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Abstract

Default correlation parameter has a material impact on the loan portfolio credit risk. Moreover, the impact is more complex than that of the default probability itself. Current study shows that the rise in default correlation can simultaneously lead to multi-directional changes in different types of risk-measures or focus on a single risk measure, but at different confidence levels. The cause for such dual impact lies in the often neglected rising impact of the default rate (DR) distribution bimodality. In general, we evidence that rise in default correlation produces a multiplicative effect of the probability of default (PD): risk measure declines for low PDs and rises for high PDs, but changes are non-proportionate for the same changes in default correlation.

Similar effects , in particular, may arise when augmenting the proportion of "green" lending. Moreover, when such a trend is associated with the decline in PD for the "green" sector and PD rise for the "brown" one, there is an overall reduction in the loan portfolio credit risk in the long-run. However, it is witnessed only after its rise in the mid-term.

The paper is accompanied with the relevant codes. They enables the interested parties to replicate the findings, as well as to derive credit risk parameters for any given DR time series and model a DR distribution for any set of distribution mixture parameters.

Key Words: default correlation; bimodal distribution; default rate; cliff effect; mixture of distribution.

JEL Codes: C34, C67, E52, H23, O44.

Contents

1. Introduction

The core competence of any bank is its capability to efficiently collect (workout) its debts. However, inadequate credit risk assessment when approving (granting) loans might require soliciting the core competence too often. This is why every bank and its regulator is interested in proper credit risk assessment. Moreover, disregarding how perfect separate credit risk parameter models might be, the bank and its regulator again are ultimately interested in the adequacy of the total risk value, i.e., at the loan portfolio level.

Probability of default (PD)) may be deemed one of the most well-known credit risk indicator. It is indeed an important risk driver. However, it is not the only one to produce material impact on the loan portfolio credit risk assessment. It is the **default correlation** which plays not least role in determining the total credit risk amount. Its importance grows fast in the modern world. The Bank for International Settlements (BIS) registered the first spike during the start of pandemics, as discussed by Aramonte and Avalos (2020). Similar effects may arise because of the world economy fragmentation, energy transition, climate change. This is why the default correlation has become, at least, an equal contributor to the total credit risk measure, as PD is, and at maximum occupied even the dominant role.

Thus, the research objective is to enhance our understanding of the impacts produced by the default correlation and its rise specifically. More importantly, we wish to know whether the impact is symmetric for the low- and high-risk segments; to what extent the credit risk value is robust to the choice of the risk-measure (its type and the confidence levels of interest).

To briefly preview the findings, we may state that the default correlation produces a multiplicative effect to the probability of default. When default correlation rises, the credit risk value declines for the low PD segments and rises for the high PD ones. Moreover, the same change in default correlation implies non-proportionate (asymmetric) change in the total portfolio default rate (DR). Moreover, rise in default correlation may simultaneously lead to the rise and decline of the measured credit risk indicators, when monitoring the evolution of the risk profile by various indicators (by different risk measures or at different confidence levels). This is the feature of the often neglected DR distribution bimodality under high default correlation.

As a result, we may suggest three recommendations to the regulators. First, it may become worth utilising multiple risk measures in the domain of the banking regulation and looking for the most conservative change in one of them. Second, when stress-testing it might be useful to replicate the path of the current study and calibrate the findings to default correlation and changes in loan portfolio structure, not limited to changes in PD. Third, it is important to accumulate the default statistics, specifically, on new lending niches. This enables the regulator to earlier identify the credit risk parameters of the segment; hence, banking regulation principles might be adjusted sooner when needed.

Last recommendation is particularly valuable when developing "green" lending (financing). There are American banks (Goldman Sachs (2020)), and Russian ones (Interfax (2022, 2023)), which stated quantitative measures for the actual or planned volumes for such lending segments. On the contrary, we have rich data on "brown" lending by 60 global banks collected in the report Ran.org (2021). This is why the revealed default correlation implications can help in better understanding the perspectives in credit risk profile changes in the such banks and the ones similar to them, when the stated objectives on rise of "green" lending are met.

To explain how we arrive at our findings, we start with the literature review in Section 2. We describe the approach to simulation in Section 3. A data simulation example is explained in Section 4. We discuss the major findings in Section 5.1. The robustness checks follow in Section 5.2. The discussion of relevant questions is given in Section 6. We conclude in Section 7.

2. Literature Review

We studied four streams of literature. The first stream is devoted to risk evaluation (to risk-measures). The second one covers the regulatory tools employed when the minimum requirements are insufficient. The third one is devoted to the climate-related regulation of the financial risks. The forth one deals specifically with the climate-credit risks' interplay.

2.1. Risk Measures

The entire financial regulation is centered around how to measure risks. Risk measure (RM) is typically a feature of the loss distribution. Historically, Jorion (2009) claims that *maturity* was the first risk measure in around 1930s. The longer the contract is, the less are the chances for the borrower to pay back, i.e., the more credit-risky it is. However, such a vision contradicts Merton (1974) who argued that the longer the debt contract is, the more opportunities the borrower has to repay on it, i.e., the less credit-risky it is.

Then *standard deviation* and *variance* became the second risk measures, according to Jorion (2009). Moreover, Markowitz (1952) extended the concept and proposed considering *semi-variance*, the variance of the negative outcomes (losses) only. The standard deviation formed a basis for the computation of the delta-normal risk measure when the asset return distribution is assumed to be a Gaussian one.

Further studies of Adam et al. (2008); Rockafellar and Uryasev (2013) have introduced the *distortion* risk measure. It weights non-proportionately the profits and losses of the same absolute amount. It may also non-proportionately weight the same marginal changes in loss. Even more advanced is the *spectral* risk measure from Adam et al. (2008). Mathematically it is integrated by the vertical axis (Lebesque integral) and not the horizontal one. Its theoretical advantage becomes evident when the loss distribution is not continuous.

However, none of these risk measures was solicited in the domain of the prudential regulation of banks. That is why, we will look at the two risk measures which became the financial industry standards in the recent decades:

- **Value-at-Risk (VaR)** was introduced in the modern practice in 1993 in (Holton, 2002, p. 20), J.P.Morgan/Reuters (1996) for the *market* risk management and adopted within the Basel I Amendment in 1996, though was already known for the portfolio *credit* risk measurement since the work by Vasicek (1987) 10 years earlier, though he applied it to the *credit* risk. VaR is often called a loss distribution *quantile*, see Equation (1);

$$VaR(\alpha) = F_X^{-1}(\alpha), \tag{1}$$

  where $F_X(x) = Pb(X \leq x) = \alpha$ — cumulative distribution function (CDF) for the random variable (r.v.) X (assume that X - is the default rate (DR), or the loss amount with an opposite sign, i.e., the maximum loss is the positive number); it is the probability that the r.v. X does not exceed the value x; $x = F_X^{-1}(\alpha)$ - inverse to the CDF, or quantile measured at the confidence level α.

- **Expected shortfall (ES)**, or tail conditional expectation, was suggested by Artzner et al. (1999) to offset the shortcomings of VaR. Specifically, ES differentiates cases with different risk realisations outside VaR, what VaR cannot disentangle by definition. Its alternative name - *Conditional Value-at-Risk (CVaR)* - was proposed by Rockafellar and Uryasev (1999). Simply saying, ES is the mean of risk realisation values which fall behind the VaR, see Equation (2).

$$ES(\alpha) = E(X|X \geq VaR(\alpha)), \tag{2}$$

  where $E(X)$ - the operator of the mathematical mean for the r.v. X.

We consider major quantiles to evaluate the two chosen risk measures (VaR, ES):

- 95%, the conventional confidence level used in statistics;

- 97.5%, the level proposed in (BCBS, 2013, p. 86, par. 181(b)) and adopted for the *market* risk capital requirements within Basel III revision pack;
- 99%, the level used for the *market* risk capital requirements in Basel II, (BCBS, 2006a, p. 115. par. 527(a); p. 195, par. 718(LXXVI)(b));
- 99.5%, the level proposed for the *credit* risk capital requirements in the draft Basel II IRB framework (BCBS, 2001, p. 36, par. 172);
- 99.9%, the finally accepted confidence level for the *credit* risk capital requirements within the IRB framework of Basel II (BCBS, 2006a, p. 64, par. 272) and Basel III BCBS (2009); it was also considered for the Basel II and *operational* risk capital requirements, (BCBS, 2006a, p. 152, par. 669(f)).

Lastly to mention, we do not neither prefer any climate scenario, nor predict global temperature for the next 200-years, as is done by Kotlikoff et al. (2021). All we are doing is the study of the credit risk distribution properties from the regulatory perspective laid done in Basel Accords.

2.2. Prudential Tools Outside Pillar I

All the financial risks can be generally classified into the capital and liquidity ones. Many countries had its risk regulation in some form, though the internationally unified capital adequacy ratio (CAR) counts its birth since the adoption of Basel I in BCBS (1988). The regulation covered capital risks (not the liquidity ones). All the risk-exposed assets are summed up and weighted by the common risk-weights to obtain the risk-weighted assets (RWA). RWAs form the denominator of the capital adequacy ratio (CAR). CAR becomes one of the most fundamental indicators of the bank's financial health.

Since Basel III in BCBS (2009), the international liquidity risk regulation was introduced in addition to regulating capital risks, while it was on the Basel Committee agenda even during its inception in 1970s, according to Goodhart (2011). Such capital and liquidity risk regulations are called the minimum, or *Pillar I*, requirements.

Though international capital risk regulation exists formally since 1988, the number of its critiques rises. One of the popular issues is a too general approach to treating various assets. For instance, corporate borrowers with quite distinct default probabilities received the same corporate risk-weight. The Basel II introduction and the shift to the external credit ratings in BCBS (2006a) was one of the solutions to differentiate asset classes and risk treatment. However, it turned out to be insufficient.

The two paths to augment the minimum requirements took place. The first one is the *micro*prudential economic capital regulation. The second one is the *macro*prudential regulation.

Economic capital regulation was formally introduced as the *Pillar II* of the Basel II. It is also known as the internal capital adequacy assessment process (ICAAP). Certain countries introduced internal liquidity adequacy assessment (ILAA) in addition to the minimum national liquidity requirements.

The ICAAP suggests that a bank has to identify additional risks and where necessary reassess the material risks according to the more accurate and more often more advanced models, than the general regulatory rules. One of the risks that required extra capital is the *credit concentration* risk. Bank of Slovenia and Bank of Spain prescribed the risk-weight add-ons when the concentration benchmarks (Herfindal-Hirschmann index, HHI) exceed the predefined thresholds in (Banco de Espana, 2008, pp. 16-17), (Banka Slovenije, 2010, p. 25). The Cyprus regulator used the loan to capital ratio instead of HHI as a concentration benchmark in (CySEC, 2012, p. 355). The larger the applicable risk-weight (RW) to a specific asset class is, the lower the bank is to offer such loans according to the regulator's intent. Inversely, the lower RW is (like it were mortgages in Basel I or loans to small-and-medium enterprises (SMEs) in Basel II), the more the bank might be willing to lend in such segments all else being equal.

The IRB risk-weight variation due to the presence of the credit concentration risk was discussed during the Basel II consultation similar to Lütkebohmert (2009); Gordy and Lütkebohmert (2013), but it was not implemented. The probable reason for not including the concentration charge is that the IRB approach is a part of the minimum Pillar I, not Pillar II requirements.

Thus, *if* the concentration risk posses threats to the banks solvability, the regulatory community prefers treating it within *Pillar II*. We put '*if*' here as there was a single paper by Jahn et al. (2013)

from the German Bundesbank who dared claiming that higher concentration is beneficial to the bank as it occurs when the bank knows well its borrowers. However, by today the position of the Bundesbank researchers is considered to be out of the mainstream of thought. WE may come across only point-wise comments supporting the statement that diversification might bring more harm, than good, see Mohamed (2023).

The alternative approach to the credit concentration risk regulation is the *macro*prudential treatment. *Macro*prudential regulation takes place when it is sector- or borrower-specific, contrary to the general *micro*prudential regulation (like, HHI- or loan-size-based rules disregarding the borrower segment). Macroprudential limits are one of the recent tools discussed in Acharya et al. (2022); Carro et al. (2022); Jurca et al. (2020). Their advantage at the first place is the absence of the direct capital consumption. They just limit the volume without an increase in the loan rates as is the case when the capital surcharges (like macroprudential risk-weight add-ons) are introduced. However, to meet profit targets limiting particular loan volume requires a bank to still upward revise the loan rates or downward revise the deposit ones, or probably do both at a time. However, we should keep limits in mind as another option to concentration risk regulation.

2.3. Climate-Related Financial Risk Regulation

Formally, the World Bank defines a green loan as "a form of financing that enables borrowers to use the proceeds to exclusively fund projects that make a substantial contribution to an environmental objective", World Bank (2021).

The Basel Committee only recently pointed out the necessity to regulate the financial risks considering the climate change perspectives and climate risks in its publications: BCBS (2021a,b). Papers contain the discussion on how to properly measure the climate risks. The advanced tools include agent-based models and the use of input-output tables.

The choice in-between the regulatory tool is not that evident and easy as it may seem from the outside. For instance, we may find two similar papers originating from the BIS by Coelho and Restoy (2022, 2023). The major difference that occurred during a year between two publications is the shift in applicable measures. Initially, Coelho and Restoy (2022) suggested that climate-related financial risk regulation should be part of the *micro*prudential Basel II Pillar II regulation of the economic capital, or ICAAP. However, one year later authors changed their position and finally recommended using *macro*prudential tools for the same objective in Coelho and Restoy (2023).

However, none of the above-cited papers (BCBS (2021a,b); Coelho and Restoy (2022, 2023)) suggest a calibration of the climate-related financial risk. Our work might help bridge this gap.

2.4. Climate-Credit Risk Relationship

Green lending as a type of lending gives rise to the credit risk primarily. That is why, we need to discuss the existing findings on the climate-credit risk relationship. It was studied by Andersen (2020); Capasso et al. (2020); Okimoto and Takaoka (2022); Kölbel et al. (2022); Blasberg et al. (2023). Most of them (Andersen (2020); Capasso et al. (2020); Okimoto and Takaoka (2022)) consider carbon dioxide (CO2) emissions as the climate risk proxy. To capture the credit risk, some of them (Okimoto and Takaoka (2022); Kölbel et al. (2022); Blasberg et al. (2023)) depart from the credit-default swap (CDS) quotes, while others (Andersen (2020); Capasso et al. (2020)) use a form of the default probability (PD) model. Disregarding the data used, all of these earlier researchers are mostly unanimous in conclusion that there is a positive climate-credit risk relationship, i.e., the lower the climate risk (the greener the project/borrower/loan) is, the lower the credit risk is.

However, the limitation of the mentioned papers studying climate-credit risk relationship is the focus on the borrower-only dependencies. The authors in Andersen (2020); Capasso et al. (2020); Okimoto and Takaoka (2022); Kölbel et al. (2022); Blasberg et al. (2023) neglected the most important - the portfolio - perspective of the credit risk.

3. Methodology: default correlation and probability of default

To better get what is going to be done, let us start by bringing a simplified (baseline) case to demonstrate what the probability of default and default correlation is in the subsection 3.1. Here it is important to see how realized portfolio default rates DR might vary given the same probability of default PD because of different default correlation ρ. Important to note that when considering basic cases - when there is a small number of borrowers in the portfolio or time spans - there are constraints on the feasible combinations of default probability and correlation. Nevertheless, when the number of borrowers grows (what is the case for the actual bank portfolios) the only limitation that resides is the non-negative values of the default correlation if we think of it as a single (common) pair-wise indicator between default/non-default flags for any two borrowers. However, except for this limitations, there are very many feasible combinations of default probabilities and default correlations left.

Having demonstrated typical dependencies between PD and ρ parameters, we will explain in the subsection 3.2, how to derive the default correlation using the actual default rate time series data. Here were are going to stress why it is important to think of an actual data as realisations from a mixture of several distributions, and not from a single one. Disregarding the properties of a distribution mixture may lead to material underestimation of the DR realisation in crisis and, hence, to credit risk underestimation.

Having proven that the actual DR data is in most cases the realisation from the distribution mixture, and not from a single one, we present the DR distribution simulation procedure in subsection 3.2.1. We describe the considered plethora of parameters in subsection 3.3 to reflect the potential scenarios for expanding "green" lending.

3.1. Baseline case

3.1.1 Feasible values for the separate parameters and their combinations

Consider a simplified loan portfolio. There are two *identical* borrowers there-in ($L1, L2$). Let them take a loan for a year each year out of the four (four time spans). Consider two cases for the default probability for each of the borrowers: 25 and 50%. The first value ($PD = 25\%$) implies that default is likely to happen only once out of four time spans (years); second one ($PD = 50\%$) - there are going to be observed two defaults out of four years. We can specify that a default is an event when the material amount of loan payment is past due for a given number of days (e.g., 30 days for the purpose of IFRS 9 or 90 days for the IRB). However, such an operational definition does not change anything in our baseline setting. That is why we continue without any loss of generality.

Right now we introduced the set-up: two borrowers take a loan each time during four time spans with one or the other default probability. Now we wish to look at when these two borrowers' defaults might occur, i.e., when they coincide and when do not happen same time. This will drive the **default correlation** (ρ) value.

As the research extension, we may think of deviations arising from applying correlation to Bernoulli r.v. and not to floating ones. But this issue falls outside the scope of the current paper.

Figure 1 contains five cases: the upper row of cases A and B corresponds to the default probability (PD) of 25% per borrower; the lower row of cases C, D, F stands for the $PD = 50\%$. In each case there are three alternatives: defaults coincide (1-1 in terms of default flags), non-defaults coincidence (0-0), or the default happen for one of the two borrowers (1-0 or 0-1). Depending on which outcomes prevail we have different default correlations. For instance, when all events occur simultaneously (there are only defaults or non-defaults), then it is +100% default correlations (cases A, C). The more the number of asynchronous cases is, the less the default correlation is. The lowest value of -100% is reached only in case E, when $PD = 50\%$. For comparison, in case B with the lower PD the default correlation is higher (-33%), than in case E, as there are more synchronous defaults in case B.

Here we may see the limitations on the feasible values and parameter combinations for the default probability and default correlation:

- Particular combinations of PD and ρ are not feasible in small samples. For instance, when the default probability equals to 50% three default correlation values are feasible: -100, 0, +100%.

Figure 1: All attainable combinations of defaults if there are only two borrowers, four time spans and two default probability values.

A. Def.cor. +100%

Time spans (1 case Pb= = 25%)	L1	L2	#D	DR
1	1	1	2	**100%**
2	0	0	0	**0%**
3	0	0	0	**0%**
4	0	0	0	**0%**
PD	25%	25%		

	mean DR	**25%**
	DR variance	**25%**
3rd worst case -	VaR (70%)	**0%**
4th worst case -	VaR (80%)	**100%**

B. Def.cor. -33%

	L1	L2	#D	DR
1	1	0	1	**50%**
2	0	0	0	**0%**
3	0	1	1	**50%**
4	0	0	0	**0%**
PD	25%	25%		

mean DR	**25%**
DR variance	**8%**
VaR (70%)	**50%**
VaR (80%)	**50%**

C. Def.cor. +100%

Time spans (1 case Pb= = 25%)	L1	L2	#D	DR
1	1	1	2	**100%**
2	1	1	2	**100%**
3	0	0	0	**0%**
4	0	0	0	**0%**
PD	50%	50%		

	mean DR	**50%**
	DR variance	**33%**
3rd worst case -	VaR (70%)	**100%**
4th worst case -	VaR (80%)	**100%**

D. Def.cor. 0%

	L1	L2	#D	DR
1	1	0	1	**50%**
2	1	1	2	**100%**
3	0	1	1	**50%**
4	0	0	0	**0%**
PD	50%	50%		

mean DR	**50%**
DR variance	**17%**
VaR (70%)	**50%**
VaR (80%)	**100%**

E. Def.cor. -100%

	L1	L2	#D	DR
1	1	0	1	**50%**
2	1	0	1	**50%**
3	0	1	1	**50%**
4	0	1	1	**50%**
PD	50%	50%		

mean DR	**50%**
DR variance	**0%**
VaR (70%)	**50%**
VaR (80%)	**50%**

However, when $PD = 25\%$, there are only two feasible values: -33 and $+100\%$. Notably, the values of -100 and 0% are not feasible.

- If the number of borrowers grows, the negative values of default correlations also evaporate. To remind, we think of default correlation as a single common parameter measured for any pair of borrowers. In other words, it can be considered as some average indicator. Now, please, imagine a portfolio with an infinite number of borrowers (a perfectly granular one) (for instance, a retail loan book of a large bank). Take a random pair of borrowers. As for them, defaults might occur in different time periods (i.e., be inversely correlated). Now add a third borrower. If its default flag inversely correlates with the first borrower out of a pair, then its defaults status is going to fully replicate that of the second borrower out of a pair. Thus, if the default correlation for the two pairs equals to -100%, it is $+100\%$ in the residual third pair.

  Such a constraint was reflected in the paper by Preisser and Qaqish (2014). If we use credit risk notations to the paper finding, we may write down the Formula (3). When the number of borrowers goes to infinity, the feasible value of the default correlation in-between any pairs of borrowers becomes non-negative.

$$\rho \geq -\frac{1}{n-1}, \tag{3}$$

  where ρ is the average (portfolio-wide) default correlation and n is the number of borrowers.

Such constraints on the feasible values and parameter combinations for the probability of default and the default correlation might have incentivized the Basel Committee to capture one-to-one dependency between the probability of default and its correlation in the Formula differentiated by asset classes. For instance, see IRB Formula 4 from Basel II BCBS (2006b) and its adjustment for the loans to G-SIBs in the Formula 5 from Basel $=$ III BCBS (2009).

$$R = 0.12 \cdot \frac{(1 - exp^{-50 \cdot PD})}{(1 - exp^{-50})} + 0.24 \cdot (1 - \frac{(1 - exp^{-50 \cdot PD})}{(1 - exp^{-50})}), \tag{4}$$

where PD is the borrower's default probability estimate.

$$R = 0.15 \cdot \frac{(1 - exp^{-50 \cdot PD})}{(1 - exp^{-50})} + 0.30 \cdot (1 - \frac{(1 - exp^{-50 \cdot PD})}{(1 - exp^{-50})}). \tag{5}$$

3.1.2 Credit risk measurement implications

Let us now focus on the properties of the loan portfolio default rate (DR) probability distribution. The historical average default rate ($meanDR$) is always equal to the probability of default in a single case ($\bar{DR} = PD$)). Same time when the default correlation is higher, the default rate variance ($DRvariance$) is higher too. We present corresponding indicators below each of five cases. For instance, when the default correlations is the lowest one (-100% in case E) the DR variance is nil. On the opposite, when the default correlation is at its peak (+100% in cases A, C), the DR variance is the largest, but depends upon PD: DR variance equals to 25% in case A and 33% - in case C. Observing such properties it was possible to derive a Formula for default correlation given the actual data in the paper Penikas (2023). We replicate it here for convenience in the Formula (6):

$$\rho = \frac{Var(DR)}{\bar{DR} \cdot (1 - \bar{DR})}, \tag{6}$$

where $\bar{DR}$ - is the historical mean default rate; $Var(DR)$ is its variance.

By comparing the five cases we can also see that under perfect default correlation there are only two feasible outcomes for the proportion (DR): zero or one. Thus, the DR distribution becomes bimodal. Notably, the height of the second mode is the single case PD.

Author from Penikas (2020) described why such a bimodal distribution might be relevant for a low default portfolio (LDP). The portfolio got the name of LDP as its was the first (left) mode in the nearby of zero which was observed historically. However, in crisis all such borrowers are to default. This means that the probability of default for an LDP portfolio can be any (including much higher than zero) when there is default correlation close to +100%.

Let us explain what the described DR distribution features mean for the credit risk evaluation. Let us start with the VaR as the basic risk measure. As we introduced only four time spans, let us take unconventional significance levels of 20 and 30%. Then we cut VaR at 80 and 70% confidence levels, respectively. As there are four time spans, one span probability of occurrence is 25%. Hence, cutting VaR at 70% is equivalent to looking at the third worst outcome, while cutting it at 80% - taking the forth (the worst) outcome. Such values are also available below each case and below DR mean and variance values at the Figure 1.

When comparing five cases from the Figure 1 we can derive the following conclusions about default probability and correlation impacts:

- When default probability (PD) grows, the credit risk value also rises. Compare cases A and C. VaR(80%)=100% for both cases. However, there is a difference at the 70%-th confidence level. Corresponding VaR equals to zero in case A with lower PD, but equals to 100% - in case C, when PD is higher.

- When default correlation (ρ) rises, credit risk value also grows. for instance, for cases C-E when the default probability is high, rise in default correlation leads to consecutive rise in the credit risk value. When transiting from E to D default correlation rises from -100 to 0%. Same time, there is a rise in VaR(80%) from 50 to 100%. The next transition from D to C default correlation increases from 0 to +100%. Here the risk measure at the lower confidence level VaR(70%) rises also from 50 to 100%.

- However, rise in default correlation may result in contradictory change in credit risk. For instance, such rise under low default probability - when shifting from B to A (from -33 to +100% in terms of default correlation) implies rise of the risk measure at the higher confidence level: VaR(80%) grows from 50 to 100%. Nevertheless, risk measure at the lower confidence level reduces: VaR(70%) declines from 50 to 0%. Thus, conclusions about the likely impact on the credit risk evaluation strongly depends upon the choice of the confidence level at which the risk is measured; and upon the risk measure type (VaR or ES). This is the result of the DR distribution bimodality under growing default correlation.

- Default rate (DR) rise in particular time spans (for example, in year 1 and 2 in case C in year 2 for case D) - is the outcome of the default correlation, and not of the probability of default. This is why the terminology used in the risk management domain needs significant improvement. For instance, it is wide-spread to distinguish Point-In-Time PD (PD PIT) and Through-The-Cycle PD (PD TTC), see (Erlenmaier, 2011, p. 46), Ozdemir and Miu (2009). It suggests that PD TTC stays mostly unchanged in time, while PD PIT rises in crisis and declines during boom times. However, we explicitly show above (focus specifically on C-D-E or A-B cases' comparison) that it is DR which changes, and not PD. Hence from the terminology point of view there exist neither PD TTC, not PD PIT. For the fairness sake, we may note that the presence of default correlation does not prevent neither PD, not default correlation (ρ) from rising in crisis. Such justifications are presented in the paper by Merika et al. (2021).

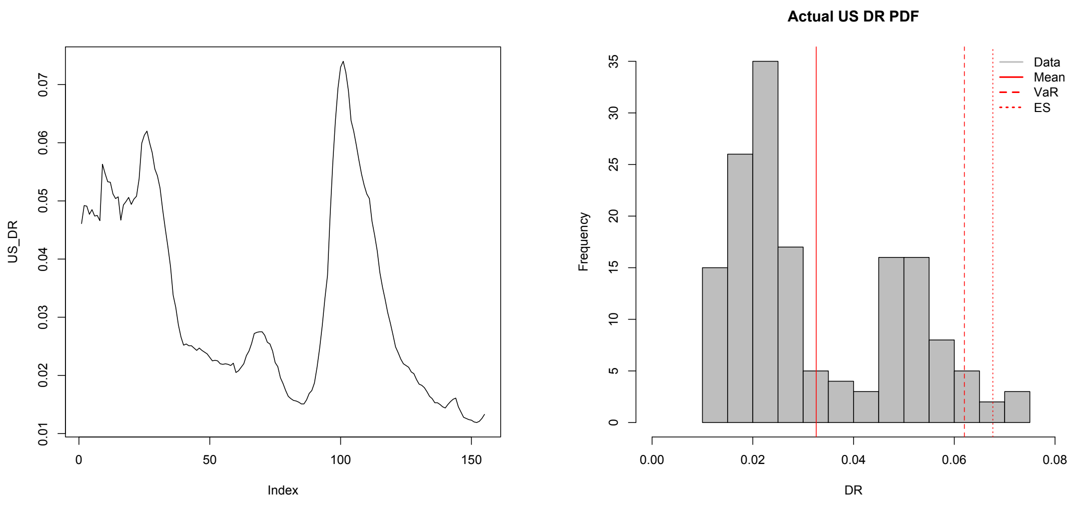
3.2. Defining the DR distribution parameters as the distribution mixture attributes

In the previous section we discussed the properties of the default rate (DR) distribution for the baseline case of two borrowers and limited number of time spans. We have shown how the observed properties are linked to the analytical Formula (6) on default correlation. Let us extrapolate our findings to the real-world data, and discuss the arisen challenges and possible remedies.[1]

3.2.1 Actual data

As an example, we take the open data on the US defaults for the total loans during 50 years since 1985 (155 quarterly data points till 3Q23). We present them on Figure 2. Average historical default rate (DR) equals to 3.26%, its variance is 0.000284. VaR and ES risk measures for the actual data equal to 6.2 and 6.8% at the 95% confidence level.

Figure 2: Actual default rate (DR) data for the US borrowers in 1985-2023.



Data source: *`https://www.federalreserve.gov/releases/chargeoff/delallsa.htm`*.

[1]The material of this subsection was published at the conference proceedings Penikas (2021a,b).

Using Formula (6) we obtain the default correlation of around 1%, which is mostly statistically indistinguishable from zero. Same time on the right panel of the Figure 2 we explicitly see two modes of the DR distribution in the nearby of DR equal to 2% and 5%. Hence, we come to an analytical contradiction. Visually the presence of the two modes signals for the presence of the statistically significant default correlation, while the computed value point out to its absence. To resolve the contradiction, we need to identify (fit) the parameters of the actual DR distribution. To do so, we need to generate data. While generating, we need to calibrate (optimize or fit) the parameters of interest which would best fit the actual data. This is why we present the typical approaches to generating distributions which we discuss. To evaluate whether the generated distribution closely matches the one from the actual data, we may use Kolmogorov-Smirnov statistics and compare it to the critical value $KS_{Critical}$ from the Formula (7):

$$KS_{Critical} = C(\alpha) \cdot \sqrt{\frac{n_1 + n_2}{n_1 \cdot n_2}}, \tag{7}$$

where $C(\alpha)$ is the critical value which depends upon the sample size and the confidence level; n_1 stands for the number of observations in the theoretical model (i.e., number of time spans), while n_2 is the number of observations in the actual data.

When the samples are equal in size ($n_1 = n_2 = n$), we may obtain at the 5% confidence level ($C(\alpha) = 0.409$, (Calixto, 2016, p. 56, Table 1.8)):

$$KS_{Critical} = C(\alpha) \cdot \sqrt{\frac{n + n}{n \cdot n}} = C(\alpha) \cdot \sqrt{\frac{2n}{n^2}} = C(\alpha) \cdot \sqrt{\frac{2}{n}} = \frac{0.409 \cdot \sqrt{2}}{\sqrt{n}} = \frac{0.58}{\sqrt{n}}, \tag{8}$$

3.2.2 Distribution simulation for the correlated Bernoulli outcomes

There are two general approaches on how to general the distribution of the sum of outcomes where each of them takes the value of zero or one (in other words, the sum of the Bernoulli r.v.): non-parametric and parametric.

Non-parametric approach is sometimes called a *genetic* one, see Kruppa et al. (2018). Initially we assign ones and zeros by objects (by columns or by borrowers). Then we start exchanging places of zeros and ones by columns. Such a replacement does not effect the probability of default (PD), but impacts the default correlation. We continues replacements until we reach the targeted correlation matrix. Here it is worth recalling the limitations on the feasible parameter values and combinations (in particular, for the default correlation). Sometimes the correlation matrix of interest may not exist. Then the algorithm cannot converge. If one of the classes is small (for zeros or for ones; for instance, for an LDP case), then the algorithm may take too much time to converge, if the solution is at all feasible. However, the advantage of the approach is that one can attain negative default correlation (if it exists).

Next, we prefer the **parametric** approach as it has no convergence problems. There are two sub-approaches. The first one is more wide-spread in the credit risk domain, while the second one - in the medical studies and biostatistics.

We may count the start of the first sub-approach from the works by Oldrich Vasicek Vasicek (1987). The sub-approach consists of two stages. First, he uses nice analytical properties of the multivariate normal distribution to model interconnected r.v. Then he assigns a threshold and all the normal r.v. realisations are censored (classified) into two groups: to zero (below the threshold) and one (above it). This is the underlying logic of IRB.

Formally Vasicek defined the asset return A_i as the linear combination of the systemic and individual factors, weighted by the degree of exposure to the systemic risk (by asset correlation, R), see Formula (9).

$$A_i = \sqrt{R} \cdot x + \sqrt{1 - R} \cdot \epsilon_i, \tag{9}$$

where $A_i \sim N(0, 1)$ is the asset return process for the company i; $x \sim N(0, 1)$ is the systemic factor (common for all the companies in the economy); $\epsilon_i \sim N(0, 1)$ is the individual (idiosyncratic) factor for the company i; R - exposure to the systemic factor (also called asset correlation; mathematics-wise, it is the second power of the ordinary Pearson product-moment correlation ρ); $N(0, 1)$ is the standard normal (Gaussian) distribution.

Long ago there was an attempt to embed multiple systemic risk drivers instead of a single one, see Pykhtin (2004). However, such an approach did not penetrate the banking regulation and IRB, while a much simpler moedl of Vasicek did.

In general, the described approach allows for the negative correlation, as the non-parametric approach does. However, the asset value should be redefined as in Formula (10), as done in the paper (Gordy, 2000, pp. 148, eq. C.4).

$$A_i = \rho \cdot x + \sqrt{1 - R} \cdot \epsilon_i, \tag{10}$$

where $rho^2 = R$. This clarification can be read as the relationship between default correlation (ρ) and asset correlation (R).

Disregarding the preferred format for the asset return - be it Formula (9) or (10) - a question on r.v. realisations cut (censoring) is left. That is why we are prone to use the second approach from the medicine and biostatistics, which immediately results in ones and zeros.

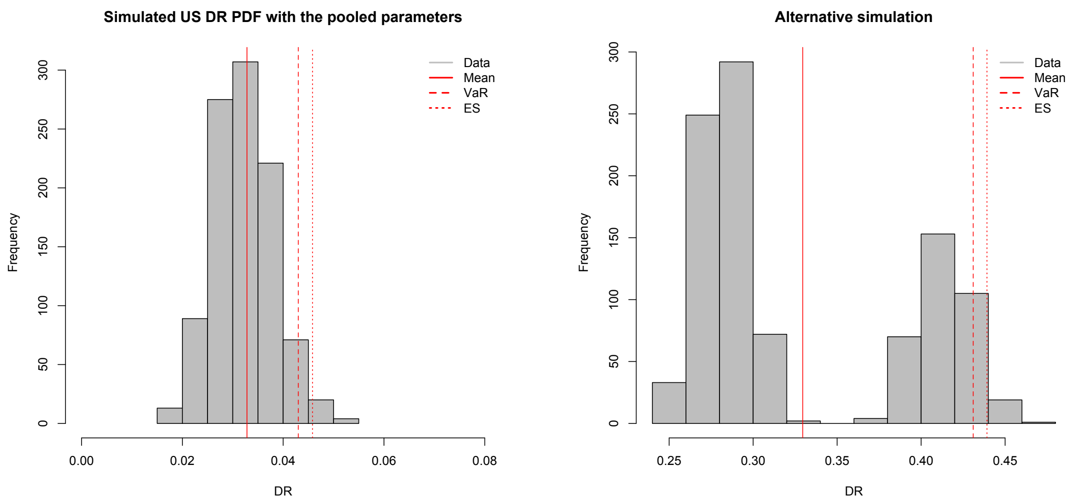
Such an approach was described by Lunn-Davies in the paper Lunn and Davies (1998). We present it in the Formula (11) below.

$$D_i = U_i \cdot Y + (1 - U_i) \cdot \theta_i, \tag{11}$$

where D_i is the default status for a borrower i, it takes value of one in default and zero otherwise; $Y \sim Bin(1, PD)$ is the Binomial (Bin) systemic factor realisation in a single trial (1 is at the first place in brackets), it takes values of one with the default probability PD and zero otherwise; $\theta_i \sim Bin(1, PD)$ is the similarly distributed, but independent of Y individual factor; $U_i \sim Bin(1, \rho)$ is the association measure, which reflects the degree of default correlation ρ of interest.

Hence, the Lunn-Davies approach enables us to reconstruct the distribution function (CDF). To do so, we are only in need of two parameters: probability of default and default correlation. As for the actual US data, the parameters are equal to 3.26% and 1.0%, respectively. DR distribution with such parameters is available at the left panel of the Figure 3.

Figure 3: Simulated DR distribution



Note: we used the DR mean and variance from the pooled actual data for the US for 1985-2023 from Figure 2.

The obtained distribution suffers from two shortcomings. First, the simulated distribution is unimodal, while the actual one is bimodal, see right panel at the Figure 2. Second, as a consequence, we have credit risk underestimation in the case of the simulated data. The VaR and ES risk-measures equal to 4.3 and 4.6% at the 95% confidence level (compare to 6.2 and 6.8% for the actual data).

Risk measures might seem comparable in the simulated and real-world data in absolute terms. However, the underesimation reaches one third (30%) in relative terms. It seams plausible to resolve the issue processing real-world data only. However, challenges arise when forecasting. If one needs to know how the risk measure is going to change if it *was not yet (earlier before) observed* given changes in the loan portfolio composition, then real-world data is unavailable. Another solution is solicited.

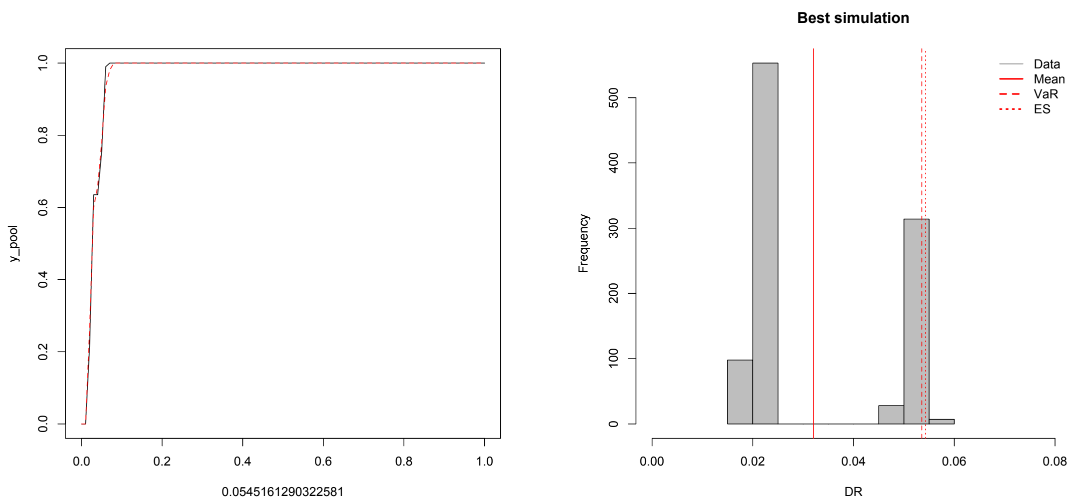
3.2.3 Simulating proportion distribution as the mixture of distributions

The range of generated distributions offered a hint how to solve the above described challenge. For instance, we can increase the actual default rate tenfold. Then the average probability of default equals to 33%, while the default correlation equals to 13% (the latter one is already statistically different from zero). Simulated distribution with such parameters is available at the right panel of Figure 3. Here the bimodality is present more explicitly. The distribution is visually closer to the actual data from the right panel of Figure 2, adjusted for the scale of the horizontal axis.

Then an idea came to mind. What if the actual data comes from several distributions (at least from the two), and not from a single one? Simply saying, one distribution might be a low -risk one (an equivalent to an LDP). Its probability of default and default correlation is close to zero. The second distribution is riskier. Then we need to decompose an observed distribution into a mixture of distributions. Such a task was common when studying the income distribution in Russia. For instance, it was exactly a representation via a mixture of distributions which allowed professor S.A.Aivazian to obtain the best fit for the actual data, see Aivazian (2012).

To define the mixture parameters given the actual default rate time series a separate programming code was prepared. It is available in Annex A.1.1. The simulated outcome is available at Figure 4. The left panel contains comparison of the CDFs for the actual and simulated distributions, while the right panel has the simulated CDF itself.

Figure 4: Simulated DR distribution: best fit



Note: we used the distribution mixture parameters, which we discovered with respect to the actual US data for 1985-2023 presented in Figure 2.

Thus, when we decomposed the actual US data for 1985-2023 into a mixture of distributions, we obtained the following parameters. Mixture consists of 91% of the distribution with low risk and low default correlation ($PD = 0\%$, $\rho = 0\%$) and of 9% of the distribution with high risk and high default correlation ($PD = 36\%$, $\rho = 34\%$). Kolmogorov-Smirnov criterion equals to 5.4%. It is above the critical level of 4.6% for the 5% significance level, but still implies close to sufficient fit to real-world data, compared to the case when no mixture of two distributions was considered.

We described above what the default correlation is; how it is linked to the probability of default; we have shown that the real-world data is more often a series of realisations from a mixture of multiple distributions with low risk (low default probability and correlation) and the high-risk distributions (with high PD and default correlation).

Now we can discuss the real-world risk profile evolution when the portfolio structure changes in general and as a result of "green" loans extension in particular. Annex A.1.2 contains the code to simulate DR distributions with the given mixture parameters.

3.3. "Green" lending specifics

Discussing the implications from increasing the proportion of "green" finance ("green" loans) is more convenient, when keeping in front of ones eyes a typical example on what such proportions might be. The data on the quantitative targets between the banks is scarce. The most vivid example and, to the best knowledge of the author, the only example is brought by an American bank. In 2019 it declared to form a portfolio of sustainable finance equal to USD 750 bn by 2030, Goldman Sachs (2020). "Green" loan form a (non-dominant) part of the stated objective.

However, at the date of announcement the objective was close to three fourths of the bank's total assets, see Table 1. Even if the entire objective is not to be formed out of the "green" loans only, it is still a material ercomposition of the loan book.

Table 1: Green Exposure Targets Adjusted to Total Assets Growth.

#	variable	2019	2020	2021	2022	2023	2024	2025	2026	2027	2028	2029	2030
1	US CPI value, pp	1.81	1.25	4.69	8	4.8	3	2.2	2.1	2.1	2	2	2
2	US CPI type	actual	actual	actual	actual	projected	projected	projected	projected	projected	projected	projected	projected
3	US CPI, cum %	1.02	1.03	1.08	1.17	1.22	1.26	1.29	1.31	1.34	1.37	1.39	1.42
4	Bank TA, $bn	992	1163	1464	1441	1510	1555	1590	1623	1657	1690	1724	1759
5	Bank sust. target, $bn	750	750	750	750	750	750	750	750	750	750	750	750
6	Bank sust. cur, ratio	0	0.2	0.4	0.55	0	0	0	0	0	0	0	0
7	Bank sust. cur, $bn	0	150	300	413	0	0	0	0	0	0	0	0
8	Bank sust. cur to actual, %	0.76	0.64	0.51	0.52	0.5	0.48	0.47	0.46	0.45	0.44	0.44	0.43

Note: CPI - consumer price index; TA - total assets; cur - current value; sust. - exposure to sustainable finance; data source for the actual 2019-22 values:
`https://www.statista.com/statistics/244983/projected-inflation-rate-in-the-united-states/`;
sources for the 2023-2030 predicted values:
`https://www.cbo.gov/system/files/2023-02/58848-Outlook.pdf#page41`.
Bank targets and current values are sources from here:
2019 - the target announced, actual exposure is nil, see Goldman Sachs (2020); 2020, 2021 - the reports (Goldman Sachs, 2021, p. 7) and (Goldman Sachs, 2022, p. 5) contain the current exposure in USD bn; 2022 - the (Goldman Sachs, 2023, p. 8) report contains the values relative to the announced target;

However, the target was set for the ten years and is planned to be reached by 2030. By that time the bank total assets are unlikely to stay the same. Conservatively, we may expect them to rise with the pace of inflation. The predicted cumulative inflation is above 40%. If the bank does not upward revise its sustainable exposure target, by 2030 the target is likely to be below 45% of the total assets, being in 2030 much less ambitious than its was at the announcement in 2019.

We confirm that the "green" loans forms just a part of the objective. However, for the illustration purposes let us take the two computed proportions (43 and 76%) as the intermediate values in the range of the considered ones, i.e., as if the "green" loans occupied such portions. In addition, we consider zero as the starting portion and 100% as the most extreme value.

To underline, there are no reliable data to reliable conclude what the typical default correlation values are for the "green" lending segment. That is why for the sake of objectivity we look at the entire range of values from zero to 100%. We do not consider negative default correlation within the segment neither from the substance perspective, nor statistically because of Lunn-Davies approach limitations.

We consider several combinations of the "green" loan subportfolio parameters:

- Two risk measures: VaR, ES;
- Five quantiles for risk measures: 95, 97.5, 99, 99.5, 99.9%;

- Four shares of the "green" exposures in the loan book: 0, 43, 76, 100%.
- The default probability (PD): 3.4%, 5%, 22%;
- The default correlation (ρ): 0%, 25%, 50%, 75%, 90%, 100%.

For the robustness check of our findings we additionally consider what is to change if we assume variation in the PD for the "green" and "brown" segments (when the proportion of "green" lending rises, let the corresponding PD to decline, while the "brown" segment PD - to rise).

4. General results

To study the properties of the credit risk distribution, we depart from the existing stylized loan book and try augmenting the new lending segment portion. We use US data (see Figure 2) as inputs for the credit risk parameters. In particular, we assume that the initial DR distribution is the mixture of the two high- and low-risk DR distributions. We add the third DR distribution to such a hypothetical loan book. let the portion of the new segment equals 43% for the purpose of the current section (alternative scenarios are available in Annex A.2.1).

We use Lunn-Davies simulation model from Formula (11) to simulate the defaults. We use 1000 time spans (artificial years) and 500 borrowers per time snapshot. Thus, the lowest considered PD of 3.4% yields an integer number of 17 defaulted borrowers out of 500.

Let us describe a simulation example in detail to better see the evolution in probabilistic features of the credit loss distribution in Figure 5. Concise parameters are available in Table 2.

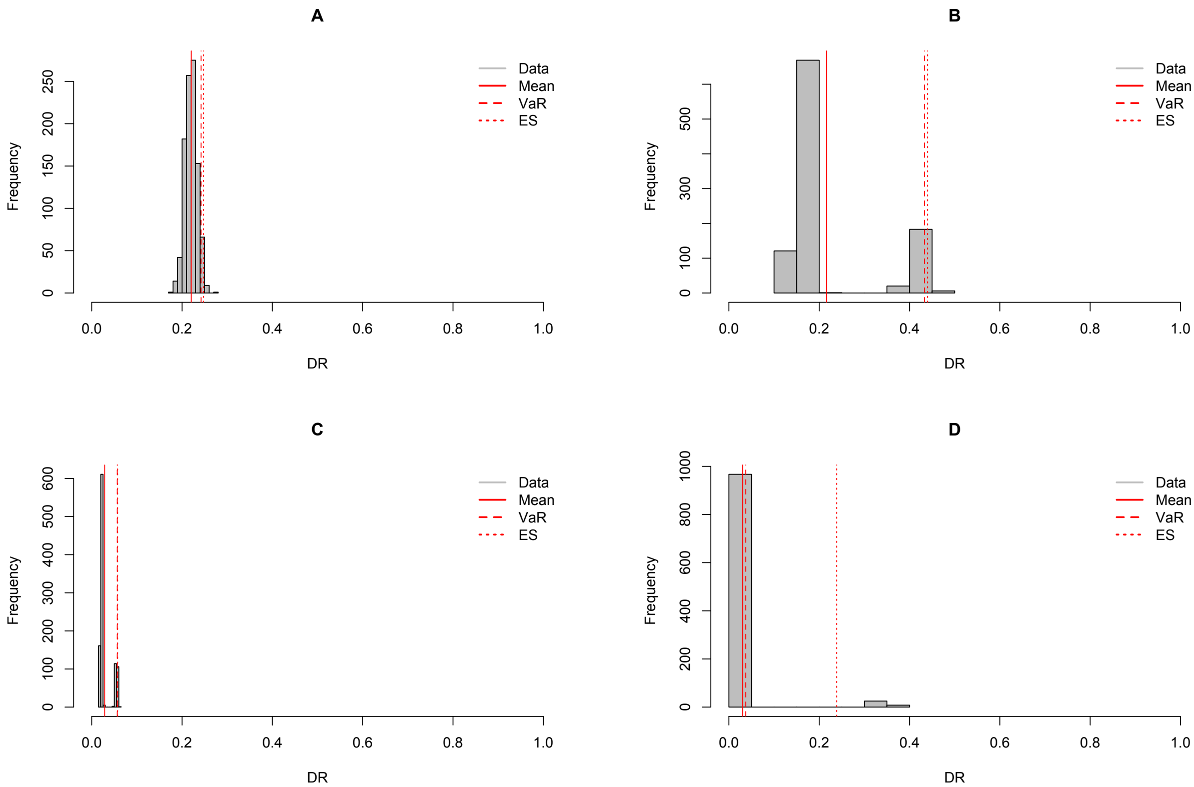
Table 2: Parameters for the illustrative scenarios.

#	Variable	A	B	C	D
	Inputs				
1	w_1			**0.87**	0.50
2	w_2	1.00	1.00	0.13	0.07
3	w_3				**0.43**
4	$\sum_{i=1}^{3} w_i$	1.00	1.00	1.00	1.00
5	PD_1			0.00	0.00
6	PD_2	0.22	0.22	0.22	0.22
7	PD_3				**0.03**
8	R_1			0.00	0.00
9	R_2	0.00	**0.26**	0.26	0.26
10	R_3				**0.75**
	Outputs				
11	$\tilde{DR}_1$			0.00	0.00
12	$\tilde{DR}_2$	0.22	0.22	0.22	0.22
13	$\tilde{DR}_3$				0.03
14	$\tilde{DR}_0$	0.22	0.22	0.03	0.03
15	VaR	0.24	0.43	0.06	0.04
16	ES	0.25	0.44	0.06	0.24

Note: w_i - weight of the i-th sub-portfolio within the total portfolio; PD_i - probability of default for the i-th sub-portfolio; R_i - default correlation for the i-th sub-portfolio; i-th sub-index refers: 0 - total portfolio; 1 - ordinary (close to risk-free, or low-default) sub-portfolio; 2 - sub-portfolio with highly risky loans; 3 - new loans' sub-portfolio. DR - default rate; Rho - default correlation.

Panel A from Figure 5 presents a high-risk low-default-correlation portfolio ($PD = 22\%$, $\rho = 0\%$). It is like the second component of the distribution mix for the US data, but with zero default correlation.

Figure 5: Risk measures' evolution illustration.



Note: risk measures (VaR, ES) are evaluated at the 95% confidence level; scenario parameters are available in Table 2. Panel C corresponds to the actually observed data from Figure 2.

As there is no default correlation, the distribution of the proportion (the share of defaults, DR) has a nice unimodal bell-shaped curve. There are no outliers, no extremes, no fat tails. The risk measures of VaR and ES are close to each other and equal to around 24% of the number of the borrowers.

Panel B from Figure 5 is a modification of Panel A. Here we increase the default correlation from zero to 26%, as was identified for the US data. Now the distribution becomes bimodal. The risk measures are also close to each other, but now they are located in the nearby of the second hump of the distribution. There is a *cliff-effect*. There is a radical change in default rate from the ordinary (boom) times on the left mode of the distribution to the crisis (bust) times on its right mode.

Such a cliff-effect situation is not welcomed by risk-managers. It means that the ordinary risk and capital consumption can be low in the nearby of the left mode of no more than 20%, but the loss deserving capital provisioning is twice more, it is around 45%. It means that disregarding the low expected DR value in good times, the bank has to allocate capital against 45% loss. We are not here to discuss the implications on how to better allocate it: via provisions or via RWA. This is a separate research issue.

However, if we dilute the panel B portfolio within a larger low-risk low-default-correlation portfolio (but also comprising of the existent exposures), even the extreme risk evaporates. This is illustrated in Panel C. It is the actual US data case when there is a mixture of the two distributions: 87% of low-risk low-default-correlation distribution and 13% for its antagonist from panel B. Here the risk measure is around 6% for the both types (VaR, ES).

Next, let us consider a case when the bank increases the proportion of the new loans to 43%. Hence the proportion of the previous mixture from panel C now occupies the residual 57%. Hence, the proportion of the high-risk existent loans from panel B is now $57\% \cdot 13\% = 7\%$.

Here we assume that the new lending segment is a low-risk high-default-correlation portfolio ($PD = 3.4\%$, $\rho = 75\%$). Multiple risk-management challenges immediately arise.

First, the overall loss distribution becomes much more dispersed. Though most of its realisations concentrate around 0% of total borrowers as default cases, a minor part is located around 35% of defaults.

Like with panel B, the bank might be delighted that sufficient capital requirements will be negligible at good times. However, risk-managers and financial regulators require bank to hold capital against crisis times. At such bad times the risk measure is several times higher than 6% which is the current value from the panel C.

Second, the risk measure estimate is not robust. In the three previous panels VaR and ES were close to each other. This signalled for the existence of no principal difference in which one to choose. For instance, such a change occurred in market risk regulation by the Basel Committee. In 1996 the Basel I amendment introduced the VaR at 99% level. The Basel III fundamental review of the trading book (FRTB) BCBS (2013) substituted it by ES at lower 97.5% to essentially preserve the gross risk estimate and capital requirements unchanged. However, as we see from panel D the choice of the risk measure might drastically change the risk estimate in the considered hypothetical case of ours.

Third, though exorbitant the ES estimate of 24% is in panel D, it is not robust in itself. It has no underlying data for that particular computed value. The value of 24% is derived as the mathematical mean in accordance with the ES definition as the average over observations in excess of the VaR. However, the feasible risk realisation which is likely to happen is around 40% where the second hump (mode) of the distribution is.

The single simulation case considered in Figure 5 gave us the following food for thought.

- Increasing the share of high-default-correlated exposure (though a low-risk one) results in significant rise of the credit risk value as the risk measure concentrates around the newly born second mode of the loss distribution;
- The credit risk estimate and capital requirement based upon them might be unreliable. The estimate depends strongly upon the chosen risk measure and the quantile. The actual loss (DR values corresponding to the second mode) might be above the mathematically obtained ES estimates;

Next, we look at the implications from increasing the proportion of "green" loans depending on the combinations of risk parameters in the segment and its relationship to the risk parameters of the "brown" one.

5. Application to "green" lending

We remind that we focus on the hypothetical loan book. It is a combination of three distributions of the correlated Bernoulli trials (defaults). Two of the distributions are based on the US data (both of them are considered to be "brown" ones). The third one corresponds to the "green" lending. By varying PD, default correlation and the proportion values of the third ("green") segment within a mixture we arrive at the total portfolio DR distribution.

Two subsections follow. First, we assume no changes in the default probabilities per segment in subsection **??**; after we adopt changes in PDs per segment in subsection **??**.

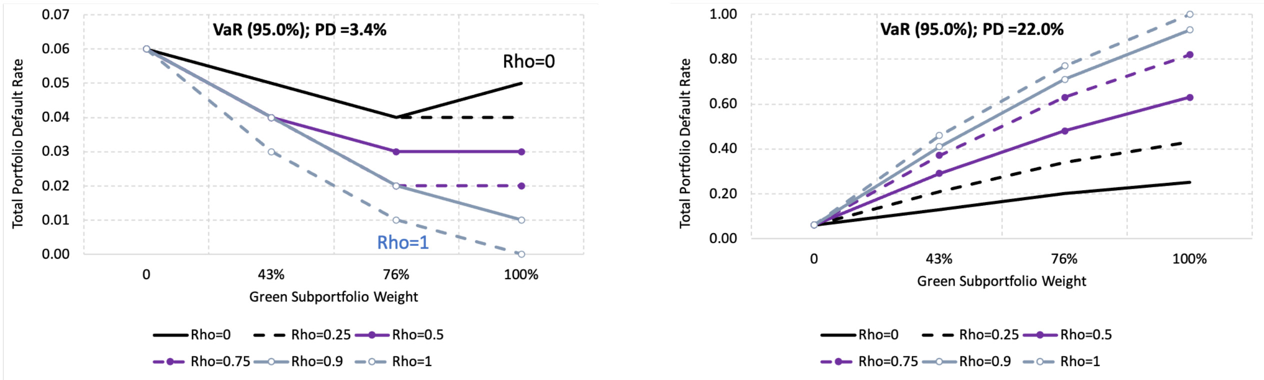
5.1. Major findings

Figures 6, 7 contain the results for the simulated scenarios. Details are available in Annex A.2.1.

Figure 6 presents the VaR risk measure at the 95% confidence level, if the "green" lending is a low risk one ($PD = 3.4\%$, see left panel) or a high risk one ($PD = 22.0\%$, see right panel). Expectedly, increasing lending in the low-risk segment always leads to credit risk decline; while the accumulation of the high-risk lending does the opposite (leads to its rise). However, and more interestingly, **default correlation multiples the effect of the default probability, i.e., higher default correlation reduces the low credit risk more for the "green" segment, than when it is nil; and vice versa, it leads to high credit risk additional rise.**

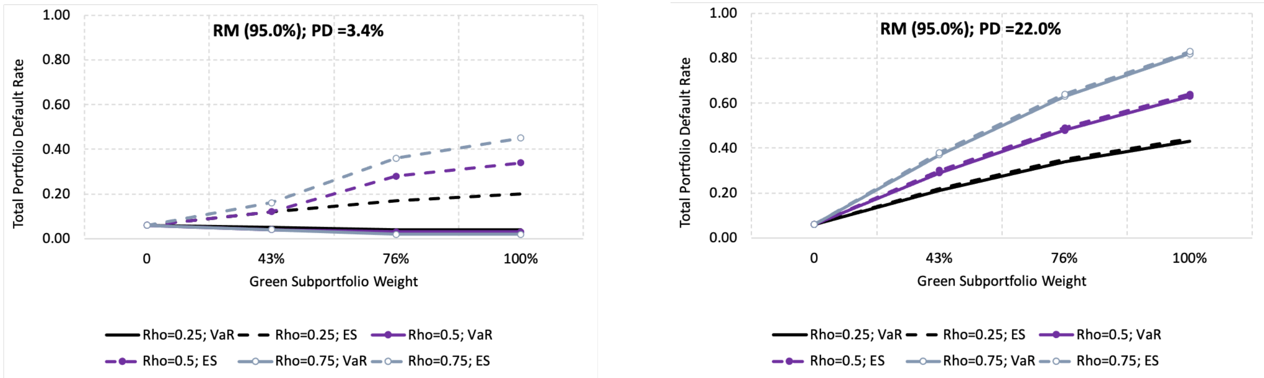
Figure **??** presents the credit risk sensitivity to the choice of the risk measure. We present the full range of default correlations in the "green" segment when it is low risk (on the left panel) and high (on the right one). In the latter case the choice of the risk measure does not make a difference. ES risk measure by definition produces higher credit risk, but not materially larger than according to VaR. Interestingly,

Figure 6: Impact of default correlation in the green segment.



Note: the legend includes the parameters for the green exposures:
Rho - default correlation; PD - probability of default.
The chart is the visual representation of the selected cases from the Table 3.

Figure 7: Impact of risk-measure choice.



Note: the legend includes the parameters for the green exposures:
Rho - default correlation; PD - probability of default.
RM - risk-measure: VaR - Value-at-Risk, ES - Expected Shortfall;
The chart is the visual representation of the selected cases from the Table 3.

if the "green" segment is a low risk one, credit risk is sensitive to the choice of the risk-measure. For instance, VaR measure is mostly close to zero for any portion of "green" loans in the total portfolio. However, when switching to the ES, the risk value jumps up to 40%. Such a cliff effect is not desirable form the risk-management perspective. The effect itself was already noticed in the baseline case when shifting between cases B and A on the Figure 1.

To additionally highlight, default correlation does not multiply the effect of risk-measure choice, as it did before depending on the level of PD.

5.2. Alternative scenarios

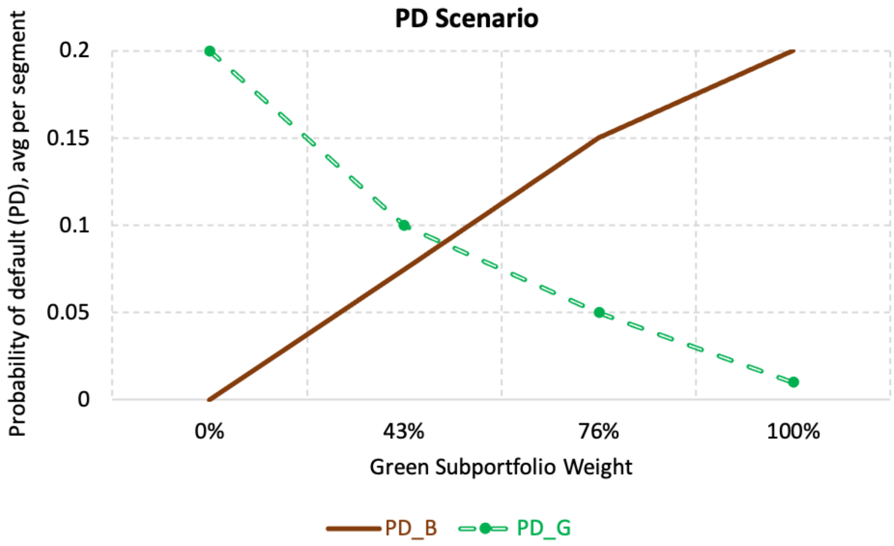
The above simulation was based on the assumption of the same default probability per segment ("green" or "brown"). However, the efforts towards promoting "green" transition might result in reduction of PDs for the "green" borrowers and its rise for the "browner" ones (a 'PD reversal' effect).[2]

To consider implications of such PD reversal, we introduce scenarios for the changes in PD. The "green" borrowers start with the 20% per annum PD and finish with the 1% value. The "brown" ones start with the zero PD (we will increase the PD for the low risky segment out of the two typical for the US economy and considered above). By the time the "green" borrowers occupy the entire loan book, the default probability of the "brown" borrowers is assumed to reach the same 20% from which the "green"

[2]Author acknowledges K.V.Yudaeva for suggesting considering such a scenario.

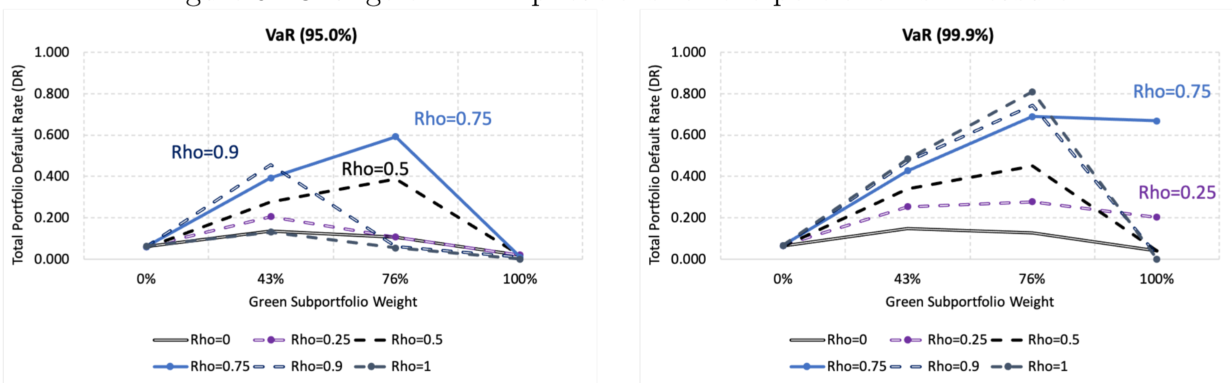
borrowers departed. For the PD scenarios' illustration, please, refer to the Figure 8.

Figure 8: PD Scenarios.



Note: PD - probability of default; B - brown segment; G - green one.

Figure 9: Change of PD implications for the portfolio risk measure.



Note: granular data is available in Annex A.2.2.

We consider the entire range of the default correlation values for the "green" segment. The simulated results are available in Annex A.2.2. Figure 9 contains illustrations for the VaR at the 95 and 99.9% confidence levels.

We may register the positive impact from accumulating "green" loans. Somewhere after the portion of "green" loans exceed half of the book, the credit risk measure start to decline (specific break-even point depends on the combination of the default probability and default correlation).

Notably, given particular combination of default probability and default correlation combination the credit risk measure may not decline when using quite high confidence levels. For instance, see the right panel of the Figure 9 where under default correlation of 25 and 75% the risk measure resides in the nearby of 20 and 70%, when the loan book consists entirely of "green" loans and the credit risk is proxied as the VaR at the 99.9% confidence level.

6. Discussion

There are four often raised questions when discussion the credit risk implications of increasing the proportion of green loans:

1. Unfortunately, the promotion of the "green" finance agenda often discovers the dishonest action called *green-washing*. The term is used to reflect activities when the actually "brown" production or initiative is positioned as a "green" one. Hence, how the presence of green-washing impact the findings?[3]

2. It seems that increasing exposure to any lending segment provokes rise in credit risk concentration and hence rise in the credit risk metrics (measures). Similarly, rise in brown lending is to result in equivalent rise in credit risk, isn't it?[4]

3. Promoting green finance and green transition overall impact the correlation between green and brown sectors. What is to happen to the credit risk measure if the negative default correlation is assumed between the two sectors?[5]

4. "Green" lending segment is no less heterogenous, than the "brown" one. Why do you then use a single default correlation parameter for the "green" portfolio?[6]

Let us comment on these questions.

First, indeed green-washing is the unfavourable tendency rising in parallel to "green" lending. De facto it means that the true share of "green" loans is lower than it is claimed to be. From the credit risk perspective studied here, the smaller the actual portion of green loans is, the lower the credit risk is. Hence, the presence of green-washing has a positive counter-balancing effect of improving the creditworthiness of banks as lenders when they augment the share of green loans.

Second, all else being equal, increasing exposure to a particular lending segment should result in credit risk concentration rise. However, the statement holds true when the two requirements are met:

- That should be a brand-new segment which was not present in the loan book before.

- To have a rise in credit risk measure portfolio-wise, the new segment should be highly default-correlated within itself.

For instance, if a bank has no green lending, it cannot boost the share of the brown lending as it is already 100% of the loan book. If the bank had some green lending, re-increasing the proportion of the brown exposures is equivalent to moving left-wards in terms of the illustrative chart 5, i.e., from panel D to panel C.

This is why the rise in brown lending leads to a decline in the share of highly default-correlated segment (when such a one is present in the loan book). When a green segment is not present, the proportion of the brown on is already 100% and cannot be raised in relative terms (though it can be done in absolute, nominal terms). Thus, increasing exposure to brown sector does not have the same implications of increasing credit risk, as it happens with the rise in green lending and reduction of the brown one.

Third, considering the negative default correlation is attractive, but not feasible within the chosen simulation framework. The Lunn-Davies approach in formula (11) takes non-negative correlation values only. Moreover, we may recall the limitation raised by Preisser and Qaqish (2014) in Formula (3). When our loan book is large the portfolio- (segment-) wide average default correlation is bounded with zero from below.

However, we may imagine that if there is indeed a negative default correlation in-between the two segment (green and brown), there should be an equivalent to U-shaped dependency in Figures 6, 7. Initial accumulation of the assets (green loans) which are negatively correlated with an existing (brown) asset, should result in diversification effect and till some portion of new (green) loans reduce the credit risk measure. Nevertheless, further focus on a single segment (offering only green loans) should evaporate the created diversification effect and amplify the credit concentration risk. Such a reduction and further rise in the credit risk should have an equivalent to a U-shaped dependency.

---

[3]Author acknowledges A.G.Morozov for drawing attention to this trend.

[4]Author acknowledges N.Turdyeva and D.Musaelyan for raising the point.

[5]Author acknowledges A.Sinyakov for putting the question to the agenda.

[6]Author acknowledges D.Musaelyan for putting this question.

Fourth, To remark, the industry-wide paradigm coined by Vasicek in Formula (9) or suggested by Lunn-Davies in Formulas (11) depart from the average default correlations. Definitely, when considering different pairs of borrowers, the applicable pair-wise default correlations might not be the same as the segment-averages derived from formulas (9) or (11). However, the model framework departs from the assumption that there is some portfolio- or segment-average default correlation, a single (common) parameter for every borrower (R in the Vasicek model or ρ in the Lunn-Davies one). The Basel Committee later differentiated the parameter R making it dependent upon PD of the borrower, but that was not done in the original theoretical papers by Vasicek. Moreover, the limitations of such a predefined link between PD and R is explained by Penikas (2023).

The key implication here is that the portfolio credit risk modeling departs from an average default correlation parameter typical for the entire sector (e.g., brown or green one). It does not mean that the sector is homogeneous, but using a single default correlation parameter is the only solution. Due to the absence of default flags for the majority of borrowers, one cannot statistically fit the default correlation parameter for every pair of borrowers (specifically, if one of the two borrowers had never experienced a default). However, as explained by Penikas (2023), one can do so at the portfolio level by observing the default rate historical variance (recall Formula (6)).

Thus, proceeding with the common default correlation parameter per segment (green or brown) assumes that the sector is heterogeneous. But a single parameter helps to compare segments in-between the rest. For instance, current paper uses the simulation default correlation parameters of 26% and 75% for the brown sub-segment and the green segment. Both of them are heterogeneous, but the green ones are more dependent upon the systemic factor (successful green transition), rather than the brown ones.

7. Conclusion

The objective of the current paper was to study the properties of the credit risk distribution (and default rate, DR, specifically) when the risk parameters vary: probability of default, default correlation, portfolio composition (weight of the particular lending segments).

One might have expected the high risk and its high concentration implications. It seems that the saying that "one should avoid putting all eggs into one basket" as a underlying risk management principle of diversification. However, individual US investors like Mohamed (2023), as well as the German Bundesbank in Jahn et al. (2013) have put it under doubt whether it is indeed worth following diversification and avoiding risk concentration.

Moreover, modern banking regulation became much complicated. As a result, general risk-management principles might find counter-intuitive reflection in the instruments used, for instance, depending on the chosen risk measure type and the confidence levels.

The current paper helped focusing on the series of features when evaluating credit risk which did not get sufficient coverage before. In particular, we find that:

- Quantitatively credit risk measures can decline, as well as rise when the default correlation rises. We show when such measures move in a single direction, and when there might be opposite trends at the same time.

- Single-direction rise in the credit risk value takes place when the default probability is low and the default correlation rises, while the credit risk value declines when PD is high and default correlation also rises.

- However, there might be changes in opposite directions for the credit risk value same time. For instance, when default correlation rises, credit risk value when measured using a simpler risk measure (e.g., VaR) or at a lower confidence level might decline, while it may rise for a more sophisticated one (e.g., ES) or at higher confidence level. We explain that this comes from the DR distribution bimodality which appears under high and rising default correlation. Implication from banks and regulators is that to preserve financial stability they may wish to use several risk-measures, not limited to a single one when regulating risk.

- As for the "green" lending we show that under any default correlation the credit risk measure declines in the longer run when banks accumulate such loans and two trends takes place: from one side, the PD for the "green" segment declines; and, from another side, PD for the "brown" one increases. However, such a total credit risk measure decline takes place after its rise in the mid-run.
- We develop two types of codes: to fit the credit risk parameters as we consider the DR distribution to originate from a mixture of distributions and to simulate a DR distribution for any combination of parameters which might form such a mixture.

Our findings are important today when the entire series of events at global scale (world economy fragmentation, energy transition, climate change) may lead to an increase in default correlation. Current paper has evidence that even under no changes in the default probability such a rise in default correlation materially changes the profiles of risks taken by the banks. This might created complications for banks and regulators, as well as offer new opportunities to develop business when they find themselves in the positive outcomes.

A. Annexes

A.1. Code in R project

A.1.1 DR distribution parameters' fit

```
#=========================================
# Fitting DR distribution
#=========================================

# --------------- Delete previous objects -------------------
rm(list=ls(all=TRUE))

##############################################

#================= Upload data ============

#============US DEFAULTS ===============
### URL: https://www.federalreserve.gov/releases/chargeoff/delallsa.htm

### UNcomment to process US ~50Y DR data
# set the path to the stored data

 Data0 <- read.table(".../US_Defaults.txt", header=T)


### start from 1985 to 2023Q3
 row1 <- 1
 row2 <- 155

#-------------------------------------------

### Residential All Loans
# col  <- 2

### Agricultural Loans
# col  <- 11

### Total Loans
 col  <- 12


#-------------------------------------------
 x01 <- Data0[row1:row2, col] / 100

#-------------------------------------------
PD_0   <- mean(x01)
var_DR <- var(x01)
Rho_0  <- var_DR / ( PD_0 * (1 - PD_0) )

# if we omitted the fact that it is a mixture of distributions
PD_0
Rho_0
```

```
### SIMULATION PARAMETERS ####################

##############################################

# the number of steps (units) we want to check between 0 (START, S) and 1 (MAX_)

Scale_1 <- 100

#--------------------------------
# number of hypothetical time spans

T <- 200

# number of hypothetical borrowers in our simulated portfolio

N <- 200

############## OTHER PARAMETERS (one can change, but see no need) ########

### number of observations per time span per borrower
### it is obviously one; but it is explicitly inputted as a constant (pasted value)
### as the data generation function allows for other values

obs  <- 1

### number of trials per time span per borrower
### it is obviously one; but it is explicitly inputted as a constant (pasted value)
### as the data generation function allows for other values

trials  <- 1

###########################################

# The algorithm objective is to derive the parameters of the DR density distribution.
# There are two models to be tested
# - Vasicek (1987): assumes correlated N(0,1)-distributed components
# - Lunn, Davies (1997): assume correlated Bernoulli trials

### Depending on which model you wish to fit, comment or relevant sections.

# The key assumption is that it comes from a mixture of correlated Bernoulli distributions.
# We assume that the mixture is composed out of two distributions.
# The code uses search on the net procedure.
# The objective function is the distance between CDFs (actual and fitted).
# There are two options: either to use KS or CvM distance.
# The objective function is minimised, the PD and Rho for the both distributions are saved.
# The progress is displayed using the CDF fit plot.

# single symbol (#) can de taken away (can be commented / UNcommented)
# triple symbol (###) provides a comment (explanation). Not for uncommenting.


###########################################
```

```
###########################################


# ----------------- Packages download --------------------

library(fBasics)
library(scatterplot3d)
library(ggplot2)

library(distr)
library(fGarch)

#library(copula)

#library(fcopulae)
library(stats19)

library(QRM)

library(sn)


library(moments)

library(pracma)

#library(CreditMetrics)

### from PC archive
# library(mlCopulaSelection)

#============================

#row1 <- 1
#row2 <- 10

#col  <- 2
#col  <- 6

#x01 <- Data0[row1:row2, col]

#=======================================

### UNcomment below to see the raw data
# Data0

# time <- Data0[row1:row2, 1]


#------------------- CHECK the uploaded data ----------------------
US_DR <- x01

plot(US_DR, type = "l")
```

```
#-------------------------------------------
# actual versus N PDFs with the same DR mean and variance, and same sample size


# hist(US_DR, ylim=c(0,50))
 hist(US_DR)

 lines(density(rnorm(n=row2, mean=mean(x01), sd=sd(x01))), col="red")



### create CDF function
 ecdf_x01 <- ecdf(x01)
 ecdf_x01

### extract CDF fn values for particular realisations

# x_cdf    <- ecdf_x01(x01)


###################### SET the parameters for algorithm #####################
### these initial values will be replaced by the derived ones


statistics_KS <- 100



 alpha10 <- 0

 beta10  <- 0


 alpha20 <- 0

 beta20  <- 0

 w00     <- 0

#-------------------------------------------

#------------ LENGTH of search vectors ----------------------
# Initially we check 0 - 1, but the range can de reduced for more precise computation.


# START

S     <- 0

#--------------------------------
```

```
# END
MAX_A   <- 1

MAX_B   <- 1

################ OTHER PARAMETERS (NOT to be changed) #################
#------------------------
Scale_2 <- Scale_1
n_obs   <- Scale_1

W       <- Scale_1


x_beta <- seq(0, 1, by = 1/n_obs)

#-----------------------
# END

M_A     <- MAX_A * Scale_1

M_B     <- MAX_B * Scale_1

#### FROM


A10 <- S

B10 <- S

A20 <- S

B20 <- S

# the weight of the first distribution in the mixture of two
# starting search value for the weight

w_0 <- S


#### TO


A11 <- M_A

B11 <- M_B

A21 <- M_A

B21 <- M_B

# end search value for the weight

w_1 <- W
```

```
##############################################

##### START A SEARCH ON THE NET CYCLE ########

##############################################


#### WE ASSUME THIS IS A LOW RISK LOW CORRELATED distribution.

 for (a1 in 0:0)
 for (b1 in 0:0)

#### UNcomment if wish to find parameters for the 1st distribution
# for (a1 in A10:A11)
# for (b1 in B10:B11)

#==============================

# DR Distribution No. 1

#==============================

#### start a cycle to generate DR series for 1st distribution

{

{

 alpha1  <- a1/Scale_1

 beta1   <- b1/Scale_1

#-------------------------------------
### probability of success (default) on each trial

pb  <- alpha1

### correlation parameter

Rho <- beta1


### auxiliary (temporary) DR series

DR1  <- rep(0, T)

#=================================

#=================================
# default quantile for the given PD (pb)
```

```
Def_q   <- qnorm(pb, mean = 0, sd = 1)

# common factor

CF  <- rep(0, T)

# individual factor

Y  <- matrix(0, nrow=T, ncol = N)

# correlation parameter to simulate dependent outcomes

U  <- matrix(0, nrow=T, ncol = N)


# asset return for Vasicek model

AR  <- matrix(0, nrow=T, ncol = N)

# outcome (default / non-default)

X  <- matrix(0, nrow=T, ncol = N)

# sequence (series) of defaults

Def  <- rep(0, T)

# default rate (DR) sequence (series)
DR  <- rep(0, T)


#--------------------------------
#--------------------------------

for (t in 1:T)

{
### Vasicek (1987) model
# CF[t] <- rnorm(obs, mean = 0, sd = 1)

### Lunn, Davies (1997) model
 CF[t] <- rbinom(obs, trials, pb)

#---------------------------------

for (n in 1:N)

{


#============================================
### Vasicek (1987) model
#--------------------------------------------
```

```
# Y[t,n] <- rnorm(obs, mean = 0, sd = 1)
# U[t,n] <- Rho

#--------------------------------
# AR[t,n] <- CF[t] * U[t,n] + ( (1 - ( (U[t,n])^2) )^(1/2) ) * Y[t,n]
# X[t, n] <- (AR[t,n] <= Def_q) * 1

#===========================================
### Lunn, Davies (1997) model
#-------------------------------------------


 Y[t,n] <- rbinom(obs, trials, pb)
 U[t,n] <- rbinom(obs, trials, Rho)

#--------------------------------

 X[t,n]    <- CF[t] * U[t,n] + (1 - U[t,n] ) * Y[t,n]

#--------------------------------
#--------------------------------


Def[t] <- sum(X[t,])

DR[t]  <- Def[t] / N


}
}

#=================================
#=================================

DR1 <- DR

#--------------------------------

ecdf_DR1 <- ecdf(DR1)
y_beta1  <- ecdf_DR1(x_beta)

#==============================

# DR Distribution No. 2

#==============================

#### start a cycle to generate DR series for 2nd distribution

#-----------------------------------------

for (a2 in A20:A21)
```

```
for (b2 in B20:B21)

{

{

 alpha2  <- a2/Scale_2

 beta2   <- b2/Scale_2

#-------------------------------

pb  <- alpha2

Rho <- beta2

### auxiliary (temporary) DR series

DR2  <- rep(0, T)

#=================================

#=================================
# default quantile for the given PD (pb)
Def_q   <- qnorm(pb, mean = 0, sd = 1)

# common factor

CF  <- rep(0, T)

# individual factor

Y  <- matrix(0, nrow=T, ncol = N)

# correlation parameter to simulate dependent outcomes

U  <- matrix(0, nrow=T, ncol = N)

# asset return for Vasicek model

AR  <- matrix(0, nrow=T, ncol = N)

# outcome (default / non-default)

X  <- matrix(0, nrow=T, ncol = N)

# sequence (series) of defaults
```

```
Def  <- rep(0, T)

# default rate (DR) sequence (series)
DR  <- rep(0, T)


#--------------------------------
#--------------------------------

for (t in 1:T)

{
### Vasicek (1987) model
#  CF[t] <- rnorm(obs, mean = 0, sd = 1)

### Lunn, Davies (1997) model

 CF[t] <- rbinom(obs, trials, pb)

#---------------------------------

for (n in 1:N)


{

#===========================================
### Vasicek (1987) model
#------------------------------------------

# Y[t,n] <- rnorm(obs, mean = 0, sd = 1)
# U[t,n] <- Rho

#--------------------------------
# AR[t,n] <- CF[t] * U[t,n] + ( (1 - ( (U[t,n])^2) )^(1/2) ) * Y[t,n]
# X[t, n] <- (AR[t,n] <= Def_q) * 1

#===========================================
### Lunn, Davies (1997) model
#------------------------------------------

 Y[t,n] <- rbinom(obs, trials, pb)
 U[t,n] <- rbinom(obs, trials, Rho)

#--------------------------------

 X[t,n]    <- CF[t] * U[t,n] + (1 - U[t,n] ) * Y[t,n]

#--------------------------------
#--------------------------------


Def[t] <- sum(X[t,])
```

```
DR[t]  <- Def[t] / N

}

}

#=================================
#=================================

DR2 <- DR

#-------------------------------------------------

ecdf_DR2 <- ecdf(DR2)

y_beta2  <- ecdf_DR2(x_beta)

####################################################

#------------------------------------------
### assume there is only 2nd distribution
# for (w  in w_0:w_0)


### UNcomment to find weights within a mixture of distributions
 for (w  in w_0:w_1)

{

 w0 <- w/W

DR_pool  <- w0 * DR1 + (1-w0) * DR2

dr <- ecdf(DR_pool)

y_pool <-dr(x_beta)

y_pool_d   <- w0 * y_beta1 + (1-w0) * y_beta2


#------------------------------------

ecdf_x01  <- ecdf(x01)

p    <- ecdf_x01(x_beta)

#------------------------------------

# Kolmogorov - Smirnov

  KS  <- max ( abs ( y_pool - p) )
```

```
# Cramer-von-Mises

# KS <- (1/ n_obs) * sum ( (y_pool - p )^2 )

#-----------------------------------

# COUNTER for the number of the current search (fit) iteration
# considers parameters of the 2nd distribution only and weight


i =
  (w   - w_0 + 1) +

+ (w_1 - w_0 + 1) * (a2  - A20) +

+ (w_1 - w_0 + 1) * (A21 - A20 + 1) * (b2  - B20) +

+ (w_1 - w_0 + 1) * (A21 - A20 + 1) * (B21 - B20 + 1) * (a2  - A20) +

+ (w_1 - w_0 + 1) * (A21 - A20 + 1) * (B21 - B20 + 1) * (A21 - A20 + 1) * (b2 - B20)


#-----------------------------------

### UNcomment to monitor progress by each step,
### BUT it slows a lot the search time


# plot(x_beta, y_pool, type = "line", xlab = i)

# lines(x_beta, p, type="line", col = "red", lty = 2)


# hist(x01, xlab = i)

# lines(x_beta, y_pool_d, col = "red")


#-----------------------------------


if ( KS < statistics_KS )

{

statistics_KS  <- KS


 alpha10  <- a1/Scale_1

 beta10   <- b1/Scale_1

 alpha20  <- a2/Scale_2
```

```
 beta20   <- b2/Scale_2

 w00    <- w/W

# R_IRB_2  <- beta20^2

# monitoring progress during improvement steps only
# comparison of actual and fitted CDFs
# horizontal axis shows the minimised KS value at the most recent step

 plot(x_beta, y_pool, type = "line", xlab = KS)

 lines(x_beta, p, type="line", col = "red", lty = 2)

# hist(x01, xlab = KS)

# lines(x_beta,y_pool_d, col = "red")

}

#------------------------------------

}

}

}

}

}

#------------------------------------

n <- row2

# KS critical level (for N > 10)

Critical = 0.58 / (n)^0.5

#################################

w00 # 1st distribution weight


 alpha10  # PD  1

 beta10   # Rho 1

 alpha20  # PD  2
```

```
 beta20   # Rho 2 (correlation)

# R_IRB_2  # Vasicek R (corr.squared)


statistics_KS # difference to actual series CDF

Critical # If it exceeds statistics_KS, fit is proper

#################################
```

A.1.2 DR distribution simulation

```
#========================================
# Simulating DR distribution
#========================================

# --------------- Delete previous objects -------------------

rm(list=ls(all=TRUE))

# set the path to store the project output
path0 <- ".../02_output/"


##############################################
############# LIBRARIES  #####################
##############################################

# install.packages("tidyverse")
# install.packages("stringr")
# install.packages("writexl")

# install.packages("distr")
# install.packages("stats19")
# install.packages("moments")


# ----------------- Packages download --------------------

library(fBasics)
library(scatterplot3d)
library(ggplot2)

library(distr)
library(fGarch)

library(stats19)

library(sn)
library(moments)

library (writexl)
library(tidyverse)

library(stringr)


##############################################
############# PARAMETERS #####################
##############################################
# number of portfolios (currently previewed for K = 3)

K <- 3
```

```
#--------------------------------
# number of hypothetical time spans
T <-  500

# number of hypothetical borrowers in our simulated portfolio
N <-  500

#******************************

#------------------------
n1_esg    <- 4

W_ESG    <- rep(0, n1_esg)
W_ESG[1]  <- 0
W_ESG[2]  <- 0.43
W_ESG[3]  <- 0.76
W_ESG[4]  <- 1

#------------------------
n2_esg    <- 3

PD_ESG    <- rep(0, n2_esg)
PD_ESG[1] <- 0.034
PD_ESG[2] <- 0.04
PD_ESG[3] <- 0.220

#------------------------
n3_esg    <- 6

R_ESG   <- rep(0, n3_esg)

 R_ESG[1] <- 0
 R_ESG[2] <- 0.25
 R_ESG[3] <- 0.5
 R_ESG[4] <- 0.75
 R_ESG[5] <- 0.90
 R_ESG[6] <- 1

#------------------------
n4_q    <- 5

quantiles    <- rep(0, n4_q)
quantiles[1] <- 0.950
quantiles[2] <- 0.975
quantiles[3] <- 0.990
quantiles[4] <- 0.995
quantiles[5] <- 0.999

#----------------------------
#Output   <- matrix(0, nrow= n2_esg * n3_esg + 1, ncol = n1_esg + 2)

Output_VaR   <- matrix(0, nrow= n4_q * n2_esg * n3_esg + 1, ncol = n1_esg + 3)
Output_ES    <- matrix(0, nrow= n4_q * n2_esg * n3_esg + 1, ncol = n1_esg + 3)
```

```
# headers
Output_VaR[1, 1] <- "Quantile"
Output_VaR[1, 2] <- "Rh_ESG"
Output_VaR[1, 3] <- "PD_ESG"
Output_VaR[1, 4] <- "W1_ESG"
Output_VaR[1, 5] <- "W2_ESG"
Output_VaR[1, 6] <- "W3_ESG"
Output_VaR[1, 7] <- "W4_ESG"

Output_ES[1, 1] <- "Quantile"
Output_ES[1, 2] <- "Rh_ESG"
Output_ES[1, 3] <- "PD_ESG"
Output_ES[1, 4] <- "W1_ESG"
Output_ES[1, 5] <- "W2_ESG"
Output_ES[1, 6] <- "W3_ESG"
Output_ES[1, 7] <- "W4_ESG"
#------------------------
##############################################
###########################################
###########################################

for (i1  in 1: n1_esg)
{
for (i2  in 1: n2_esg)
{
for (i3  in 1: n3_esg)
{

###############################################
# the weight of the first distribution in the mixture of two
# starting search value for the weight

w01 <- 0.87       # LDP, w/o ESG
w02 <- 1 - w01   # risky portfolio , w/o ESG

#--------------
w_3 <- W_ESG[i1]    # ESG portfolio
w12 <- 1 - w_3 # sum of weights for other 2 portfolios

w_1 <- w01 * w12 # LDP
w_2 <- w02 * w12 # risky portfolio

#-------------------------------------
### probability of success (default) on each trial
##### if you wish to have LDP, better put minimal, but non-zero PD value

  alpha1 <- 0.00        # PD 1
  alpha2 <- 0.22         # PD 2
  alpha3 <- PD_ESG[i2]    # PD 3

### correlation parameter
```

```
beta1  <- 0.00         # Rho 1
beta2  <- 0.26         # Rho 2
beta3  <- R_ESG[i3]     # Rho 3


##############################################

# the number of steps (units) we want to check between 0 (START, S) and 1 (MAX_)

Scale_1 <- 100


############## OTHER PARAMETERS (one can change, but see no need) ########

### number of observations per time span per borrower
### it is obviously one; but it is explicitly inputted as a constant (pasted value)
### as the data generation function allows for other values

obs  <- 1

### number of trials per time span per borrower
### it is obviously one; but it is explicitly inputted as a constant (pasted value)
### as the data generation function allows for other values

trials  <- 1

###########################################

# The algorithm objective is to derive the parameters of the DR density distribution.
# There are two models to be tested
# - Vasicek (1987): assumes correlated N(0,1)-distributed components
# - Lunn, Davies (1997): assume correlated Bernoulli trials

### Depending on which model you wish to fit, comment or uncomment relevant sections.

# The key assumption is that it comes from a mixture of correlated Bernoulli distributions.
# We assume that the mixture is composed out of two distributions.
# The code uses search on the net procedure.
# The objective function is the distance between CDFs (actual and fitted).
# There are two options: either to use KS or CvM distance.
# The objective function is minimised, the PD and Rho for the both distributions are saved.
# The progress is displayed using the CDF fit plot.

# single symbol (#) can de taken away (can be commented / UNcommented)
# triple symbol (###) provides a comment (explanation). Not for uncommenting.

#==============================

x_beta <- seq(0, 1, by = 1/Scale_1)


#####################################################
```

```
Proba    <- rep(0, K)
Correl    <- rep(0, K)
Weight    <- rep(0, K)

#----------------------------
Proba[1]  <- alpha1
Proba[2]  <- alpha2
Proba[3]  <- alpha3

#----------------------------
Correl[1] <- beta1
Correl[2] <- beta2
Correl[3] <- beta3

#----------------------------


Weight[1] <- w_1
Weight[2] <- w_2
Weight[3] <- w_3

#----------------------------
DefRate   <- matrix(0, nrow=T, ncol = K)

#==============================

# DR CYCLE for N portfolios

#==============================


for (k in 1:K)
{

pb  <- Proba[k]
Rho <- Correl[k]

#=================================

### auxiliary (temporary) DR series
# DR2  <- rep(0, T)

#=================================
# default quantile for the given PD (pb)
Def_q   <- qnorm(pb, mean = 0, sd = 1)

# common factor
CF  <- rep(0, T)

# individual factor
Y  <- matrix(0, nrow=T, ncol = N)

# correlation parameter to simulate dependent outcomes
```

```
U  <- matrix(0, nrow=T, ncol = N)

# asset return for Vasicek model
AR  <- matrix(0, nrow=T, ncol = N)

# outcome (default / non-default)
X  <- matrix(0, nrow=T, ncol = N)

# sequence (series) of defaults
Def  <- rep(0, T)

# default rate (DR) sequence (series)
DR  <- rep(0, T)

#--------------------------------
#--------------------------------

for (t in 1:T)

{
### Vasicek (1987) model
#  CF[t] <- rnorm(obs, mean = 0, sd = 1)

### Lunn, Davies (1997) model

 CF[t] <- rbinom(obs, trials, pb)

#---------------------------------

for (n in 1:N)

{

#===========================================
### Vasicek (1987) model
#-------------------------------------------

# Y[t,n] <- rnorm(obs, mean = 0, sd = 1)
# U[t,n] <- Rho

#--------------------------------
# AR[t,n] <- CF[t] * U[t,n] + ( (1 - ( (U[t,n])^2) )^(1/2) ) * Y[t,n]
# X[t, n] <- (AR[t,n] <= Def_q) * 1

#===========================================
### Lunn, Davies (1997) model
#-------------------------------------------

 Y[t,n] <- rbinom(obs, trials, pb)
 U[t,n] <- rbinom(obs, trials, Rho)

#--------------------------------
```

```
 X[t,n]    <- CF[t] * U[t,n] + (1 - U[t,n] ) * Y[t,n]

#--------------------------------
#--------------------------------


Def[t] <- sum(X[t,])

DR[t]  <- Def[t] / N


}

}

#=================================
#=================================

#DR2 <- DR

DefRate[,k] <- DR

#-------------------------------------------------

#ecdf_DR2 <- ecdf(DR2)
#y_beta2  <- ecdf_DR2(x_beta)

}

####################################################

DR_pool  <- Weight[1] * DefRate[,1] + Weight[2] * DefRate[,2] + Weight[3] * DefRate[,3]

dr       <- ecdf(DR_pool)
y_pool   <- dr(x_beta)
#------------------------------------
# y_pool_d  <- w_1 * y_beta1 + w_2 * y_beta2 + w_3 * y_beta3

#------------------------------------
Risk <- DR_pool

# simulated DR versus N PDFs with the same DR mean and variance, and same sample size

hist(Risk, main = "")

# hist(Risk, ylim=c(0,T))

# hist(Risk)
# lines(density(rnorm(n=T, mean=mean(Risk), sd=sd(Risk))), col="red")

#------------------------------------
#------------------------------------
#------------------------------------
```

```
for (i4  in 1: n4_q)
{

Q_nn <- quantiles[i4]

###############################################
n_row <- (i4-1) * n3_esg * n2_esg + (i3-1) * n2_esg + i2 + 1
n_col <- i1 + 3

#---------------------------------

Output_VaR[n_row,1] <-  Q_nn
Output_VaR[n_row,2] <-  beta3
Output_VaR[n_row,3] <- alpha3

Output_ES[n_row,1]  <-  Q_nn
Output_ES[n_row,2]  <-  beta3
Output_ES[n_row,3]  <- alpha3

#---------------------------------

VaR  <- quantile(Risk, Q_nn)
ES   <-mean(Risk [Risk >= VaR] )

Output_VaR[n_row, n_col] <- round(VaR, digits = 3)
Output_ES[n_row, n_col]  <- round(ES , digits = 3)

}

##################################

}
}
}

#------------------------------------
var      <- c(path0, "VaR", ".xlsx")
path_var <- str_c(var, collapse = "")

es       <- c(path0, "ES", ".xlsx")
path_es  <- str_c(es, collapse = "")

df_var <- data.frame(Output_VaR)
df_es  <- data.frame(Output_ES)

write_xlsx(df_var, path_var)
write_xlsx(df_es , path_es )

#========================================
```

A.2. *Simulated output*

A.2.1 *Fixed* PDs per sector

Table 3: VaR calibration
(assume **no** changes in PD per segments).

				W^G			
RM	Quantile	ρ^G	PD^G	0%	43%	76%	100%
VaR	0.95	0	0.03	0.06	0.05	0.04	0.05
VaR	0.95	0	0.04	0.06	0.05	0.05	0.06
VaR	0.95	0	0.22	0.06	0.13	0.2	0.25
VaR	0.95	0.25	0.03	0.06	0.05	0.04	0.04
VaR	0.95	0.25	0.04	0.06	0.05	0.05	0.06
VaR	0.95	0.25	0.22	0.06	0.21	0.34	0.43
VaR	0.95	0.5	0.03	0.06	0.04	0.03	0.03
VaR	0.95	0.5	0.04	0.06	0.04	0.03	0.04
VaR	0.95	0.5	0.22	0.06	0.29	0.48	0.63
VaR	0.95	0.75	0.03	0.06	0.04	0.02	0.02
VaR	0.95	0.75	0.04	0.06	0.04	0.03	0.02
VaR	0.95	0.75	0.22	0.06	0.37	0.63	0.82
VaR	0.95	0.9	0.03	0.06	0.04	0.02	0.01
VaR	0.95	0.9	0.04	0.06	0.04	0.02	0.01
VaR	0.95	0.9	0.22	0.06	0.41	0.71	0.93
VaR	0.95	1	0.03	0.06	0.03	0.01	0
VaR	0.95	1	0.04	0.06	0.03	0.01	0
VaR	0.95	1	0.22	0.06	0.46	0.77	1
VaR	0.98	0	0.03	0.06	0.05	0.05	0.05
VaR	0.98	0	0.04	0.06	0.05	0.05	0.06
VaR	0.98	0	0.22	0.06	0.13	0.2	0.25
VaR	0.98	0.25	0.03	0.06	0.13	0.21	0.27
VaR	0.98	0.25	0.04	0.06	0.13	0.22	0.28
VaR	0.98	0.25	0.22	0.06	0.22	0.34	0.44
VaR	0.98	0.5	0.03	0.06	0.04	0.39	0.51
VaR	0.98	0.5	0.04	0.06	0.14	0.22	0.52
VaR	0.98	0.5	0.22	0.06	0.3	0.49	0.64
VaR	0.98	0.75	0.03	0.06	0.04	0.56	0.75
VaR	0.98	0.75	0.04	0.06	0.34	0.57	0.75
VaR	0.98	0.75	0.22	0.06	0.38	0.64	0.83
VaR	0.98	0.9	0.03	0.06	0.4	0.69	0.9
VaR	0.98	0.9	0.04	0.06	0.4	0.69	0.9
VaR	0.98	0.9	0.22	0.06	0.42	0.72	0.93
VaR	0.98	1	0.03	0.06	0.44	0.77	1
VaR	0.98	1	0.04	0.06	0.44	0.01	1
VaR	0.98	1	0.22	0.06	0.46	0.77	1
VaR	0.99	0	0.03	0.06	0.05	0.05	0.05
VaR	0.99	0	0.04	0.06	0.05	0.06	0.06
VaR	0.99	0	0.22	0.06	0.14	0.21	0.26
VaR	0.99	0.25	0.03	0.06	0.14	0.22	0.28
VaR	0.99	0.25	0.04	0.06	0.15	0.23	0.3
VaR	0.99	0.25	0.22	0.06	0.22	0.35	0.45

				W^G			
RM	Quantile	ρ^G	PD^G	0%	43%	76%	100%
VaR	0.99	0.5	0.03	0.06	0.23	0.41	0.53
VaR	0.99	0.5	0.04	0.06	0.23	0.41	0.54
VaR	0.99	0.5	0.22	0.06	0.31	0.5	0.64
VaR	0.99	0.75	0.03	0.06	0.34	0.58	0.76
VaR	0.99	0.75	0.04	0.06	0.35	0.59	0.77
VaR	0.99	0.75	0.22	0.06	0.38	0.64	0.83
VaR	0.99	0.9	0.03	0.06	0.41	0.7	0.91
VaR	0.99	0.9	0.04	0.06	0.41	0.7	0.91
VaR	0.99	0.9	0.22	0.06	0.43	0.72	0.94
VaR	0.99	1	0.03	0.06	0.44	0.77	1
VaR	0.99	1	0.04	0.06	0.46	0.77	1
VaR	0.99	1	0.22	0.06	0.46	0.77	1
VaR	1	0	0.03	0.06	0.05	0.05	0.05
VaR	1	0	0.04	0.06	0.06	0.06	0.06
VaR	1	0	0.22	0.06	0.14	0.21	0.26
VaR	1	0.25	0.03	0.06	0.15	0.23	0.3
VaR	1	0.25	0.04	0.06	0.15	0.24	0.31
VaR	1	0.25	0.22	0.06	0.22	0.35	0.46
VaR	1	0.5	0.03	0.06	0.24	0.41	0.54
VaR	1	0.5	0.04	0.06	0.24	0.42	0.55
VaR	1	0.5	0.22	0.06	0.31	0.5	0.65
VaR	1	0.75	0.03	0.06	0.35	0.59	0.78
VaR	1	0.75	0.04	0.06	0.35	0.59	0.78
VaR	1	0.75	0.22	0.06	0.38	0.65	0.84
VaR	1	0.9	0.03	0.06	0.42	0.7	0.92
VaR	1	0.9	0.04	0.06	0.42	0.71	0.92
VaR	1	0.9	0.22	0.06	0.43	0.73	0.94
VaR	1	1	0.03	0.06	0.46	0.77	1
VaR	1	1	0.04	0.06	0.46	0.77	1
VaR	1	1	0.22	0.06	0.46	0.77	1
VaR	1	0	0.03	0.06	0.06	0.05	0.06
VaR	1	0	0.04	0.06	0.06	0.06	0.07
VaR	1	0	0.22	0.06	0.14	0.22	0.26
VaR	1	0.25	0.03	0.06	0.16	0.24	0.31
VaR	1	0.25	0.04	0.06	0.15	0.25	0.31
VaR	1	0.25	0.22	0.06	0.23	0.36	0.46
VaR	1	0.5	0.03	0.06	0.26	0.43	0.55
VaR	1	0.5	0.04	0.06	0.25	0.43	0.56
VaR	1	0.5	0.22	0.06	0.31	0.51	0.65
VaR	1	0.75	0.03	0.06	0.36	0.6	0.79
VaR	1	0.75	0.04	0.06	0.37	0.59	0.79
VaR	1	0.75	0.22	0.06	0.39	0.65	0.84
VaR	1	0.9	0.03	0.06	0.43	0.71	0.92
VaR	1	0.9	0.04	0.06	0.42	0.71	0.93
VaR	1	0.9	0.22	0.06	0.43	0.73	0.94
VaR	1	1	0.03	0.06	0.46	0.77	1
VaR	1	1	0.04	0.06	0.46	0.77	1
VaR	1	1	0.22	0.06	0.47	0.77	1

				W^G			
RM	Quantile	ρ^G	PD^G	0%	43%	76%	100%
ES	0.95	0	0.03	0.06	0.05	0.05	0.05
ES	0.95	0	0.04	0.06	0.05	0.05	0.06
ES	0.95	0	0.22	0.06	0.14	0.2	0.25
ES	0.95	0.25	0.03	0.06	0.12	0.17	0.2
ES	0.95	0.25	0.04	0.06	0.12	0.19	0.28
ES	0.95	0.25	0.22	0.06	0.22	0.35	0.44
ES	0.95	0.5	0.03	0.06	0.12	0.28	0.34
ES	0.95	0.5	0.04	0.06	0.14	0.23	0.47
ES	0.95	0.5	0.22	0.06	0.3	0.49	0.64
ES	0.95	0.75	0.03	0.06	0.16	0.36	0.45
ES	0.95	0.75	0.04	0.06	0.31	0.53	0.39
ES	0.95	0.75	0.22	0.06	0.38	0.64	0.83
ES	0.95	0.9	0.03	0.06	0.39	0.48	0.53
ES	0.95	0.9	0.04	0.06	0.32	0.59	0.58
ES	0.95	0.9	0.22	0.06	0.42	0.72	0.93
ES	0.95	1	0.03	0.06	0.32	0.53	0.04
ES	0.95	1	0.04	0.06	0.38	0.35	0.05
ES	0.95	1	0.22	0.06	0.46	0.77	1
ES	0.98	0	0.03	0.06	0.05	0.05	0.05
ES	0.98	0	0.04	0.06	0.06	0.06	0.06
ES	0.98	0	0.22	0.06	0.14	0.21	0.26
ES	0.98	0.25	0.03	0.06	0.14	0.22	0.29
ES	0.98	0.25	0.04	0.06	0.14	0.23	0.29
ES	0.98	0.25	0.22	0.06	0.22	0.35	0.45
ES	0.98	0.5	0.03	0.06	0.19	0.41	0.53
ES	0.98	0.5	0.04	0.06	0.23	0.41	0.54
ES	0.98	0.5	0.22	0.06	0.3	0.5	0.64
ES	0.98	0.75	0.03	0.06	0.27	0.58	0.76
ES	0.98	0.75	0.04	0.06	0.35	0.58	0.77
ES	0.98	0.75	0.22	0.06	0.38	0.64	0.83
ES	0.98	0.9	0.03	0.06	0.41	0.7	0.91
ES	0.98	0.9	0.04	0.06	0.41	0.7	0.91
ES	0.98	0.9	0.22	0.06	0.43	0.72	0.94
ES	0.98	1	0.03	0.06	0.45	0.77	1
ES	0.98	1	0.04	0.06	0.45	0.71	1
ES	0.98	1	0.22	0.06	0.46	0.77	1
ES	0.99	0	0.03	0.06	0.06	0.05	0.06
ES	0.99	0	0.04	0.06	0.06	0.06	0.06
ES	0.99	0	0.22	0.06	0.14	0.21	0.26
ES	0.99	0.25	0.03	0.06	0.15	0.23	0.3
ES	0.99	0.25	0.04	0.06	0.15	0.24	0.31
ES	0.99	0.25	0.22	0.06	0.23	0.36	0.46
ES	0.99	0.5	0.03	0.06	0.25	0.42	0.54
ES	0.99	0.5	0.04	0.06	0.24	0.42	0.55
ES	0.99	0.5	0.22	0.06	0.31	0.51	0.65
ES	0.99	0.75	0.03	0.06	0.35	0.59	0.78
ES	0.99	0.75	0.04	0.06	0.36	0.59	0.78
ES	0.99	0.75	0.22	0.06	0.39	0.65	0.83
ES	0.99	0.9	0.03	0.06	0.42	0.7	0.92

				W^G			
RM	Quantile	ρ^G	PD^G	0%	43%	76%	100%
ES	0.99	0.9	0.04	0.06	0.42	0.71	0.92
ES	0.99	0.9	0.22	0.06	0.43	0.73	0.94
ES	0.99	1	0.03	0.06	0.46	0.77	1
ES	0.99	1	0.04	0.06	0.46	0.77	1
ES	0.99	1	0.22	0.06	0.46	0.77	1
ES	1	0	0.03	0.06	0.06	0.05	0.06
ES	1	0	0.04	0.06	0.06	0.06	0.07
ES	1	0	0.22	0.06	0.14	0.21	0.26
ES	1	0.25	0.03	0.06	0.16	0.24	0.31
ES	1	0.25	0.04	0.06	0.15	0.25	0.31
ES	1	0.25	0.22	0.06	0.23	0.36	0.46
ES	1	0.5	0.03	0.06	0.25	0.42	0.55
ES	1	0.5	0.04	0.06	0.25	0.43	0.56
ES	1	0.5	0.22	0.06	0.31	0.51	0.65
ES	1	0.75	0.03	0.06	0.36	0.6	0.78
ES	1	0.75	0.04	0.06	0.37	0.59	0.79
ES	1	0.75	0.22	0.06	0.39	0.65	0.84
ES	1	0.9	0.03	0.06	0.43	0.7	0.92
ES	1	0.9	0.04	0.06	0.42	0.71	0.92
ES	1	0.9	0.22	0.06	0.43	0.73	0.94
ES	1	1	0.03	0.06	0.46	0.77	1
ES	1	1	0.04	0.06	0.46	0.77	1
ES	1	1	0.22	0.06	0.47	0.77	1
ES	1	0	0.03	0.06	0.06	0.05	0.07
ES	1	0	0.04	0.06	0.06	0.06	0.07
ES	1	0	0.22	0.06	0.14	0.22	0.26
ES	1	0.25	0.03	0.06	0.17	0.25	0.33
ES	1	0.25	0.04	0.06	0.15	0.26	0.31
ES	1	0.25	0.22	0.06	0.24	0.37	0.47
ES	1	0.5	0.03	0.06	0.26	0.44	0.55
ES	1	0.5	0.04	0.06	0.25	0.43	0.56
ES	1	0.5	0.22	0.06	0.32	0.51	0.65
ES	1	0.75	0.03	0.06	0.36	0.6	0.79
ES	1	0.75	0.04	0.06	0.38	0.6	0.79
ES	1	0.75	0.22	0.06	0.39	0.65	0.84
ES	1	0.9	0.03	0.06	0.43	0.71	0.92
ES	1	0.9	0.04	0.06	0.42	0.71	0.93
ES	1	0.9	0.22	0.06	0.43	0.73	0.94
ES	1	1	0.03	0.06	0.46	0.77	1
ES	1	1	0.04	0.06	0.46	0.77	1
ES	1	1	0.22	0.06	0.47	0.77	1

A.2.2 *Varying* PDs per sector

RM	Quantile	ρ^G	PD^B	PD^G	DR
VaR	0.95	0	0	0.2	0.06
VaR	0.95	0	0.075	0.1	0.135
VaR	0.95	0	0.15	0.05	0.105
VaR	0.95	0	0.2	0.01	0.02
VaR	0.95	0.25	0	0.2	0.06
VaR	0.95	0.25	0.075	0.1	0.206
VaR	0.95	0.25	0.15	0.05	0.107
VaR	0.95	0.25	0.2	0.01	0.02
VaR	0.95	0.5	0	0.2	0.055
VaR	0.95	0.5	0.075	0.1	0.278
VaR	0.95	0.5	0.15	0.05	0.387
VaR	0.95	0.5	0.2	0.01	0.02
VaR	0.95	0.75	0	0.2	0.06
VaR	0.95	0.75	0.075	0.1	0.393
VaR	0.95	0.75	0.15	0.05	0.592
VaR	0.95	0.75	0.2	0.01	0.01
VaR	0.95	0.9	0	0.2	0.061
VaR	0.95	0.9	0.075	0.1	0.457
VaR	0.95	0.9	0.15	0.05	0.061
VaR	0.95	0.9	0.2	0.01	0.01
VaR	0.95	1	0	0.2	0.064
VaR	0.95	1	0.075	0.1	0.13
VaR	0.95	1	0.15	0.05	0.054
VaR	0.95	1	0.2	0.01	0
VaR	0.975	0	0	0.2	0.062
VaR	0.975	0	0.075	0.1	0.14
VaR	0.975	0	0.15	0.05	0.109
VaR	0.975	0	0.2	0.01	0.03
VaR	0.975	0.25	0	0.2	0.06
VaR	0.975	0.25	0.075	0.1	0.219
VaR	0.975	0.25	0.15	0.05	0.113
VaR	0.975	0.25	0.2	0.01	0.03
VaR	0.975	0.5	0	0.2	0.06
VaR	0.975	0.5	0.075	0.1	0.309
VaR	0.975	0.5	0.15	0.05	0.411
VaR	0.975	0.5	0.2	0.01	0.02
VaR	0.975	0.75	0	0.2	0.064
VaR	0.975	0.75	0.075	0.1	0.4
VaR	0.975	0.75	0.15	0.05	0.642
VaR	0.975	0.75	0.2	0.01	0.015
VaR	0.975	0.9	0	0.2	0.062
VaR	0.975	0.9	0.075	0.1	0.463
VaR	0.975	0.9	0.15	0.05	0.384
VaR	0.975	0.9	0.2	0.01	0.01
VaR	0.975	1	0	0.2	0.065
VaR	0.975	1	0.075	0.1	0.471
VaR	0.975	1	0.15	0.05	0.445
VaR	0.975	1	0.2	0.01	0

RM	Quantile	ρ^G	PD^B	PD^G	DR
VaR	0.99	0	0	0.2	0.062
VaR	0.99	0	0.075	0.1	0.141
VaR	0.99	0	0.15	0.05	0.121
VaR	0.99	0	0.2	0.01	0.03
VaR	0.99	0.25	0	0.2	0.061
VaR	0.99	0.25	0.075	0.1	0.232
VaR	0.99	0.25	0.15	0.05	0.243
VaR	0.99	0.25	0.2	0.01	0.042
VaR	0.99	0.5	0	0.2	0.061
VaR	0.99	0.5	0.075	0.1	0.317
VaR	0.99	0.5	0.15	0.05	0.442
VaR	0.99	0.5	0.2	0.01	0.03
VaR	0.99	0.75	0	0.2	0.066
VaR	0.99	0.75	0.075	0.1	0.425
VaR	0.99	0.75	0.15	0.05	0.679
VaR	0.99	0.75	0.2	0.01	0.027
VaR	0.99	0.9	0	0.2	0.064
VaR	0.99	0.9	0.075	0.1	0.464
VaR	0.99	0.9	0.15	0.05	0.711
VaR	0.99	0.9	0.2	0.01	0.02
VaR	0.99	1	0	0.2	0.066
VaR	0.99	1	0.075	0.1	0.484
VaR	0.99	1	0.15	0.05	0.804
VaR	0.99	1	0.2	0.01	0
VaR	0.995	0	0	0.2	0.064
VaR	0.995	0	0.075	0.1	0.144
VaR	0.995	0	0.15	0.05	0.124
VaR	0.995	0	0.2	0.01	0.035
VaR	0.995	0.25	0	0.2	0.064
VaR	0.995	0.25	0.075	0.1	0.243
VaR	0.995	0.25	0.15	0.05	0.261
VaR	0.995	0.25	0.2	0.01	0.131
VaR	0.995	0.5	0	0.2	0.062
VaR	0.995	0.5	0.075	0.1	0.329
VaR	0.995	0.5	0.15	0.05	0.446
VaR	0.995	0.5	0.2	0.01	0.035
VaR	0.995	0.75	0	0.2	0.066
VaR	0.995	0.75	0.075	0.1	0.427
VaR	0.995	0.75	0.15	0.05	0.685
VaR	0.995	0.75	0.2	0.01	0.384
VaR	0.995	0.9	0	0.2	0.064
VaR	0.995	0.9	0.075	0.1	0.468
VaR	0.995	0.9	0.15	0.05	0.73
VaR	0.995	0.9	0.2	0.01	0.02
VaR	0.995	1	0	0.2	0.067
VaR	0.995	1	0.075	0.1	0.485
VaR	0.995	1	0.15	0.05	0.806
VaR	0.995	1	0.2	0.01	0
VaR	0.999	0	0	0.2	0.065
VaR	0.999	0	0.075	0.1	0.147

RM	Quantile	ρ^G	PD^B	PD^G	DR
VaR	0.999	0	0.15	0.05	0.126
VaR	0.999	0	0.2	0.01	0.039
VaR	0.999	0.25	0	0.2	0.066
VaR	0.999	0.25	0.075	0.1	0.253
VaR	0.999	0.25	0.15	0.05	0.277
VaR	0.999	0.25	0.2	0.01	0.202
VaR	0.999	0.5	0	0.2	0.062
VaR	0.999	0.5	0.075	0.1	0.34
VaR	0.999	0.5	0.15	0.05	0.45
VaR	0.999	0.5	0.2	0.01	0.039
VaR	0.999	0.75	0	0.2	0.066
VaR	0.999	0.75	0.075	0.1	0.428
VaR	0.999	0.75	0.15	0.05	0.689
VaR	0.999	0.75	0.2	0.01	0.669
VaR	0.999	0.9	0	0.2	0.064
VaR	0.999	0.9	0.075	0.1	0.471
VaR	0.999	0.9	0.15	0.05	0.744
VaR	0.999	0.9	0.2	0.01	0.02
VaR	0.999	1	0	0.2	0.067
VaR	0.999	1	0.075	0.1	0.485
VaR	0.999	1	0.15	0.05	0.809
VaR	0.999	1	0.2	0.01	0
ES	0.95	0	0	0.2	0.062
ES	0.95	0	0.075	0.1	0.141
ES	0.95	0	0.15	0.05	0.115
ES	0.95	0	0.2	0.01	0.022
ES	0.95	0.25	0	0.2	0.061
ES	0.95	0.25	0.075	0.1	0.228
ES	0.95	0.25	0.15	0.05	0.171
ES	0.95	0.25	0.2	0.01	0.07
ES	0.95	0.5	0	0.2	0.06
ES	0.95	0.5	0.075	0.1	0.316
ES	0.95	0.5	0.15	0.05	0.424
ES	0.95	0.5	0.2	0.01	0.024
ES	0.95	0.75	0	0.2	0.064
ES	0.95	0.75	0.075	0.1	0.41
ES	0.95	0.75	0.15	0.05	0.65
ES	0.95	0.75	0.2	0.01	0.038
ES	0.95	0.9	0	0.2	0.062
ES	0.95	0.9	0.075	0.1	0.464
ES	0.95	0.9	0.15	0.05	0.452
ES	0.95	0.9	0.2	0.01	0.012
ES	0.95	1	0	0.2	0.065
ES	0.95	1	0.075	0.1	0.474
ES	0.95	1	0.15	0.05	0.504
ES	0.95	1	0.2	0.01	0
ES	0.975	0	0	0.2	0.063
ES	0.975	0	0.075	0.1	0.143
ES	0.975	0	0.15	0.05	0.119
ES	0.975	0	0.2	0.01	0.032

RM	Quantile	ρ^G	PD^B	PD^G	DR
ES	0.975	0.25	0	0.2	0.063
ES	0.975	0.25	0.075	0.1	0.237
ES	0.975	0.25	0.15	0.05	0.213
ES	0.975	0.25	0.2	0.01	0.07
ES	0.975	0.5	0	0.2	0.061
ES	0.975	0.5	0.075	0.1	0.323
ES	0.975	0.5	0.15	0.05	0.436
ES	0.975	0.5	0.2	0.01	0.024
ES	0.975	0.75	0	0.2	0.066
ES	0.975	0.75	0.075	0.1	0.418
ES	0.975	0.75	0.15	0.05	0.674
ES	0.975	0.75	0.2	0.01	0.26
ES	0.975	0.9	0	0.2	0.064
ES	0.975	0.9	0.075	0.1	0.467
ES	0.975	0.9	0.15	0.05	0.71
ES	0.975	0.9	0.2	0.01	0.012
ES	0.975	1	0	0.2	0.067
ES	0.975	1	0.075	0.1	0.482
ES	0.975	1	0.15	0.05	0.803
ES	0.975	1	0.2	0.01	0
ES	0.99	0	0	0.2	0.065
ES	0.99	0	0.075	0.1	0.148
ES	0.99	0	0.15	0.05	0.127
ES	0.99	0	0.2	0.01	0.04
ES	0.99	0.25	0	0.2	0.066
ES	0.99	0.25	0.075	0.1	0.255
ES	0.99	0.25	0.15	0.05	0.28
ES	0.99	0.25	0.2	0.01	0.22
ES	0.99	0.5	0	0.2	0.062
ES	0.99	0.5	0.075	0.1	0.342
ES	0.99	0.5	0.15	0.05	0.45
ES	0.99	0.5	0.2	0.01	0.04
ES	0.99	0.75	0	0.2	0.066
ES	0.99	0.75	0.075	0.1	0.428
ES	0.99	0.75	0.15	0.05	0.69
ES	0.99	0.75	0.2	0.01	0.74
ES	0.99	0.9	0	0.2	0.064
ES	0.99	0.9	0.075	0.1	0.472
ES	0.99	0.9	0.15	0.05	0.748
ES	0.99	0.9	0.2	0.01	0.02
ES	0.99	1	0	0.2	0.068
ES	0.99	1	0.075	0.1	0.485
ES	0.99	1	0.15	0.05	0.809
ES	0.99	1	0.2	0.01	0
ES	0.995	0	0	0.2	0.065
ES	0.995	0	0.075	0.1	0.148
ES	0.995	0	0.15	0.05	0.127
ES	0.995	0	0.2	0.01	0.04
ES	0.995	0.25	0	0.2	0.066
ES	0.995	0.25	0.075	0.1	0.255

RM	Quantile	ρ^G	PD^B	PD^G	DR
ES	0.995	0.25	0.15	0.05	0.28
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ES	0.995	0.75	0	0.2	0.066
ES	0.995	0.75	0.075	0.1	0.428
ES	0.995	0.75	0.15	0.05	0.69
ES	0.995	0.75	0.2	0.01	0.74
ES	0.995	0.9	0	0.2	0.064
ES	0.995	0.9	0.075	0.1	0.472
ES	0.995	0.9	0.15	0.05	0.748
ES	0.995	0.9	0.2	0.01	0.02
ES	0.995	1	0	0.2	0.068
ES	0.995	1	0.075	0.1	0.485
ES	0.995	1	0.15	0.05	0.809
ES	0.995	1	0.2	0.01	0
ES	0.999	0	0	0.2	0.065
ES	0.999	0	0.075	0.1	0.148
ES	0.999	0	0.15	0.05	0.127
ES	0.999	0	0.2	0.01	0.04
ES	0.999	0.25	0	0.2	0.066
ES	0.999	0.25	0.075	0.1	0.255
ES	0.999	0.25	0.15	0.05	0.28
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ES	0.999	0.9	0.2	0.01	0.02
ES	0.999	1	0	0.2	0.068
ES	0.999	1	0.075	0.1	0.485
ES	0.999	1	0.15	0.05	0.809
ES	0.999	1	0.2	0.01	0

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