detailed in Subsection 3.2.

The use of semi-structural models of this type is a global practice and makes it possible to combine the use of current mathematical methods and the close correlation of key macroeconomic indicators through multiple channels of mutual influence whose duration and impact are widely varied. The key advantages of such models include their practical focus, that is their ability to interpret the forecast by components, the relative ease of including additional factors (from a technical point of view), and the analysis of an unlimited number of possible scenarios for the development of factors included in the model.

4. Concluding remarks

The presented DEMUR model is a modification of the neo-Keynesian model of a small open economy. It has the following key features characterising the Ural MR's economy:

1. Stochastic output growth is driven by a neutral technological progress.
2. The global economy is not modelled as a result of an equilibrium that is beyond the scope of the Bank of Russia's behaviour under the Taylor rule. Even more so, the Ural MR is too small to have any significant impact on the global economy.
3. The influence of the public sector on the economy is implemented through the use of consolidated budget spending indicators for the Ural MR (although more research is needed in this area).
4. In addition to Brent price dynamics, changes in oil production are also included (according to EAI and OPEC data). As OPEC+ output quotas are regularly taken into account, we can present a more accurate explanation for output changes, and forecast output while taking into account current and planned restrictions.
5. Accounting for the dynamics of global prices for base metals mined and/or processed in Ural: iron, copper, aluminium, zinc, and titanium. We believe that the cycle of global metal price changes should reasonably correspond with the investment cycles of major metal processing companies and influence changes in physical output.
6. Russia's inflation and output data are divided into two components: *the Ural MR* and *Russia without the Ural MR*, which makes it possible to account for regional inflationary expectations, regional inflation and the regional output gap.
7. The national currency is under a floating exchange rate regime, and its changes are determined by uncovered interest rate parity.

The proposed approach enables us to determine the consequences of planned and/or unforeseen changes in key macroeconomic indicators both for Russia as a whole and the Ural MR, as well as to quantify the contribution of the Ural MR to changes in Russian economic indicators. Moving forward, the model could be developed horizontally (for example, by including the other six macroregions, with Russia presented as the sum of seven regions) and vertically (for example, by analysing and forecasting the leading GRP indicator by five components).

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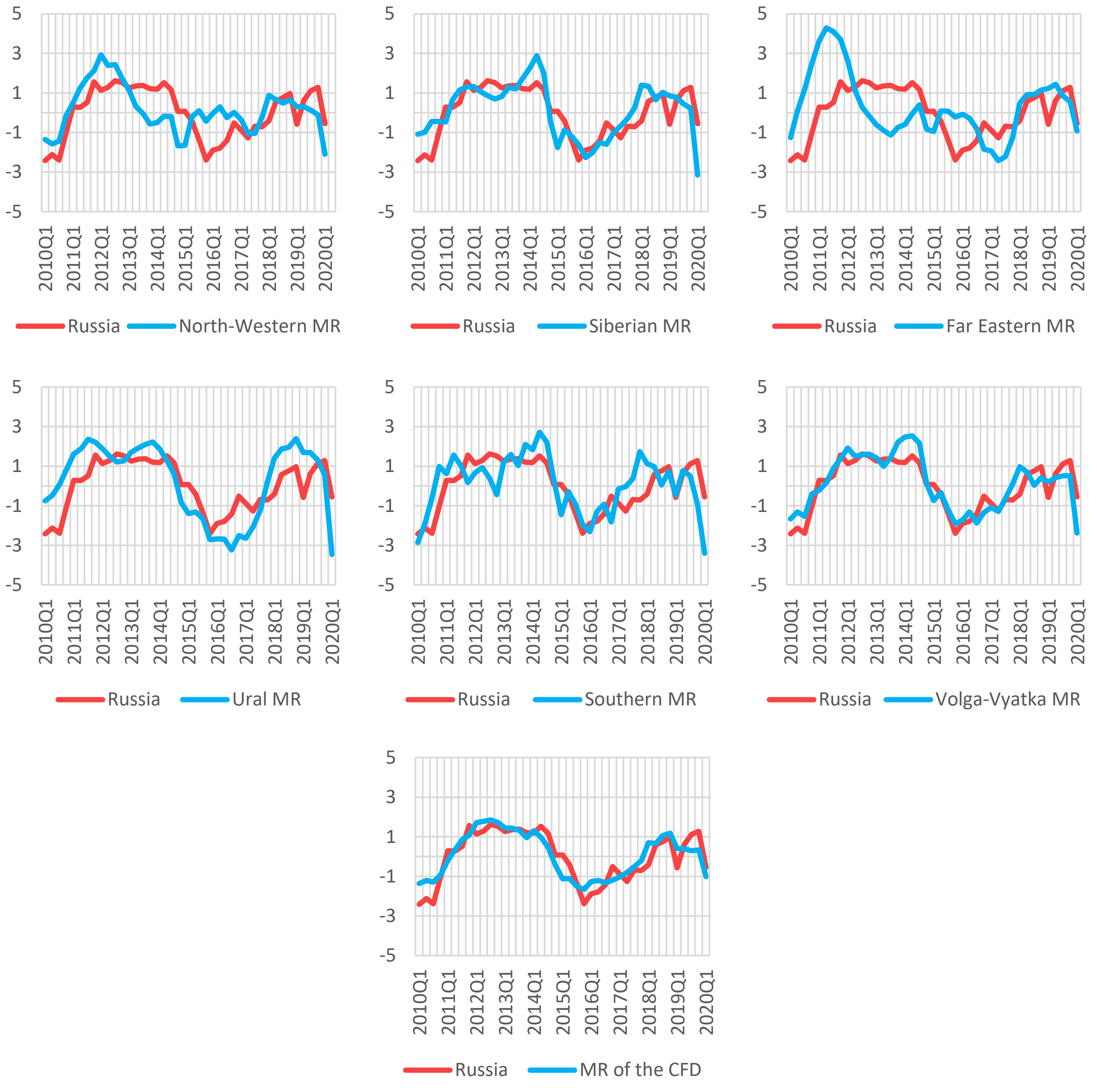
Annexes

Annex 1

GRP gaps for Russia and macroregions compared

Figure 1. Dynamics of GRP gaps for Russia and macroregions compared



Annex 2

Model equation coefficients

Table 1 Calibrated values of the model coefficients

Legend	Value	Coefficient interpretation	Equation
$b1_{URAL}$	0.5564	The Ural MR's output resilience	(1)
$b2_{URAL}$	0.1111	Monetary policy impact on the Ural MR's output	(1)
$b3_{URAL}$	0.2412	External demand impact on the Ural MR's output	(1)
$b_{GDP_{RUSwoURAL}}$	0.2449	Impact of Russia's output gap (excluding Ural) on the Ural MR's output gap	(1)
$w_{GDP_{URAL}}$	0.1889	Share of the Ural MR's GRP in Russia's GRP	(1.8)
$b9_{URAL}$	0.2108	Impact of the OCI on the Ural MR's output gap	(1)
$b7$	0.6451	Impact of the MBI on the Ural MR's output gap	(1)
$b12$	0.7043	Impact of the gap in expenditure of the Ural MR's consolidated budgets on Ural MR's output gap	(1)
$g1_{URAL}$	0.6892	The Ural MR's interest rate resilience	(2)
$g3_{URAL}$	0.2949	Impact of the output gap on the Ural MR's interest rate	(2)
$b4_{URAL}$	0.8566	Real interest rate weight in the Ural MR's monetary conditions	(4)
$a1_{URAL}$	0.3618	The Ural MR's inflation resilience	(5)
$a4_{URAL}$	0.2279	Impact of the Ural MR's output gap on the Ural MR's inflation	(5)
$a55$	0.7039	Impact of Russian inflation's deviation (excluding the Ural MR) from the Ural MR's inflation on the Ural MR's inflation	(5)
$w_{CPI_{URAL}}$	0.1334	Weight of the Ural MR inflation in nationwide inflation	(5)
$b1_{RUS}$	0.7994	Resilience of Russia's output	(8)
$b2_{RUS}$	0.2280	Monetary policy impact on Russia's output	(8)
$b3_{RUS}$	0.5558	Impact of external demand on Russia's output	(8)
w_{CPIE}	0.0441	Weight of fuel inflation in the CPI	(9)
w_{CPIF}	0.3681	Weight of food inflation in the CPI	(9)
$a1_{RUS}$	0.4568	Resilience of Russia's inflation	(10)
$a2_{RUS}$	0.0410	Impact of marginal costs on Russia's inflation	(10)

$a3_{RUS}$	0.3786	Share of domestic goods in marginal costs of Russian firms	(11)
$a21_{RUS}$	0.4316	Resilience of Russia's food inflation	(13)
$a22_{RUS}$	0.1143	Impact of global food prices on Russia's food inflation	(13)
$a23_{RUS}$	0.5506	Impact of the business cycle on Russia's food inflation	(14)
$a31_{RUS}$	0.4935	Resilience of Russia's fuel inflation	(16)
$a33_{RUS}$	0.3936	Impact of the business cycle on Russia's fuel inflation	(17)
$b6$	0.7914	Weight of oil prices in OCI	(24)
$b4_{RUS}$	0.6378	Real interest rate weight in Russia's monetary conditions	(20)
$g1_{RUS}$	0.8503	Resilience of Russia's interest rate	(21)
$g2_{RUS}$	0.4180	Weight of inflation in the Taylor Rule	(21)
$g3_{RUS}$	0.4412	Weight of the output gap in the Taylor Rule	(21)
$e1$	0.5509	Resilience of the exchange rate of the national currency	(29)
b_{IRON}	0.2000	Iron price factor in the Metal Bundle Index (MBI)	(25)
b_{COPPER}	0.2000	Copper price factor in the MBI	(25)
$b_{TITANIUM}$	0.2000	Titanium price factor in the MBI	(25)
$b_{ALUMINUM}$	0.2000	Aluminium price factor in the MBI	(25)
b_{ZINC}	0.2000	Zinc price factor in the MBI	(25)
$b5_{URAL}$	0.7381	Resilience of the Ural MR's credit premium	(6)
$b5_{RUS}$	0.6930	Resilience of Russia's credit premium	(23)
$\rho_{CPI_{RW}^{QoQ\ SAAR}}$	0.3177	Resilience of global inflation	(27)
a_{RW}	0.4423	Impact of the global output gap on global inflation	(27)

Annex 3

Impulse response functions

Figure 2. Response of the variables to Russia's output gap

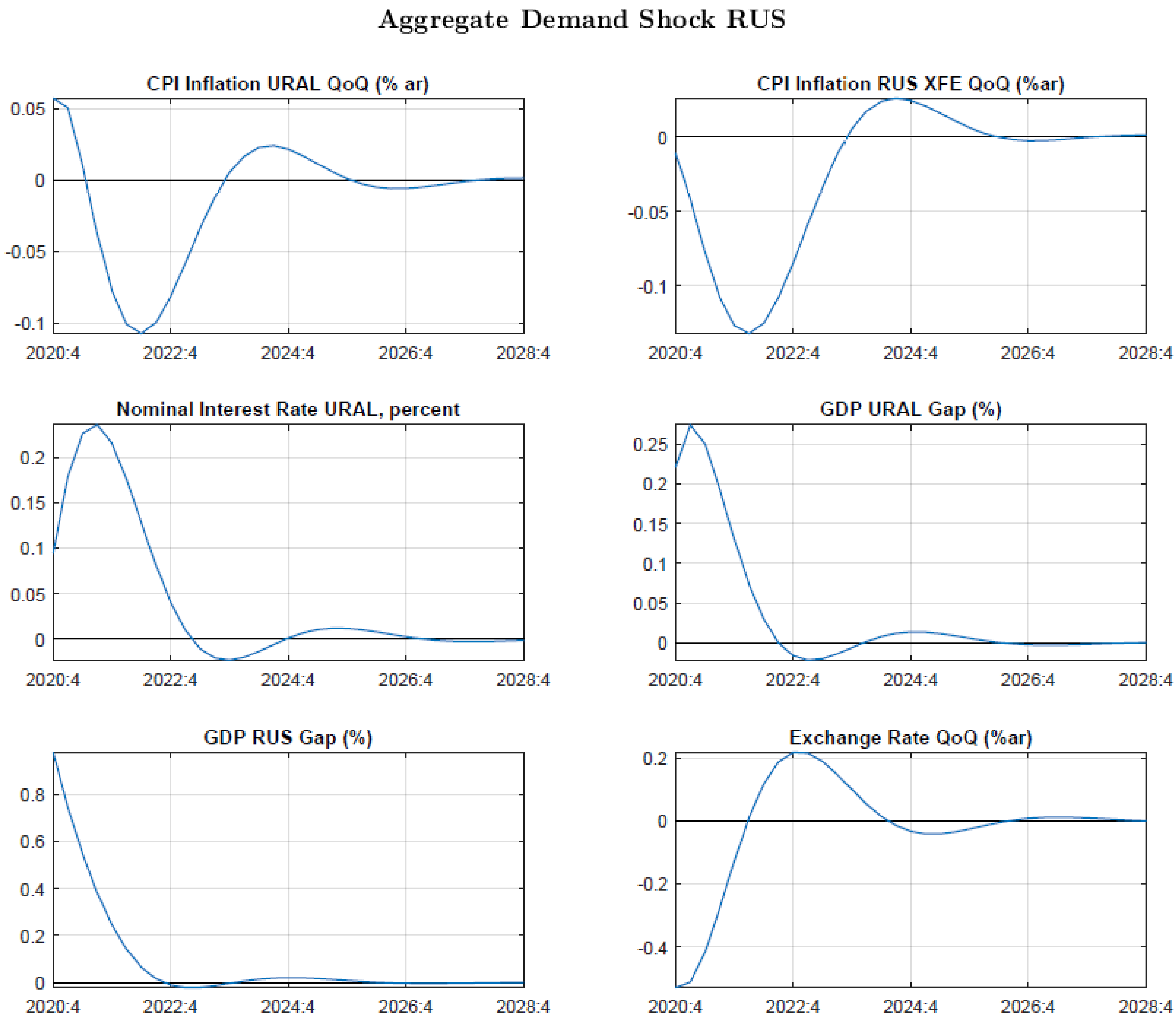


Chart 3. Response of the variables to the output gap of the rest of the world

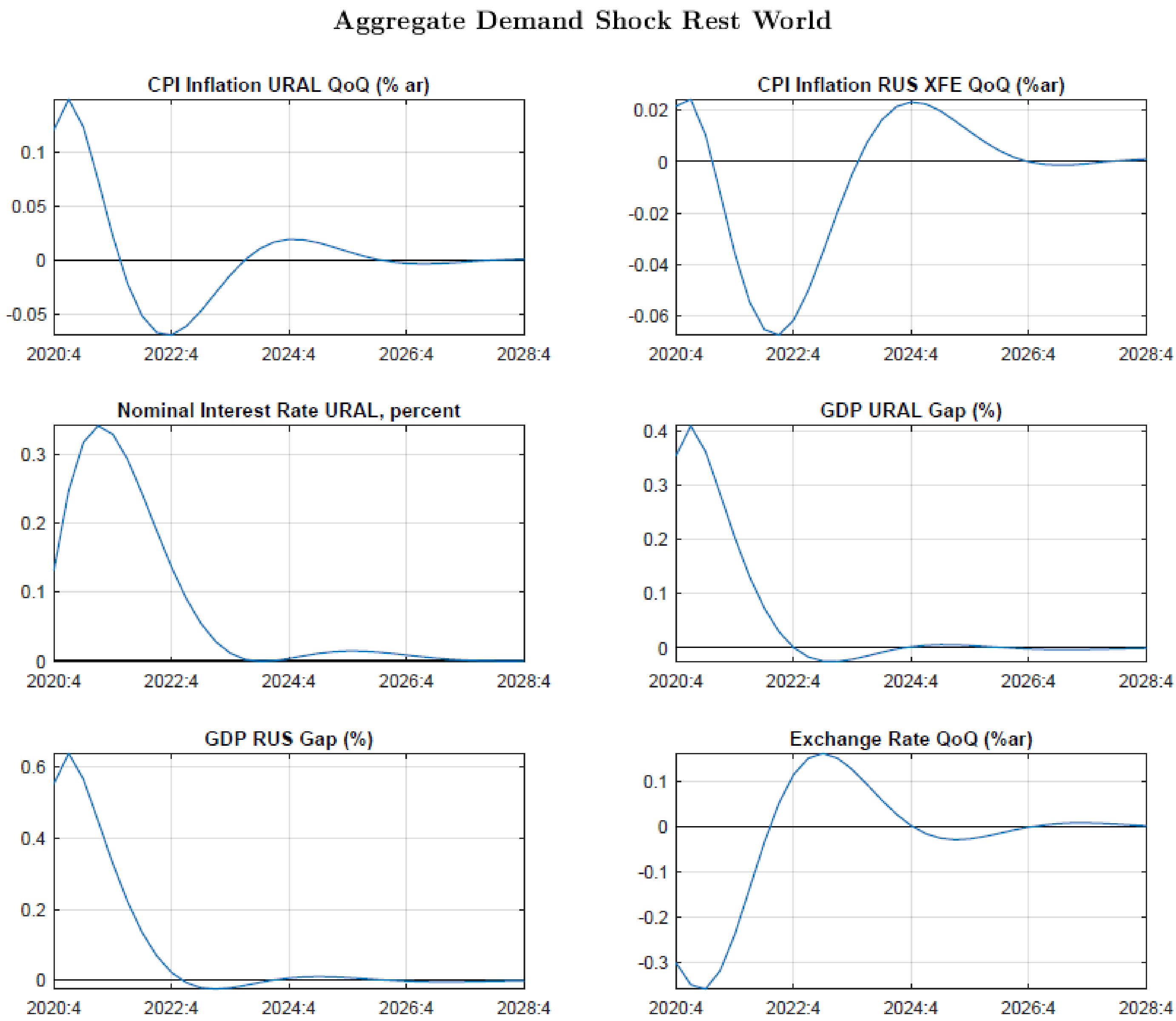


Chart 4. Response of the variables to the monetary policy (interest rate) shock