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# Do We Need Taylor-type Rules in DSGE?

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## ABSTRACT

The small-scale open economy dynamic stochastic general equilibrium (DSGE) models are estimated with a second-order approximation. The models differ in monetary policy rules. Optimal policy under commitment is best according to marginal likelihood. The conventional Taylor-type rule performs better in short-term forecasting but loses to other policies in long-term forecasting. Monetary policy rules heavily influence the dynamic and estimated parameters of models. They may produce a "price puzzle" and easily lead to the absence of inflation anchoring. The most interesting results relate to the performance of different rules in economies estimated with other rules. Very hawkish policies in a usual economy lead to a non-unique solution. An explosive trajectory is produced by the usual policy in an economy with a fiscal authority that does not care about debts/assets. Only the optimal policy under commitment can work in each of them. However, it may lead to a worse loss function than that produced by simple rules.

**Keywords**: DSGE; monetary policy; estimated optimal policy under commitment

**JEL-classification**: E31, E32, E37, E52.

## 1. INTRODUCTION

The aim of this paper is to investigate the consequences of changing the monetary policy rule. Taylor-type rules are the default in DSGE literature [Portier et al. (2023)]. However, there are many discussions about anchoring inflation expectations [Guler (2021); Carvalho et al. (2023)]. This can be formulated as an alternative rule of monetary policy, which can be interpreted as "inflation expectation targeting."

Some authors suggest alternatives to the Taylor rule, but these rules are still Taylor-type rules [Covas and Zhang (2010); Drumond et al. (2022)]. They change variables in the Taylor rule: price level instead of inflation [Covas and Zhang (2010)], and capacity utilization instead of the output gap [Drumond et al. (2022)]. They may also use a nonlinear form of Taylor-type rules. However, they keep the Taylor-type form of the rule, which is the dependence of the interest rate on inflation and some other variables. After that, they investigate the properties of the rule. Some authors use random walks instead of fixed coefficients in the Taylor rule [Gurkaynak et al. (2022)]. This is used to demonstrate the consequences of weak monetary policy (with a low response to inflation).

The more conventional use of the Taylor rule includes a few important details. The first is the choice of inflation. It is not about the choice of the inflation measure despite the existence of such papers [Junicke (2019)]. It is the choice between current and expected inflation [Portier et al. (2023)]. This choice has a powerful effect on model dynamics and the estimation of other parameters. The next important detail is related to the output gap. The more common approach is the deviation of output from the potential level based on TFP (total factor productivity). It is often suggested that TFP is stationary [Junicke (2019)]. Some authors use stationary TFP in model equations and suggest the existence of a trend in the equation for observed variables [Smets and Wouters (2007)]. Sometimes, it is based on a unit root exogenous process [Diebold et al. (2017); Ivashchenko (2022)]. The alternative is the deviation from flexible economy output [Smets and Wouters (2007)]. Sometimes, it is output growth instead of the gap [Diebold et al. (2017)].

Inflation anchoring is a very important topic that is often investigated without DSGE models. Some papers use regressions to check the influence (anchoring is interpreted as the absence of influence) of current variables on long-term observed expectations [Nautz et al. (2017)]. Anchoring is a continuous variable (deviation of inflation expectations from target) according to other papers [Guler (2021)]. They estimate the influence of credibility on this variable.

There are papers that investigate anchoring with DSGE or half-structured models with non-rational expectations. It may be constant expectations (with time-changing shares of different constants) within half-structured models [Lustenhouwer (2021)]. This leads to multiple steady-states due to the rule of expectations shares changing. Another non-rational form is the usage of the learning approach [Carvalho et al. (2023)]. Firms could learn the inflation target (rational steady-state) with a statistical test (if the target is constant or time-variant). This allows receiving a time-variant answer about the anchoring question.

The aim of this paper requires starting from a simple DSGE model. The next step is to modify it according to the new MP-rule. Russian historical data should be analyzed with each modification (this requires estimation of the models). Changing our opinion about the MP-rule would affect the estimation of other parameters. Some important properties of the model could be affected by this change. An additional layer is the influence of the rule on the central bank's "utility function" and the utility of households. The next layer is how different policies look (how close simple econometric estimation would be).

The Russian economy experiences extraordinarily large shocks. The importance of nonlinear effects becomes larger when state variables are far from steady-state and/or shock magnitude is large. It is obvious that such topics require a nonlinear DSGE model. It should be estimated with a nonlinear approximation. However, the model should be relatively simple for more tractable results.

The rest of the paper is organized as follows. The next section describes the DSGE model. The data and results are described after that. This includes estimation results, forecasting quality, anchoring tests, IRFs, and others. The last section is the conclusion.

## 2. MODEL

The small-scale DSGE model of an open economy includes four types of agents: households, firms, government, and the rest of the world.

### *Households*

Households maximize the expected utility function (1) with a budget restriction (2).

$$U_t = E\left(\sum_{s=0}^{\infty}\beta^s\left(\frac{Z_{C,t+s}\left(\left(C_{t+s}/Z_{trY,t+s}\right)/\left(C_{h,t+s-1}/Z_{trY,t+s-1}\right)^h\right)^{1-\omega_c}}{\left(1-\omega_c\right)}+\frac{\mu_M\left(M_{t+s}/Z_{trY,t+s}\right)^{1-\omega_M}}{\left(1-\omega_M\right)}-\frac{\mu_L\left(L_{t+s}\right)^{1+\omega_l}}{\left(1+\omega_l\right)}\right)\right)=$$

$$=\left(\left(\frac{Z_{C,t}\left(\left(C_t/Z_{trY,t}\right)/\left(C_{h,t-1}/Z_{trY,t-1}\right)^h\right)^{1-\omega_c}}{\left(1-\omega_c\right)}+\frac{\mu_M\left(M_t/(P_tZ_{trY,t})\right)^{1-\omega_M}}{\left(1-\omega_M\right)}-\frac{\mu_L\left(L_t\right)^{1+\omega_l}}{\left(1+\omega_l\right)}\right)\right)+E_t\beta U_{t+1}\rightarrow\max_{C,L,K} \quad (1)$$

$$P_tC_t+M_t+B_{H,t}/R_t+FX_tB_{WH,t}/R_{W,t}=(1-\tau)W_tL_t+M_{t-1}+B_{H,t-1}+FX_tB_{WH,t-1}+T_t \quad (2)$$

$C_t$ is consumption, $C_{h,t}$ is habit (equal to consumption but not controlled by individual households), $L_t$ is labor, $M_t$ is money, $W_t$ is wage, $R_t$ is the interest rate in domestic currency, $B_{H,t}$ is bond/deposit savings in domestic currency, $B_{WH,t}$ is bond/deposit savings in foreign currency, $R_{W,t}$ is the interest rate in foreign currency, $FX_t$ is the exchange rate (units of domestic currency per unit of foreign), $T_t$ are transfers from the government, $Z_{trY,t}$ is exogenous process of TFP growth, $Z_{C,t}$ is exogenous demand shock process.

The formula (1) uses an alternative form of habit. Conventional habit formation with subtraction (instead of division) can lead to complex values of the utility function when variables follow a normal distribution. Dividing, which makes the first term equal to $exp(z_{c,t}+(1-\omega_c)(c_t-c_{h,t-1}h))/(1-\omega_c)$, avoids this issue and results in a similar dependence on the lag of consumption. Situation when current consumption is below habit related level could happen with near-zero probability (taking into account approximation errors). Suggested approach produce similar effects to conventional one: dependence on previous period consumption and higher nonlinear effects (but this effect may be much smaller).

An additional detail is the existence of a stochastic trend with drift in all real variables, originating from an exogenous unit root TFP process. All components of the utility function should be cointegrated. Therefore, it is impossible to have $C_t$ without dividing by $Z_{trY,t}$. Omitting the stochastic trend from the model is bad practice as it would eliminate the microeconomic foundation, which is one of the main advantages of DSGE models.

***Firms***

Firms operate under monopolistic competition and solve problems (3)-(6). They maximize the expected discounted flow of dividends with price rigidity effects in the Rotemberg form (3). The constraints are: budget (4), production function (5), and demand (6) derived from CES (constant elasticity of substitution) aggregation.

$$E\left(\sum_{t=0}^{\infty}\left(\underset{k=0}{\overset{t-1}{\Pi}}R_k\right)^{-1}\left(D_{f,t}-e^{\varphi_p}P_{F,t}Y_{D,t}\left(\frac{P_{f,t}}{P_{f,t-1}}-e^{\bar{p}}\right)^2\right)\right)\rightarrow\max_{D,L,Y} \quad (3)$$

$$D_{f,t}+W_tL_{f,t}=P_{f,t}Y_{f,t}+T_{W,t} \quad (4)$$

$$Y_{f,t}=Z_{trY,t}Z_{Y,t}\left(L_{f,t}\right)^{1-\alpha_k} \quad (5)$$

$$Y_{f,t}=\left(\frac{P_{f,t}}{P_{F,t}}\right)^{-z_{\theta,t}}Y_{D,t} \quad (6)$$

$D_{f,t}$ – are the dividends of firm *f*, $L_{f,t}$ is the amount of labor used by firm *f*, $P_{f,t}$ is the price of goods by firm *f*, $Y_{f,t}$ is the output of firm *f*, $T_{w,t}$ – is the transfer with the foreign part of the firm, $P_{F,t}$ is the price level of domestic firms, $Y_{D,t}$ is the demand for domestic firms' output, $Z_{Y,t}$ is an exogenous stationary TFP process, $Z_{\theta,t}$ is an exogenous process of demand elasticity.

There are two important details related to the problem of firms. The first is the discounting factor. The conventional method uses a stochastic discount factor based on the household's Lagrange multiplier of the budget restriction. This is equivalent in the case of linear approximation and models without financial rigidities. However, it creates problems for generalization of the model: whose Lagrange multiplier should be used if heterogeneous agents own the firm? Moreover, firms may be owned by foreign agents or the government. Using the interest rate eliminates these problems.

The second detail is Rotemberg rigidity. Many authors use real costs of price change, which produces an additional term in the GDP formula that has no analog in the national account system. Using moral costs solves this problem. These two approaches are equivalent in the case of a first-order approximation.

***Government***

The government operates under the budget restriction (7). The monetary policy rule is of the Taylor type (9a), and the rule for transfers is (8). Two variables are crucial for these rules. The first is future inflation expectation $p_{EXP,t}$, described by rule (10). This allows control over which inflation period is more influential for the Taylor rule (next period or future one). The second variable is households' domestic currency assets $A_{H,t}$, described by (11). This represents the government's liabilities (minus assets) affecting fiscal policy. Such a variable reduces the number of state variables.

$$B_{t-1}+T_t=D_t+M_t-M_{t-1}+B_t/R_t+\tau(W_tL_t) \quad (7)$$

$$T_t/(P_tZ_{trY,t})=\gamma_{tr}T_{t-1}/(P_{t-1}Z_{trY,t-1})+(1-\gamma_{tr})(\gamma_{try}(y_{D,t}-\overline{y_D})+\gamma_{trA}(a_{H,t}-\overline{a_H})+z_{tr,t}) \quad (8)$$

$$r_{H,t} = \gamma_r r_{H,t-1} + (1-\gamma_r)(\gamma_{rp} E_t(p_{EXP,t+1} - \overline{p}) + \gamma_{ry}(y_{D,t} - \overline{y_D}) + z_{r,t}) \tag{9a}$$

$$p_{EXP,t} = \gamma_{\exp} p_t + (1-\gamma_{\exp}) E_t p_{EXP,t+1} \tag{10}$$

$$a_{H,t} = \frac{A_{H,t}}{P_t Z_{trY,t}} = \frac{B_{H,t} + M_{H,t}}{P_t Z_{trY,t}} = b_{H,t} + m_{H,t} \tag{11}$$

Small letters denote stationary variables. The transformation depends on the variable: real variables (e.g., $y_{D,t}$) are the logs of the ratio of the initial variable to the common trend ($Z_{trY,t}$); nominal variables are divided by the price level and common trend as in (11); interest rates ($R_t$, $R_{W,t}$) are stationary positive variables transformed by logs; the real exchange rate ($fx_t$) is the log of the ratio the initial variable corrected by domestic and foreign price levels, and so on.

An important detail is that the government receives all dividends from firms. This is specific to Russia, where the government owns a large share of firms. Rule (8) reflects three ideas of fiscal policy. The first is smoothing, meaning slow changes in government transfers (controlled by $\gamma_{tr}$). The second is cyclical dependence: fiscal policy can be pro- or counter-cyclical depending on the sign of $\gamma_{try}$. The third idea is budget balancing: a higher level of government debts (which are households' assets, counting money as part of government debts) should lead to lower government transfers. The negative value of $\gamma_{trA}$ reflects this mechanism of government asset control. However, there is a question of how close to zero it can be without explosiveness. The Blanchard-Kahn condition is the main restriction for non-explosive trajectories of all variables, including government assets.

There are a few alternative policy rules. The default is the Taylor rule (9a). An alternative rule is “inflation expectation targeting” (9b). This rule results from an absolutely hawkish monetary policy, suggesting that any deviation of inflation expectations from the target is inappropriate. At the same time, a monetary policy shock is desired for the model, making it the unique source of deviation of long-term inflation expectations from the target. This policy ensures perfect inflation expectation anchoring.

$$E_t(p_{EXP,t+1} - \overline{p}) = z_{r,t} - \overline{z_r} \tag{9b}$$

The third alternative is the optimal policy under commitment, which minimizes the central bank's cost function (9c) with the restriction of all other economic rules. It targets three goals: inflation expectation, financial stability (smoothing interest rates), and the output gap. The monetary policy shock is interpreted as fluctuations in time preferences. An additional suggestion for the optimal policy under commitment is an unchanged steady-state. A change in the steady-state of $z_{R,t}$ would influence the nominal interest rate and inflation steady-state, affecting the steady-state of other variables. This influence is eliminated, so the steady-state for each monetary policy remains the same (conditional on parameter values). It is important that the government's discount factor differs from the households' one. If it is very large, the policy under commitment would be equivalent to discretionary policy.

$$E_t \sum_{s=0}^{\infty} e^{\gamma_\beta + z_{r,t+s} - \overline{z_r}} \left( (p_{EXP,t+s} - \overline{p})^2 + \gamma_r (r_{H,t+s} - r_{H,t+s-1})^2 + \gamma_y (y_{D,t+s} - \overline{y_D})^2 \right) \to \min \tag{9c}$$

It is important to highlight that the optimal policy under commitment has a different aim than household welfare. The policy aim should differ for a much better fit of the data [Chen et al. (2017)]. It is common to use the same three components (or part of them) of the penalty function in both open and closed economies [Chen et al. (2017), Liu et al. (2020), Clarida (2014)]. Moreover, the optimal policy (according to such a penalty) within a simple DSGE model is equivalent to a Taylor-type rule with inflation expectation and productivity [Clarida (2014)]. Such transformations of the optimal policy to short relationships between endogenous variables are usually used for very simple models due to complexity. The resulting relationship may be too complicated for interpreting results. However, deriving the FOC (first order conditions) for optimal policy under commitment is much simpler.

Combining all policy versions into a single general rule would be very beneficial but complicated. However, the multimodal distribution of the posterior density makes implementing such a general rule very difficult. Different rules require different other parameters for a unique non-explosive rational expectation solution (as will be described later). This makes it highly likely that the posterior mean breaks Blanchard-Kahn conditions, and the posterior mode is non-informative about other modes. This complexity discourages such an exercise.

***Rest of the world***

The rest of the world operates under a budget restriction typically called the balance of payments (12).

$$B_{WH,t-1} FX_t + EX_t P_t + T_{W,t} = FX_t B_{WH,t} / R_{W,t} + IM_t P_{im,t} \tag{12}$$

The rest of the world is described by exogenous rules. Its inflation is described by (13). The interest rate (for households) is described by (14), which includes some dependence on the foreign bond position of households. If households try to increase their foreign debts, the interest rate will increase. Import prices are described by (15). If the coefficient equals one, it means that exogenous prices for imports are in foreign currency. However, if the coefficient differs, it indicates some mechanical restriction on the exchange rate pass-through to import prices. Equation (16) describes exports, which depend on the real exchange rate. It is assumed that export goods and domestic consumption are the same (thus, their prices are the same too). This is a conventional simplification (especially for small-scale models) that prevents the creation of an additional group of firms [Junicke (2019)]. Medium-scale models often have specific parameters of CES aggregation for export goods, but they do not use different currencies for price rigidity [Adolfson et al. (2011); Ivashchenko (2022)]. Equation (17) comes from CES aggregation of import and domestic goods into the final product.

$$p_{W,t} = z_{pw,t} \tag{13}$$

$$r_{W,t} = \gamma_{rw,bw}\left(b_{WH,t} - \overline{b_{WH}}\right) + z_{rw,t} \tag{14}$$

$$p_{IM,t} = \gamma_{pim,fx}\left(fx_t - \overline{fx}\right) + z_{pim,t} \tag{15}$$

$$ex_t = \gamma_{ex,fx}\left(fx_t - \overline{fx}\right) + z_{ex,t} \tag{16}$$

$$e^{im_t} = \left(1 - w_c\right)\left(e^{c_t} + e^{ex_t}\right)e^{-\theta_C\left(p_{im,t} - p_{c,t}\right)} \tag{17}$$

***Balance***

The model includes balance equations. The first (18) describes conventional GDP. The next (19) describes demand for intermediate goods, which comes from the same CES aggregation as imports. The price aggregation (20) describes the relationship between domestic firms' prices ($p_{F,t}$), import prices ($p_{im,t}$), and the aggregate price level ($P_t$). This also comes from the CES aggregation. The last equation (21) describes the denomination of domestic currency units. The price of the final goods basket in terms of the domestic goods basket is fixed.

$$e^{y_t} = e^{c_t} + e^{ex_t} - e^{im_t} \tag{18}$$

$$e^{y_{D,t}} = \left(w_c\right)\left(e^{c_t} + e^{ex_t}\right)e^{-\theta_C\left(p_{F,t} - p_{c,t}\right)} \tag{19}$$

$$1 = \left(w_c\right)e^{(1-\theta_C)\left(p_{F,t} - p_{c,t}\right)} + \left(1 - w_c\right)e^{(1-\theta_C)\left(p_{IM,t} - p_{c,t}\right)} \tag{20}$$

$$p_{c,t} = \overline{p_c} \tag{21}$$

The model includes only one source of domestic demand for simplicity. Introducing investments and government consumption (44% and 35% of consumption in 2019) makes the model much more complicated (additional state variable of capital, investment rigidity, increasing importance of financial rigidity, different deflators for different GDP components, and so on). Introducing only government consumption (without investments) creates additional problems of dividing GDP by two components of domestic demand. Thus, having a single source of domestic demand is a significant simplification of the model that allows a deep focus on monetary policy.

Exogenous Process Rules:

$$\log(Z_{*,t}) = z_{*,t} = \eta_{0,*} + \varepsilon_{*,t} \quad (22)$$

$$\log(Z_{trY,t}/Z_{trY,t-1}) = z_{trY,t} = \eta_{0,trY} + \varepsilon_{trY,t} \quad (23)$$

## 3. DATA AND RESULTS

***Data***

The small model is estimated with Russian quarterly data from 2011Q1 to 2022Q4. The variables used include GDP growth rate, personal consumption deflator, interest rate RUONIA, BIS nominal and real exchange rate indexes growth. The first four quarters are used as a pre-sample. The model is estimated with a second-order pruned approximation, which prevents an explosive trajectory of the nonlinear approximation and may be interpreted as a perturbation with respect to all shocks over the history [Lombardo and Uhlig (2018)]. The Quadratic Kalman filter is used [Ivashchenko (2014)]. It is possible to increase the number of observed variables, but this would require a more complicated model to capture important features, as seen in [Adolfson et al. (2011)]. This is much more computationally costly in the case of a nonlinear model. Therefore, the number of observed variables is slightly larger than in other models that estimate policy targets [Chen et al. (2017); Liu et al. (2020)]. It is conventional to have 5-7 observed variables for small-scale open economy models [Cermeno et al. (2012), Junicke (2019), Kreptsev and Seleznev (2018)].

A more complicated question that may affect the robustness of estimation is the dataset period. The conventional DSGE framework suggests the absence of structural breaks (even in the future). The small probability of parameter changes (in the future) would affect the solution of the model with rational expectations (an example is volatility change in regime-switching DSGE [Benchimol and Ivashchenko (2021)]). It is possible to create some techniques that take this problem into account, such as [Ivashchenko (2022)]. However, this would make the estimation technique much more complicated and computationally costly in the nonlinear case. Moreover, many papers estimate DSGE models with periods that include potential structural breaks and outliers [Kreptsev and Seleznev (2018), Junicke (2019), Chen et al. (2017)]. Nevertheless, robustness check (for estimation period) is done.

Priors are presented in Table 1. The parameters for the monetary policy rules differ depending on the model, as marked in the table. The number of parameters differs only for the model with rule (9b). However, the meaning of the parameters for (9c) is very different from (9a). The importance of financial stability (smoothing interest rates) and the output gap are related to $\gamma_r$ and $\gamma_y$ in both versions, but these are measured differently. The importance of inflation $\gamma_p$ in (9a) is substituted by time-preferences $\gamma_\beta$ in (9c). All computations are realized in modified Dynare [Adjemian et al. (2011)]. Version “a” uses rule (9a). Version “aa” uses the same rule (9a) but has quite different priors for its parameters. Version “b” uses rule (9b), which means a smaller number of parameters. Version “c” uses rule (9c). Very hawkish monetary policy leads to breaking the Blanchard-Kahn condition: the non-explosive solution becomes non-unique. This motivates changing the prior for other variables.

Table 1. Parameters prior

| Parameter | Lower bound | Upper bound | Density | Prior mean | Prior std |
|---|---|---|---|---|---|
| stderr $\varepsilon_C$ | 0.0003 | 10 | inv_gamma_pdf | 0.01 | 3 |
| stderr $\varepsilon_{ex}$ | 0.0003 | 10 | inv_gamma_pdf | 0.01 | 3 |
| stderr $\varepsilon_{pim}$ | 0.0003 | 10 | inv_gamma_pdf | 0.01 | 3 |
| stderr $\varepsilon_{pw}$ | 0.0003 | 10 | inv_gamma_pdf | 0.01 | 3 |
| stderr $\varepsilon_R$ | 0.0003 | 10 | inv_gamma_pdf | 0.01 | 3 |
| stderr $\varepsilon_{rw}$ | 0.0003 | 10 | inv_gamma_pdf | 0.01 | 3 |
| stderr $\varepsilon_{\theta,F}$ | 0.0003 | 10 | inv_gamma_pdf | 0.01 | 3 |
| stderr $\varepsilon_{tr}$ | 0.0003 | 10 | inv_gamma_pdf | 0.01 | 3 |
| stderr $\varepsilon_{tr,W}$ | 0.0003 | 10 | inv_gamma_pdf | 0.01 | 3 |
| stderr $\varepsilon_{trY}$ | 0.0003 | 10 | inv_gamma_pdf | 0.01 | 3 |
| stderr $\varepsilon_{YF}$ | 0.0003 | 10 | inv_gamma_pdf | 0.01 | 3 |
| $\alpha_K$ | 0.3 | 0.8 | normal_pdf | 0.6 | 0.05 |
| ln(β) | -0.01 | -0.00001 | normal_pdf | 0.005 | 0.005e-0 |
| $\varphi_{PF}$ | -5 | 5 | normal_pdf | 0 | 10 |
| $\gamma_{exp}$ | 0.001 | 0.999 | normal_pdf | 0.5 | 0.25 |
| $\gamma_r$ (version a) | 0.6 | 0.999 | normal_pdf | 0.8 | 0.15 |
| $\gamma_{rp}$ (version a) | 1 | 5 | normal_pdf | 1.5 | 0.5 |
| $\gamma_{ry}$ (version a) | -1 | 1 | normal_pdf | 0 | 0.15 |
| $\gamma_r$ (version aa) | 0.6 | 0.999 | normal_pdf | 0.8 | 0.15 |
| $\gamma_{rp}$ (version aa) | 1000 | 5000 | normal_pdf | 1500 | 0.5 |
| $\gamma_{ry}$ (version aa) | -1 | 1 | normal_pdf | 0 | 0.15 |
| $\gamma_\beta$ (version c) | -0.01 | -0.00001 | normal_pdf | -0.005 | 0.005e-0 |
| $\gamma_r$ (version c) | 0 | 2 | normal_pdf | 0 | 0.15 |
| $\gamma_y$ (version c) | 0 | 2 | normal_pdf | 0 | 0.15 |
| $\gamma_{tr}$ | 0.6 | 0.999 | normal_pdf | 0.8 | 0.15 |
| $\gamma_{trA}$ (version a,c) | -1 | 0 | normal_pdf | -0.1 | 0.15 |
| $\gamma_{trA}$ (version aa,b) | -1 | 0 | normal_pdf | -0.1e-9 | 0.15 |
| $\gamma_{try}$ | -1 | 1 | normal_pdf | 0 | 0.15 |
| $\gamma_{ex,fx}$ | 0 | 5 | normal_pdf | 0 | 1.5 |
| $\gamma_{pim,fx}$ | 0 | 2 | normal_pdf | 1 | 0.5 |
| $\gamma_{rw,bw}$ | -5.001 | 0 | normal_pdf | 0 | 0.5 |
| H | 0 | 0.999 | normal_pdf | 0.7 | 0.15 |
| $\mu_L$ | -5 | 5 | normal_pdf | 0 | 10 |
| $\mu_M$ | -5 | 5 | normal_pdf | 0 | 10 |
| $\eta_{0,C}$ | -5 | 5 | normal_pdf | 0 | 2 |
| $\eta_{0,ex}$ | -5 | 5 | normal_pdf | 0 | 2 |
| $\eta_{0,pim}$ | -5 | 5 | normal_pdf | 0 | 2 |
| $\eta_{0,pw}$ | 0 | 0.02 | normal_pdf | 0.005 | 0.005 |

| | | | | | |
|---|---|---|---|---|---|
| $\eta_{0,R}$ | 0.001 | 0.03 | normal_pdf | 0.015 | 0.005 |
| $\eta_{0,\theta,F}$ | 4 | 12 | normal_pdf | 8 | 2 |
| $\eta_{0,tr}$ | -5 | 5 | normal_pdf | 0 | 10 |
| $\eta_{0,tr,W}$ | -5 | 5 | normal_pdf | 0 | 2 |
| $\eta_{0,trY}$ | -0.01 | 0.02 | normal_pdf | 0.01 | 0.01 |
| $\eta_{0,YF}$ | -10 | 10 | normal_pdf | 0 | 10 |
| $\omega_C$ | 1 | 5 | normal_pdf | 1.5 | 1.5e-1 |
| $\omega_L$ | 1 | 5 | normal_pdf | 1.5 | 1.5e-1 |
| $\omega_M$ | 1 | 5 | normal_pdf | 1.5 | 1.5e-1 |
| Tax | 0 | 0.8 | normal_pdf | 0.4 | 0.5e-1 |
| $\theta_C$ | 4 | 12 | normal_pdf | 8 | 2 |
| $w_C$ | 0.4 | 0.9 | normal_pdf | 0.7 | 0.1 |
| b_WH_SS | -5 | 5 | normal_pdf | 0 | 2 |
| c_H_SS | -5 | 5 | normal_pdf | 0 | 1 |

Priors for parameters are in line with the literature. All shocks have the same priors with very standard deviations. Some parameters are hard to compare across different models. The priors for such parameters are wider. For example, steady-state for consumption (c_H_SS), foreign bonds (b_WH_SS)), government transfers ($\eta_{0,tr}$), foreign transfers ($\eta_{0,tr,W}$), export ($\eta_{0,ex}$), import prices ($\eta_{0,pim}$), and others have very large standard deviations due to the units of the corresponding steady-state. Rigidity ($\phi_{PF}$) and the relative importance of money and labor ($\mu_M$ and $\mu_L$) also have large variances. Some parameters have narrow priors. For example, labor-related elasticity ($\omega_L$) may be compared with prior means and standard deviations from other papers: 1±0 [Adolfson et al. (2011)], 2±1 [Kreptsev and Seleznev (2018)], 2.5±0.25 [Chen et al. (2017)]. Taylor-rule related coefficients $\gamma_r$, $\gamma_{rp}$ and $\gamma_Y$ are 0.8±0.05, 1.7±0.1, 0.125±0.05 [Adolfson et al. (2011)], 0.7±0.05, 1.5±0.2, 0.0±0.00 [Kreptsev and Seleznev (2018)], 0.75±0.1, 1.5±0.25, 0.12±0.05 [Smets and Wouters (2007)]. Some parameters related to the steady-state growth rate ($\eta_{0,trY}$) and steady-state interest rate ($\eta_{0,R}$) have very narrow priors.

***Estimation results***

The posterior estimates are presented in Table 2. The estimated values of parameters significantly differ depending on the assumptions about monetary policy. The most notable changes are related to inflation expectations. The parameter $\gamma_{exp}$ defines the "duration" of inflation expectations targeted by the central bank. The conventional Taylor rule (version a) and the optimal policy (version c) yield lower values corresponding to a "duration" of about 1.5-1.6 quarters (formula (24)). However, hawkish (tight) monetary policy, as indicated by versions aa and b, leads to a "duration" of 0.57 and $10^{-3}$ quarters respectively. This means that hawkish monetary policy focuses on the immediate future and achieves results in each period. Conversely, fiscal policy appears indifferent to the future ($\gamma_{trA}$) is near zero for versions aa and b). Moreover, versions a and c imply countercyclical fiscal policy ($\gamma_{try}<0$), while versions aa and b imply pro-cyclical fiscal policy ($\gamma_{try}>0$). This is related to the Blanchard-Kahn (BK) condition. Experiments without checking the BK condition do not produce significant changes in the estimated values of parameters.

$$\text{"}duration\text{"} = \sum_{t=0}^{\infty} t\gamma_{\exp}(1-\gamma_{\exp})^{t} \tag{24}$$

It is noteworthy that the standard deviation of monetary policy shocks is almost the same across models (just two times smaller for version b), despite the very different implications of these shocks.

Table 2. Estimation results. Posterior mode and standard deviation

| Parameter | Version a | | Version aa | | Version b | | Version c | |
|---|---|---|---|---|---|---|---|---|
| | Mode | std | Mode | std | mode | std | Mode | Std |
| $stderr\ \varepsilon_C$ | 2.31E-02 | 4.64E-03 | 5.11E-02 | 9.65E-03 | 3.87E-02 | 6.30E-03 | 4.81E-03 | 2.13E-03 |
| $stderr\ \varepsilon_{ex}$ | 4.61E-03 | 1.87E-03 | 4.38E-03 | 2.31E-03 | 4.60E-03 | 2.72E-03 | 4.61E-03 | 1.88E-03 |
| $stderr\ \varepsilon_{pim}$ | 4.60E-03 | 1.86E-03 | 4.50E-03 | 2.51E-03 | 3.32E-03 | 1.21E-03 | 2.06E-02 | 2.57E-03 |
| $stderr\ \varepsilon_{pw}$ | 8.51E-03 | 8.87E-04 | 8.56E-03 | 1.27E-03 | 8.58E-03 | 1.28E-03 | 8.49E-03 | 8.96E-04 |
| $stderr\ \varepsilon_R$ | 4.20E-03 | 8.83E-04 | 4.60E-03 | 2.66E-03 | 2.12E-03 | 4.79E-04 | 4.55E-03 | 1.80E-03 |
| $stderr\ \varepsilon_{rw}$ | 1.21E-01 | 6.58E-03 | 1.01E-01 | 1.69E-02 | 1.14E-01 | 1.70E-02 | 1.14E-01 | 8.71E-03 |
| $stderr\ \varepsilon_{\theta,F}$ | 4.61E-03 | 1.87E-03 | 4.61E-03 | 2.66E-03 | 4.61E-03 | 2.66E-03 | 4.60E-03 | 1.87E-03 |
| $stderr\ \varepsilon_{tr}$ | 4.61E-03 | 1.87E-03 | 5.15E-03 | 3.98E-03 | 4.74E-03 | 2.93E-03 | 4.61E-03 | 1.87E-03 |
| $stderr\ \varepsilon_{tr,W}$ | 4.61E-03 | 1.87E-03 | 5.25E-03 | 4.29E-03 | 4.75E-03 | 2.95E-03 | 4.61E-03 | 1.87E-03 |
| $stderr\ \varepsilon$ | 6.00E-03 | 2.27E-03 | 5.64E-03 | 1.25E-03 | 6.98E-03 | 1.49E-03 | 1.41E-02 | 2.59E-03 |
| $stderr\ \varepsilon_{YF}$ | 4.82E-02 | 6.22E-03 | 1.88E-02 | 4.64E-03 | 1.62E-02 | 3.03E-03 | 1.86E-02 | 2.69E-03 |
| $\alpha_K$ | 6.41E-01 | 1.59E-03 | 3.94E-01 | 1.03E-01 | 5.30E-01 | 2.56E-02 | 5.58E-01 | 1.82E-03 |
| $ln(\beta)$ | -2.12E-03 | 9.38E-05 | -9.95E-03 | 2.76E-06 | -9.95E-03 | 3.81E-06 | -2.84E-03 | 2.71E-04 |
| $\varphi_{PF}$ | 4.71E+00 | 3.33E-02 | 3.14E+00 | 2.91E-01 | 3.09E+00 | 2.98E-02 | 2.18E+00 | 6.72E-03 |
| $\gamma_{exp}$ | 3.82E-01 | 2.79E-04 | 6.35E-01 | 3.12E-01 | 9.99E-01 | 7.40E-03 | 3.96E-01 | 1.22E-03 |
| $\gamma_r$ *(version a)* | 6.24E-01 | 3.63E-04 | | | | | | |
| $\gamma_{rp}$ *(version a)* | 1.31E+00 | 3.43E-03 | | | | | | |
| $\gamma_{ry}$ *(version a)* | 1.36E-02 | 2.06E-04 | | | | | | |
| $\gamma_r$ *(version aa)* | | | 7.00E-01 | 1.76E-01 | | | | |
| $\gamma_{rp}$ *(version aa)* | | | 1.50E+03 | 5.70E-01 | | | | |
| $\gamma_{ry}$ *(version aa)* | | | 3.20E-04 | 2.34E-01 | | | | |
| $\gamma_\beta$ *(version c)* | | | | | | | -1.06E-03 | 3.14E-04 |
| $\gamma_r$ *(version c)* | | | | | | | 3.51E-01 | 1.14E-03 |
| $\gamma_y$ *(version c)* | | | | | | | 1.87E-01 | 5.94E-04 |
| $\gamma_{tr}$ | 8.00E-01 | 1.50E-01 | 7.95E-01 | 2.22E-01 | 7.99E-01 | 2.37E-02 | 7.96E-01 | 2.51E-03 |
| $\gamma_{trA}$ *(version a,c)* | -1.00E-01 | 1.71E-01 | | | | | -7.46E-02 | 3.40E-04 |
| $\gamma_{trA}$ *(version aa,b)* | | | -1.43E-11 | 3.71E-08 | -3.44E-09 | 7.25E-08 | | |
| $\gamma_{try}$ | -2.60E-05 | 1.50E-01 | 1.43E-02 | 2.24E-01 | 2.54E-02 | 2.83E-02 | -2.09E-03 | 3.38E-04 |
| $\gamma_{ex,fx}$ | 1.77E-01 | 9.75E-03 | 2.22E-01 | 1.81E-01 | 2.17E+00 | 2.83E-02 | 1.36E-03 | 3.27E-04 |
| $\gamma_{pim,fx}$ | 1.48E+00 | 6.88E-03 | 2.13E-10 | 5.16E-07 | 1.96E-02 | 2.43E-02 | 2.40E-01 | 7.64E-04 |
| $\gamma_{rw,bw}$ | -2.92E-01 | 9.96E-03 | -1.21E-01 | 1.26E-01 | -1.38E-01 | 3.28E-02 | -9.77E-03 | 3.40E-04 |
| $H$ | 9.14E-01 | 1.15E-03 | 8.24E-01 | 1.73E-01 | 7.54E-01 | 2.83E-02 | 3.64E-01 | 1.09E-03 |

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| $\mu_L$ | 3.25E+00 | 1.74E-03 | -4.65E+00 | 7.53E-01 | 2.49E+00 | 2.83E-02 | -4.49E-01 | 1.36E-03 |
| $\mu_M$ | 1.32E-03 | 1.74E+00 | -5.00E+00 | 5.99E-01 | -5.00E+00 | 2.91E-10 | 1.40E-01 | 4.70E-04 |
| $\eta_{0,C}$ | 4.61E+00 | 1.95E-03 | 2.31E+00 | 5.84E-01 | 1.63E+00 | 2.83E-02 | -1.72E+00 | 5.35E-03 |
| $\eta_{0,ex}$ | -1.40E+00 | 7.43E-03 | 5.48E-01 | 2.87E-01 | -1.54E-01 | 2.83E-02 | 1.04E+00 | 3.43E-03 |
| $\eta_{0,pim}$ | -5.02E-01 | 4.18E-03 | -1.71E-01 | 8.16E-01 | -6.38E-01 | 2.40E-02 | -1.25E+00 | 3.86E-03 |
| $\eta_{0,pw}$ | 5.63E-03 | 1.25E-03 | 5.74E-03 | 1.78E-03 | 5.80E-03 | 1.79E-03 | 5.54E-03 | 1.24E-03 |
| $\eta_{0,R}$ | 1.03E-02 | 9.82E-04 | 1.95E-02 | 2.62E-03 | 2.03E-02 | 2.70E-03 | 2.09E-02 | 3.45E-04 |
| $\eta_{0,\theta,F}$ | 6.99E+00 | 2.11E-02 | 8.42E+00 | 6.47E-01 | 8.10E+00 | 2.83E-02 | 4.16E+00 | 1.23E-02 |
| $\eta_{0,tr}$ | 2.07E-03 | 1.96E+00 | 4.39E-01 | 4.33E-01 | 2.50E+00 | 2.77E-02 | -8.45E-02 | 3.36E-04 |
| $\eta_{0,tr,W}$ | -3.55E-01 | 1.78E-03 | -6.66E-01 | 5.54E-01 | -2.07E-01 | 2.77E-02 | 1.06E+00 | 3.45E-03 |
| $\eta_{0,trY}$ | 2.99E-03 | 8.33E-04 | 2.28E-03 | 1.23E-03 | 2.35E-03 | 1.50E-03 | 7.82E-04 | 3.23E-04 |
| $\eta_{0,YF}$ | 3.76E+00 | 1.10E-03 | 4.85E-01 | 1.54E-01 | 2.60E+00 | 2.83E-02 | 4.21E+00 | 9.91E-03 |
| $\omega_C$ | 1.72E+00 | 3.25E-03 | 1.50E+00 | 1.66E-01 | 1.55E+00 | 2.83E-02 | 1.73E+00 | 5.65E-03 |
| $\omega_L$ | 1.41E+00 | 3.78E-03 | 1.85E+00 | 2.20E-01 | 1.39E+00 | 2.83E-02 | 1.71E+00 | 5.89E-03 |
| $\omega_M$ | 1.50E+00 | 1.50E-01 | 1.48E+00 | 2.40E-01 | 1.48E+00 | 2.83E-02 | 1.49E+00 | 4.69E-03 |
| *Tax* | 3.84E-01 | 1.09E-03 | 4.59E-01 | 7.82E-02 | 3.88E-01 | 2.83E-02 | 4.04E-01 | 1.24E-03 |
| $\theta_C$ | 9.15E+00 | 1.84E-02 | 4.83E+00 | 9.30E-01 | 7.50E+00 | 2.83E-02 | 9.44E+00 | 2.96E-02 |
| $w_C$ | 5.08E-01 | 1.53E-03 | 4.24E-01 | 2.55E-01 | 7.00E-01 | 2.73E-02 | 6.38E-01 | 2.03E-03 |
| *b_WH_SS* | 2.89E-01 | 8.93E-03 | 7.00E-01 | 6.59E-01 | 5.36E-01 | 2.86E-02 | 1.10E+00 | 3.45E-03 |
| *c_H_SS* | 3.61E+00 | 1.53E-03 | 1.13E+00 | 2.48E-01 | 2.11E+00 | 2.47E-02 | 3.58E+00 | 8.66E-03 |

The changes also affect non-government parameters. The relative importance of labor and money for household utility ($\mu_L$ and $\mu_M$) varies significantly. Average transfers and transfers from abroad ($\eta_{0,tr}$ and $\eta_{0,tr,W}$) also show considerable differences. Sensitivity of import prices and export to the exchange rate ($\gamma_{pim,fx}$ and $\gamma_{ex,fx}$) varies greatly. Version aa suggests that import prices are almost insensitive to the exchange rate, while version b suggests low sensitivity. This parameter is closer to the most hawkish monetary policy. However, in the case of export, the smallest sensitivity is for version c, and the highest is for version b. Thus, there is no monotonic dependence of estimation results on the "hawkish" level of monetary policy.

These large differences result in variations in marginal likelihood (computed using Laplace approximation, meaning a normal approximation of the posterior is used for integration over the parameter space) and the value of the posterior. Version a gives a log-marginal likelihood (log-ML) of 417.873158 and a posterior mode of 627.7705. Version aa gives a log-ML of 414.4 and a posterior mode of 559.438651. Version b gives a log-ML of 339.475205 and a posterior mode of 540.7267. Version c gives a log-ML of 598.823292 and a posterior mode of 846.6288. Thus, version c with optimal policy provides the best fit, while version b is the worst. Despite the large differences, versions a and aa have a similar level of data description.

***Forecasting***

Marginal likelihood describes the overall fit of the model, incorporating the influence of parameter uncertainty. The posterior mode considers how far parameters are from their prior mean. To further evaluate the model's performance, we analyze forecasting performance for the best parameter values corresponding to the posterior mode. Table 3 presents the RMSE (Root-Mean-Squared-Errors) of the DSGE model relative to an AR model (in-sample) for different forecasting horizons. The best values are in bold.

Table 3. RMSE DSGE/RMSE AR for posterior mode (in-sample)

| Version | Forecasting horizon | 1 | 2 | 4 | 8 | 12 |
|---|---|---|---|---|---|---|
| A | nominal exchange rate growth | **0.845** | **0.978** | **0.979** | **0.986** | **0.993** |
| | Inflation | **1.369** | 1.131 | 1.191 | 1.285 | 1.273 |
| | real exchange rate growth | **0.830** | **0.979** | **0.973** | **0.984** | 0.999 |
| | GDP growth | **1.006** | 0.999 | 0.999 | 1.001 | 1.001 |
| | interest rate | **1.156** | **1.082** | 1.245 | 1.533 | 1.635 |
| | Mean | **1.041** | **1.034** | 1.077 | 1.158 | 1.180 |
| | Median | **1.006** | **0.999** | 0.999 | 1.001 | 1.001 |
| Aa | nominal exchange rate growth | 0.996 | 0.990 | 0.980 | 0.993 | 0.995 |
| | Inflation | 1.397 | 1.301 | 1.245 | 1.246 | 1.212 |
| | real exchange rate growth | 0.967 | 1.000 | 0.984 | 0.996 | 1.003 |
| | GDP growth | 1.009 | **0.991** | 1.000 | 1.000 | **1.000** |
| | interest rate | 2.110 | 1.184 | **1.046** | **1.002** | **0.990** |
| | Mean | 1.296 | 1.093 | 1.051 | 1.048 | 1.040 |
| | Median | 1.009 | 1.000 | 1.000 | 1.000 | 1.000 |
| B | nominal exchange rate growth | 0.982 | 1.003 | 0.985 | 0.988 | 0.996 |
| | Inflation | 1.397 | 1.264 | 1.210 | 1.212 | 1.180 |
| | real exchange rate growth | 0.924 | 1.021 | 0.995 | 0.995 | 1.007 |
| | GDP growth | 1.066 | 0.999 | 1.002 | 1.001 | 1.001 |
| | interest rate | 2.338 | 1.195 | 1.054 | 1.011 | 1.000 |
| | Mean | 1.341 | 1.096 | 1.049 | 1.041 | 1.037 |
| | Median | 1.066 | 1.021 | 1.002 | 1.001 | 1.001 |
| C | nominal exchange rate growth | 1.240 | 1.002 | 0.991 | 0.990 | 0.994 |
| | Inflation | 1.493 | **1.122** | **1.030** | **1.012** | **0.993** |
| | real exchange rate growth | 1.156 | 1.013 | 0.994 | 0.993 | **0.994** |
| | GDP growth | 1.069 | 1.022 | **0.998** | **1.000** | 1.001 |
| | interest rate | 1.249 | 1.436 | 1.194 | 1.096 | 1.180 |
| | Mean | 1.241 | 1.119 | **1.042** | **1.018** | **1.032** |
| | Median | 1.240 | 1.022 | **0.998** | **1.000** | **0.994** |

It can be seen that version a is the best in short-term forecasting, while version c performs better in the long-term. However, the interest rate forecasting of the model with optimal policy (version c) is weak, potentially due to large errors in some periods.

Table 4 shows the MAE (Median-Absolute-Error) of the DSGE model relative to an AR model (in-sample) for different forecasting horizons. The best values are in bold. MAE is less sensitive to outliers than RMSE.

Table 4.MAE DSGE/MAE AR for posterior mode (in-sample)

| Version | Forecasting horizon | 1 | 2 | 4 | 8 | 12 |
|---|---|---|---|---|---|---|
| A | nominal exchange rate growth | 1.032 | **0.786** | 0.955 | **0.864** | 0.893 |
| | Inflation | 1.410 | 0.962 | 0.765 | 0.771 | 0.708 |
| | real exchange rate growth | 0.999 | **0.660** | 0.930 | **1.032** | **1.074** |
| | GDP growth | **1.460** | 2.051 | 1.223 | **0.903** | **0.925** |
| | interest rate | **0.983** | **1.330** | **1.480** | 1.305 | 1.127 |
| | Mean | **1.177** | **1.158** | **1.071** | **0.975** | 0.946 |
| | Median | **1.032** | **0.962** | **0.955** | **0.903** | **0.925** |
| Aa | nominal exchange rate growth | **0.955** | 0.987 | 0.899 | 0.922 | **0.854** |
| | Inflation | **1.114** | 0.849 | 0.712 | 0.725 | **0.627** |
| | real exchange rate growth | **0.956** | 0.962 | 1.063 | 1.182 | 1.101 |
| | GDP growth | 2.344 | 1.347 | 1.057 | 1.033 | 0.984 |
| | interest rate | 4.424 | 2.118 | 1.792 | 1.103 | **1.032** |
| | Mean | 1.959 | 1.253 | 1.105 | 0.993 | **0.920** |
| | Median | 1.114 | 0.987 | 1.057 | 1.033 | 0.984 |
| B | nominal exchange rate growth | 1.119 | 1.202 | 0.975 | 1.000 | 0.916 |
| | Inflation | 1.208 | **0.776** | **0.676** | **0.688** | 0.656 |
| | real exchange rate growth | 1.009 | 1.183 | 1.126 | 1.117 | 1.112 |
| | GDP growth | 2.065 | **1.177** | **0.976** | 1.073 | 1.040 |
| | interest rate | 4.733 | 1.715 | 1.680 | **1.092** | 1.042 |
| | Mean | 2.027 | 1.211 | 1.086 | 0.994 | 0.953 |
| | Median | 1.208 | 1.183 | 0.976 | 1.073 | 1.040 |
| C | nominal exchange rate growth | 1.889 | 0.921 | **0.873** | 0.920 | 0.914 |
| | Inflation | 1.944 | 1.136 | 0.990 | 0.902 | 0.942 |
| | real exchange rate growth | 1.612 | 0.891 | **0.872** | 1.119 | 1.084 |
| | GDP growth | 2.267 | 2.091 | 1.324 | 1.361 | 1.450 |
| | interest rate | 1.867 | 2.798 | 2.939 | 1.564 | 1.494 |
| | Mean | 1.916 | 1.567 | 1.400 | 1.173 | 1.177 |
| | Median | 1.889 | 1.136 | 0.990 | 1.119 | 1.084 |

Using MAE, version a has a significant advantage, producing better point forecasts than the AR(1) model. Version b becomes the leader in inflation forecasting, indicating that the optimal policy version's advantage is related to outlier periods.

Table 5 provides the log-predictive-score (LPS) of forecasting for different horizons. This measure is the mean log density of the normal distribution, with mean and variance corresponding to the moments of the forecast. The best values are in bold.

The LPS indicates that the conventional Taylor-type rule (version a) is best for short-term forecasting. Version aa is the best for long-term forecasting, with optimal policy (version c) excelling in long-term inflation forecasting.

Table 5. LPS DSGE for posterior mode (in-sample)

| Version | Forecasting horizon | 1 | 2 | 4 | 8 | 12 |
|---|---|---|---|---|---|---|
| A | nominal exchange rate growth | **1.191** | 0.919 | 0.897 | 0.850 | 0.849 |
| | Inflation | **2.895** | 2.752 | 2.559 | 2.316 | 2.212 |
| | real exchange rate growth | **1.220** | 0.956 | 0.926 | 0.862 | 0.839 |
| | GDP growth | **2.526** | 2.396 | 2.176 | 1.859 | 1.717 |
| | interest rate | **-1.980** | **-2.382** | -2.570 | -2.814 | -2.926 |
| | all variables | **8.248** | **7.386** | **6.861** | 6.632 | 6.408 |
| Aa | nominal exchange rate growth | 0.944 | **0.945** | **0.931** | **0.871** | **0.879** |
| | Inflation | 2.749 | 2.741 | 2.734 | 2.694 | 2.718 |
| | real exchange rate growth | 1.018 | **0.972** | **0.964** | **0.907** | **0.905** |
| | GDP growth | 2.498 | **2.416** | **2.394** | **2.354** | **2.307** |
| | interest rate | -2.591 | -2.520 | **-2.538** | **-2.568** | **-2.582** |
| | all variables | 6.830 | 6.898 | 6.856 | **6.812** | **6.794** |
| B | nominal exchange rate growth | 0.969 | 0.917 | 0.911 | 0.867 | 0.868 |
| | Inflation | 2.720 | 2.726 | 2.721 | 2.684 | 2.703 |
| | real exchange rate growth | 1.067 | 0.942 | 0.941 | 0.900 | 0.895 |
| | GDP growth | 2.438 | 2.354 | 2.339 | 2.307 | 2.255 |
| | interest rate | -2.834 | -2.771 | -2.791 | -2.824 | -2.841 |
| | all variables | 6.555 | 6.558 | 6.538 | 6.502 | 6.487 |
| C | nominal exchange rate growth | 0.707 | 0.818 | 0.814 | 0.793 | 0.803 |
| | Inflation | 2.570 | **2.786** | **2.807** | **2.787** | **2.753** |
| | real exchange rate growth | 0.826 | 0.864 | 0.863 | 0.845 | 0.856 |
| | GDP growth | 2.393 | 2.199 | 2.142 | 2.036 | 2.003 |
| | interest rate | -2.241 | -2.750 | -2.807 | -2.866 | -2.949 |
| | all variables | 7.203 | 6.470 | 6.311 | 6.286 | 6.271 |

Different measures produce different pictures of model fit. All versions agree that the conventional Taylor-type rule (version a) is the best for short-term forecasting, explaining its widespread use. However, other versions have advantages, particularly in long-term forecasting.

***Anchoring***

Regressions are a common method for identifying inflation expectation anchoring, but the typical sample length can make this challenging, especially with the complexities in identifying expectations accurately. Using the estimated models, we simulate an extended dataset of 1000 periods to address two key topics: Taylor rule regressions and the anchoring of inflation expectations.

First, we examine how the Taylor rule appears when analyzed using OLS technique on the simulated data, with results presented in Table 6. The data is generated from a pruned squared approximation, which includes inflation expectations. Versions a and aa, which incorporate the Taylor rule, show some differences in implied coefficients despite the presence of observed expected inflation in the simulated data. These differences are substantial, exceeding the standard deviation of the regression.

Table 6. Regression of usual Taylor rule for different simulated data

| | Version a | | Version aa | | Version b | | Version c | |
|---|---|---|---|---|---|---|---|---|
| | Est. | t-ratio | Est. | t-ratio | Est. | t-ratio | Est. | t-ratio |
| r(-1) | 0.635 | 194.192 | 0.685 | 118.453 | -0.149 | -6.372 | 0.961 | 63.466 |
| P_exp(+1) | 0.489 | 155.015 | 436.641 | 194.740 | 3.060 | 21.157 | -1.166 | -35.213 |
| Y | 0.002 | 1.461 | 0.001 | 0.339 | 0.287 | 21.183 | -0.005 | -0.607 |
| C | -0.008 | -1.275 | -3.159 | -198.532 | -0.600 | -21.263 | 0.038 | 1.329 |
| R2 | 0.995 | | 0.976 | | 0.465 | | 0.820 | |

Version b shows a significantly lower R² and suggests an unconventional smoothing coefficient. Version c is particularly notable for implying a "price puzzle" in a pronounced form, a phenomenon sometimes observed in Russia [Pestova et al. (2019)]. It also indicates higher persistence, than in other studies [Kreptsev and Seleznev (2018); Ivashchenko (2022)]. Thus, an optimal policy under commitment could generate effects that are observable yet difficult to explain with the Taylor rule alone.

Next, we evaluate the anchoring of inflation expectations by regressing future inflation expectations ($E_t p_{EXP,t+1}$) on current inflation ($p_t$) or current smoothed inflation expectations ($p_{EXP,t}$). The results are shown in Table 7. Full anchoring of inflation expectations is observed only in version b, which features hawkish targeting of inflation expectations. Despite the long sample length (1000 quarters), the anchoring hypothesis is not far from the significance level. Versions a and c strongly reject the hypothesis, indicating that perfect anchoring of inflation expectations is specific to certain monetary policies. Even with longer inflation expectations, the correlation with current inflation is smaller but remains nonzero.

Table 7. Regression of inflation expectation ($E_t p_{EXP,t+1}$) for different simulated data

| | A | | Aa | | B | | C | |
|---|---|---|---|---|---|---|---|---|
| | Est. | t-ratio | Est. | t-ratio | Est. | t-ratio | Est. | t-ratio |
| $p_{EXP}$ | 0.761 | 123.026 | 0.000 | 2.382 | 0.006 | 1.727 | 0.230 | 8.477 |
| C | 0.001 | 6.069 | 0.007 | 8256.892 | 0.008 | 107.226 | 0.013 | 21.858 |
| R2 | 0.938 | | 0.006 | | 0.006 | | 0.067 | |
| P | 0.492 | 61.344 | 0.000 | 2.357 | 0.000 | 1.723 | -0.096 | -9.046 |
| C | 0.002 | 6.240 | 0.007 | 9120.228 | 0.007 | 107.243 | 0.019 | 43.157 |

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| R2 | 0.000 | | 0.006 | | 0.006 | | 0.076 | |

Thus, Taylor-type rule regressions shows: Versions a and aa closely align with Taylor rules of models. Version b shows a much lower $R^2$ and unconventional smoothing, while version c suggests a "price puzzle" and higher persistence. Only version b shows full anchoring of inflation expectations. Versions a and c reject the anchoring hypothesis, suggesting that perfect anchoring is specific to certain monetary policies. These findings illustrate the varied impacts of different monetary policy rules on inflation expectation anchoring and the behavior of Taylor rule coefficients.

***IRFs***

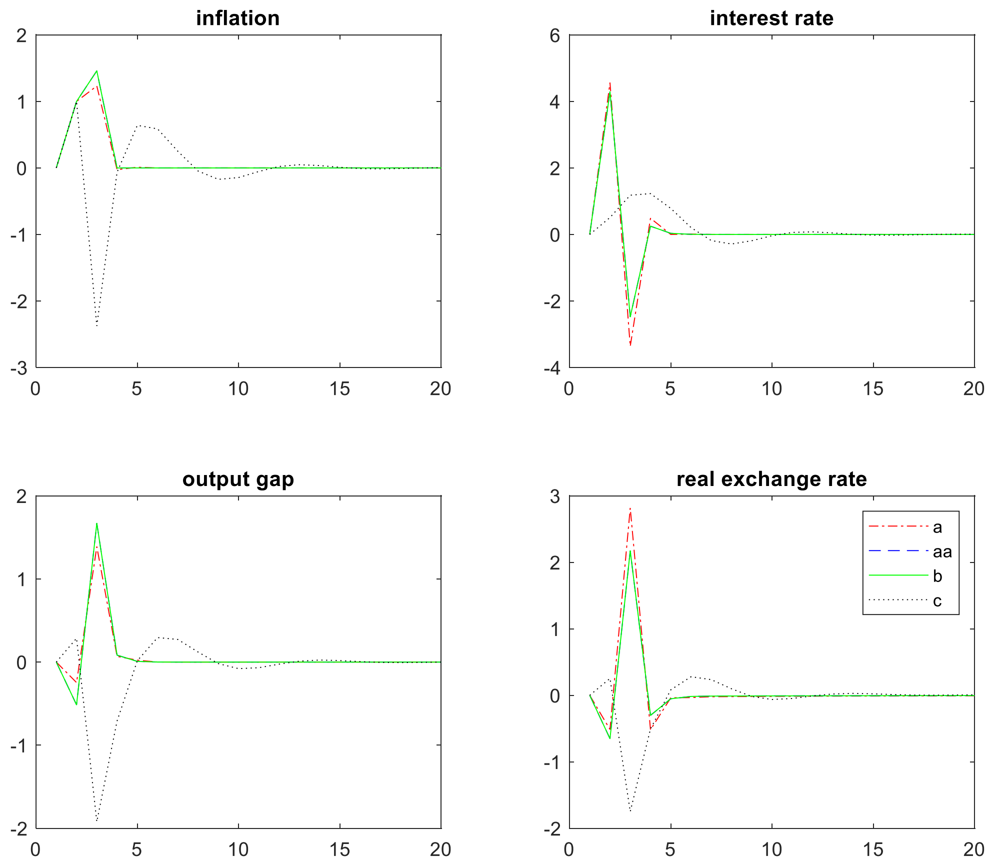
The nonlinear models produce IRFs that are state-dependent and shocks magnitude-dependent. This leads to differences between alternative IRFs definitions. The one used here is from [Benchimol and Ivashchenko (2021)]. Formula (25) defines the IRF of variable v at period t with parameters θ on shock i (which occurs at period 1) with a magnitude of s standard deviations. It is assumed that there are 100,000 random draws of variable trajectories. The IRF is the difference between a random trajectory and a trajectory with the same shocks except for one period (when it is increased by a fixed number of standard deviations). This difference is normalized by the magnitude value. The model starts from steady-state, but the first 100 periods are pre-sample (the shock of interest happens after that).

$$IRF(v,i,s,t,\theta)=\begin{pmatrix} E\left(x_v \mid \varepsilon_{1,i} \sim N\left(s\sigma(\varepsilon_i),\sigma(\varepsilon_i)\right);\theta\right) \\ -E\left(x_v \mid \varepsilon_{1,i} \sim N\left(0,\sigma(\varepsilon_i)\right);\theta\right) \end{pmatrix} / s \tag{25}$$

The different monetary policies produce quite different impulse responses. These differences are easier to understand for monetary policy shocks. Figure 1 presents normalized impulse responses for monetary policy shocks. An MP shock in each model produces inflation growth up to 1 (due to normalization, shock magnitude is 1 std.) in the first period. Versions a, aa, and b produce almost the same responses. All versions produce a similar response in inflation, with some differences in the responses of other variables. Temporary easing of monetary policy leads to expectations of its return, which increases inflation expectations and interest rates. Households temporarily decrease their assets to increase consumption and leisure, leading to higher costs and prices. However, one period later, inflation expectations drop, interest rates become much lower, and the situation reverses. The optimal policy version produces a much smaller response in interest rates. This demonstrates that a complicated monetary policy with full trust (implied by rational expectations) can achieve results without significant real actions, just by influencing expectations.

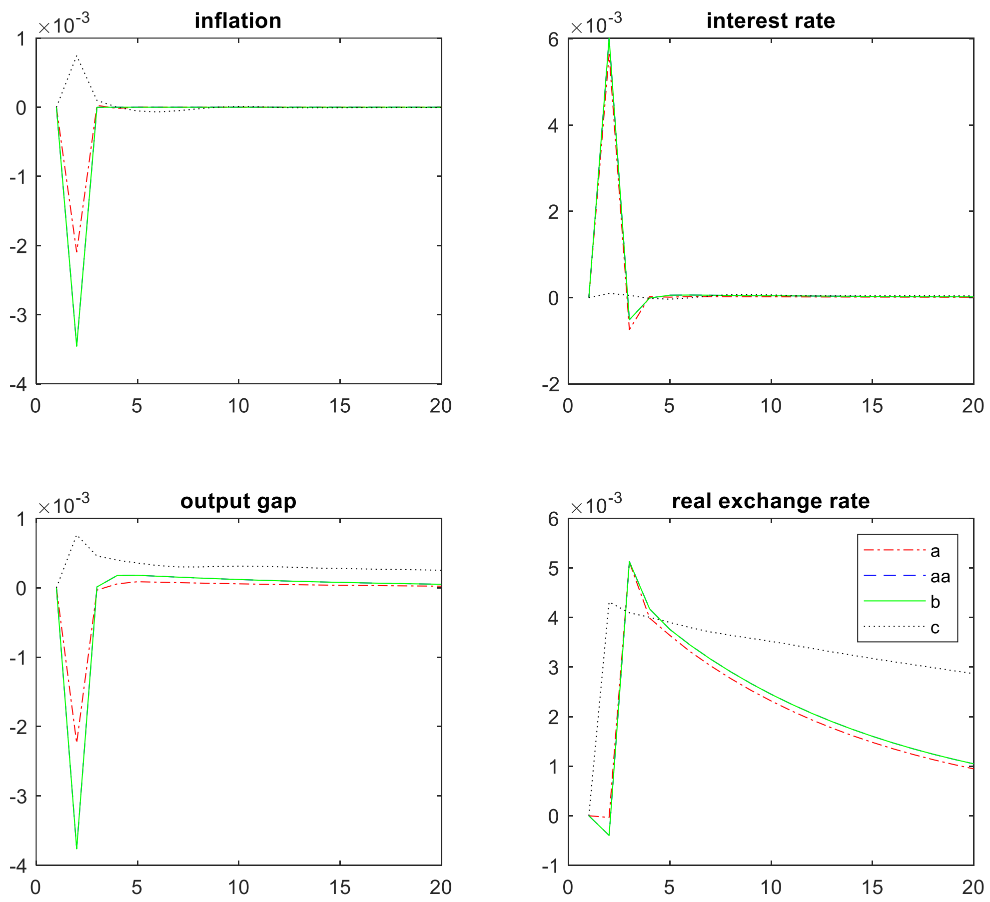
The same sign of inflation and interest rate changes after a monetary policy shock is called a “price puzzle.” This can happen due to different mechanisms within DSGE models [Ali and Anwar (2018); Henzel et al. (2009)]. Many authors choose priors that prevent such a situation, but this may lead to a worse fit for the model. Moreover, it is predictable that version aa should produce a price puzzle. A large coefficient with inflation makes it almost impossible for inflation and interest rates to move in opposite directions. Thus, monetary policy easing would lead to growth in both inflation and interest rates. Therefore, it was decided that special priors to prevent the price puzzle are not needed for other versions of the model.

**Figure 1. Normalized(to inflation growth) IRFs for monetary policy shock (1std)**



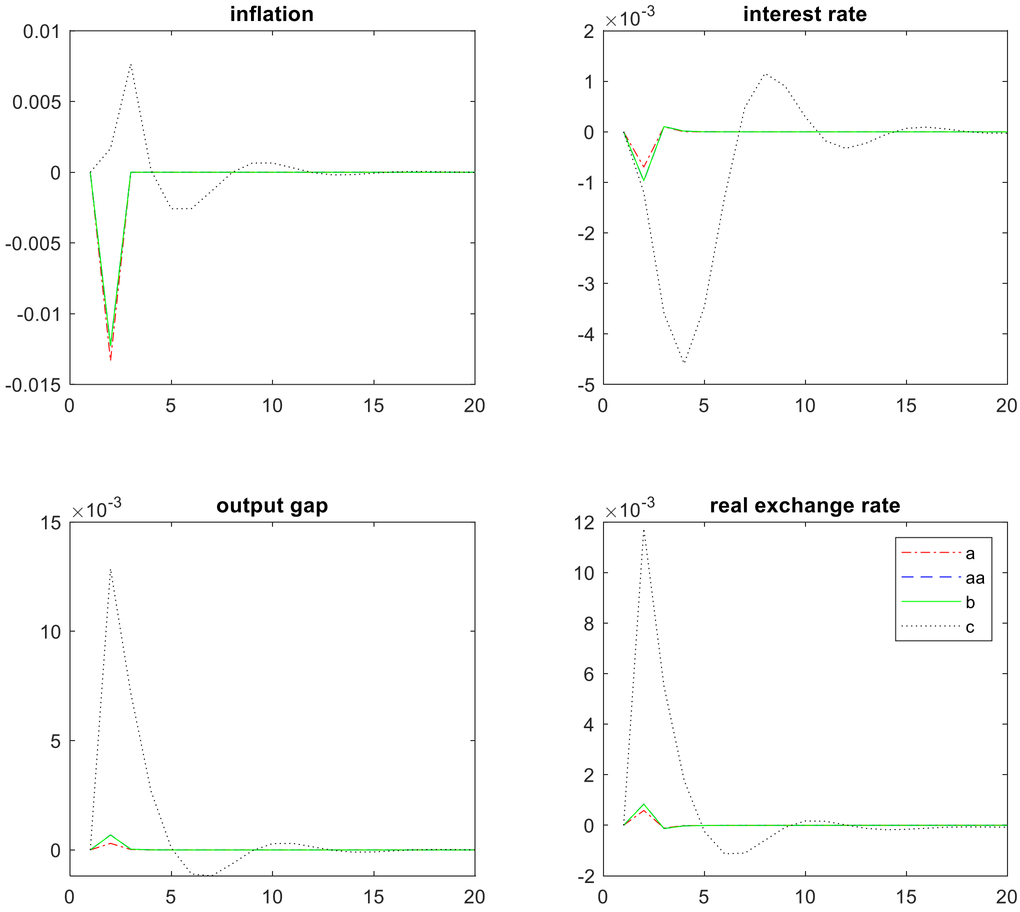

TFP shocks produce a very interesting picture depending on the type of TFP shock. Figure 2 demonstrates the consequences of a trend TFP shock. Figure 3 shows the consequences of a stationary TFP shock. Versions a, aa, and b produce similar responses for stationary TFP shocks (slightly smaller magnitude for version a). It suggests that productivity growth leads to higher production and export, making the national currency more expensive. Interest rates decline due to near-zero changes in inflation expectations and the output gap ($\gamma_{ry}$ is positive for versions a and aa). The inflation drop is limited to a single period. Optimal policy produces a much larger and more persistent decline in interest rates, and this overreaction causes inflation to grow.

**Figure 2. IRFs for TFP trend shock (1 std.)**



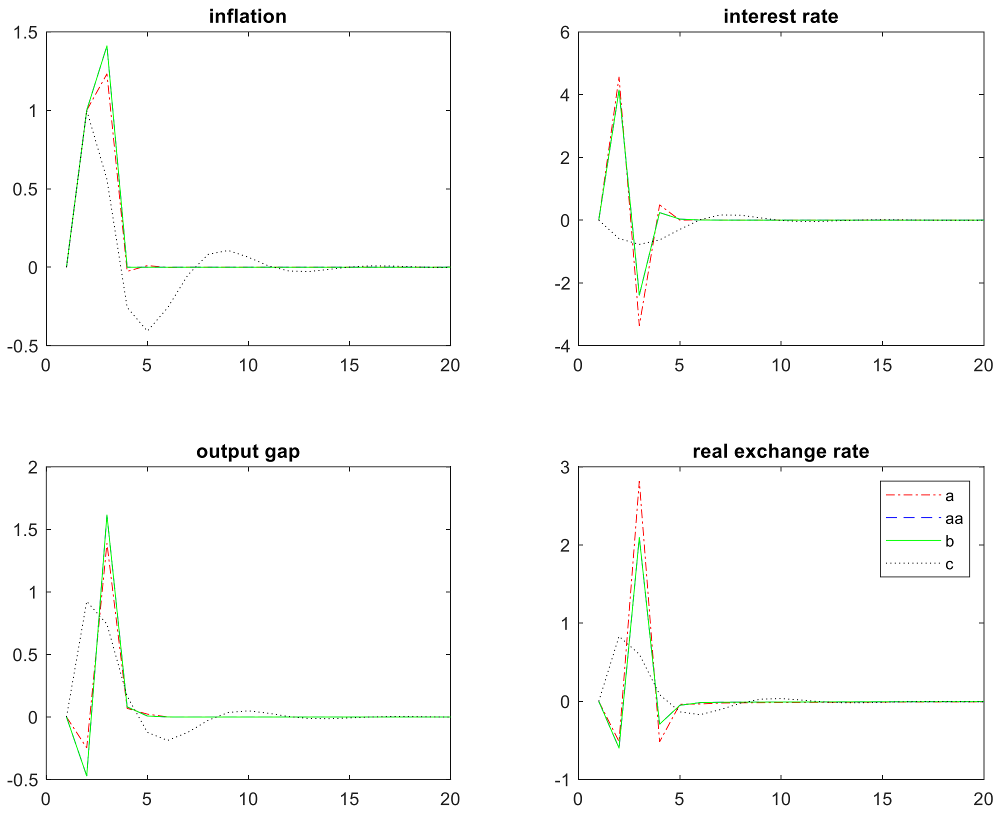

A permanent shock (trend shock of TFP) produces quite different reactions. Versions a, aa, and b are similar. They suggest a decline in inflation and the output gap due to higher production efficiency. The output gap drops due to the growth of potential output. Inflation expectations remain almost unchanged. However, interest rates grow due to the output gap. The national currency becomes more expensive for a long time. The optimal policy suggests almost zero reaction of interest rates to a TFP trend shock, which leads to growth in both inflation and output. This happens due to additional demand for cheaper goods, increasing the optimal level of production and thus higher marginal costs. Additionally, household assets drop below the steady-state, requiring transfers from fiscal authorities, stimulating further consumption while assets drift back to the steady-state. The optimal policy's reaction to a stationary TFP shock is much stronger, causing inflation to grow instead of decline. Thus, optimal policy shows very different responses to different shocks.

**Figure 3. IRFs for stationary TFP shock (1 std.)**




Optimal policy is much more nonlinear. Figure 4 shows normalized consequences of a 5 std. monetary policy shock. Versions a, aa, and b are almost the same as in figure 1. However, optimal policy changes to the opposite. It decreases interest rates, producing smaller fluctuations in inflation and opposite dynamics for the output gap. This makes the implementation of optimal policy much more complicated.

**Figure 4. Normalized IRFs for monetary policy shock (5std)**




### *Additional experiments*

The perspective on monetary policy greatly influences the estimated parameters and dynamics of the model. An open question is the consequences of mixing monetary policy from one version (parameters of equation (9), but not the inflation expectation length) with parameters from another. Is it possible to implement a policy estimated for another economy? What would be the consequences? This highlights how different the effects of the same policies can be in different economies. Such changes (policy switches) affect the Blanchard-Kahn conditions, meaning the model can become explosive or have a non-unique, non-explosive solution. In these cases, the solution with the smallest (in absolute values) eigenvalues is chosen. Table 8 presents the mean value of the central bank penalty function and its standard deviation. The letter "e" denotes an explosive solution, "~~u~~" denotes a non-unique solution, and "r" denotes Ricardian equivalence. If the amount of domestic currency assets owned by households does not influence observed variables (inflation, GDP growth, interest rate, nominal and real exchange rate growth), then Ricardian equivalence holds. Otherwise, "~~nr~~" is used.

Table 8. Penalty function value, (its std), BK-condition and Ricardian equivalence

| | parameters from version a | parameters from version aa | parameters from version b | parameters from version c |
|---|---|---|---|---|
| MP from version a | 3.763 (4.819%),r | e | e | 4.207 (1.858%),r |
| MP from version aa | 3.762 (4.803%)~~u~~,r | 0.485 (1.877%),~~nr~~ | 2.605 (1.617%),~~nr~~ | 4.207 (1.848%)~~u~~,r |
| MP from version b | 3.762 (4.803%)~~u~~,r | 0.486 (1.879%),~~nr~~ | 2.605 (1.615%),~~nr~~ | 4.207 (1.848%)~~u~~,r |
| MP from version c | 6.992 (0.447%),r | 8.419 (0.476%),~~nr~~ | 8.096 (0.481%),~~nr~~ | 4.161 (0.456%),r |

The results reflect several important ideas. Switching from one type of policy to another may be impossible. If the economy corresponds to parameters from versions aa or b (with a fiscal policy that does not care about debts), it is impossible to use conventional monetary policy a, as it leads to an explosive trajectory. The opposite case is unpredictable. If the economy has parameters corresponding to a or c, then using rules aa or b leads to a non-unique stable solution. It may work similarly (in terms of the penalty function) if the most stable solution is chosen by everyone in the economy. However, there are alternative solutions. Moreover, it is a very complicated theoretical question what should happen in an economy with multiple solutions and rational expectations. Thus, the consequences are highly unpredictable.

Optimal policy always works. It does not create explosive or non-unique solutions. However, optimal policy under commitment may be less efficient than following rules. This corresponds to the original view of [Taylor (1993)]. However, it should be noted that these results are from a second-order approximation of the solution, while optimal policy is computed fully nonlinear (corresponding first-order conditions are used as equations). This means that the disadvantage of optimal policy may be an error of approximation. An additional detail is that optimal policy produces a much less volatile value of the penalty function and yields the best results for parameters from version c.

The Ricardian equivalence is a very interesting detail. The model is constructed so that it should hold. However, parameters from versions aa and b lead to a break of it (independent of monetary policy). This highlights how fragile this property is. Therefore, it should not be assumed without strong proof for a particular case.

***Robustness check***

The estimation period (2011Q1 to 2022Q4) includes outliers (such as COVID-19) and potential structural breaks (such as the change in the monetary policy rule in 2014). These factors may influence the estimation results. A robustness check was performed. All models were re-estimated on a shorter dataset: 2015Q1–2019Q4. This stable period is only 20 quarters, which is less than half of the overall sample.

The posterior changes significantly. However, the version with optimal policy remains the best according to marginal likelihood (its advantage becomes much larger). Version *a* gives a log-marginal likelihood (log-ML) of -93.282 and a posterior mode of 105.491. Version *aa* gives a log-ML of -123.803 and a posterior mode of 91.593. Version *b* gives a log-ML of -94.267 and a posterior mode of 98.523. Version *c* gives a log-ML of 522.052 and a posterior mode of 773.156. Thus, version *c* with optimal policy provides the best fit, while version *aa* performs the worst.

The estimated parameter values also differ significantly for the short sample (see Appendix, Table A1). These estimates differ substantially from those based on the full sample. However, what is more important for the conclusions is that the parameter estimates differ across versions. A similar pattern is observed for fiscal policy: hawkish monetary policy is combined with fiscal policy that disregards debt. A comparable pattern of large differences across many parameters is maintained (e.g., μL and μM, η0,tr, and η0,tr,W). There are also some changes in the "duration of inflation." Optimal policy focuses on longer-term inflation in the short sample estimation. However, the duration of inflation targets does not change dramatically for other policies.

Forecasting abilities also change (see Appendix, Tables A2–A4). The short-term forecasting advantage of the conventional Taylor rule disappears for most measures (the all-variable LPS is an exception). Version *aa* becomes one of the leading models in long-term forecasting. Optimal policy (version *c*) becomes the leading approach overall but loses its advantage in inflation forecasting. Thus, different measures present varied pictures of model fit in the shorter sample case.

Attempts to implement monetary policy estimated under one regime in an economy estimated with a different monetary policy often break the BK conditions (see Appendix, Table A5). The shorter sample leads to additional explosive and non-unique solutions. The largest eigenvalue (of the dynamic matrix) for explosive solutions is 1.01 (MP *a*, economy *b*, and economy *aa*). In this case, it was possible to compute a simulation of the trajectory from the steady-state for 1,000 periods. The combination of MP *c* and economy *aa* has an eigenvalue close to 1. The non-unique solution could not be successfully computed for the short sample using the same approach as for the full sample.

The fragility of Ricardian equivalence remains robust. The finding that optimal policy works in every case is almost robust to the sample (the explosiveness value of $1+8\times10^{-8}$ is below the usual threshold). Optimal policy under commitment may still be less efficient than following rules for the short sample. Switching from one type of policy to another may be impossible. Only a few combinations of fiscal policy rule types (considering debt levels or not) and monetary policy types (normal or super-hawkish) are feasible. Thus, the main results are robust despite the high influence of the sample on estimation results.

## 4. CONCLUSIONS

This paper examines the consequences of switching from a conventional Taylor rule to alternative monetary policies using small-scale open economy DSGE models estimated with second-order pruned approximation at steady-state. The models differ in their monetary policy rules: a conventional Taylor-type rule, a very hawkish version of the Taylor-type rule (with a very large coefficient on inflation expectations), an absolutely hawkish policy of "inflation expectation targeting," and an optimal policy under commitment.

The models are compared in terms of fit and in-sample forecasting quality. The optimal policy under commitment achieves the best marginal likelihood (Laplace approximation), while the conventional Taylor rule, although significantly worse, performs much better than the other two policies. However, short-term point and density forecasting with the best parameter values (posterior mode) is superior for the conventional policy. The choice of the leader in long-term forecasting quality depends on the measure used. This indicates that the conventional Taylor-type rule is suitable for describing the Russian economy (especially with narrow priors), but its advantage is fragile (especially with wide priors).

The analysis shows significant differences in the dynamics and parameters of the models. Both the conventional policy and the optimal policy under commitment should lead to the absence of inflation expectation anchoring (when tested in a very simple form). The "price puzzle" may arise when a Taylor-type rule is estimated while using an optimal policy under commitment. Optimal policy under commitment differs from other policies by implying very different reactions to shocks that are hard to distinguish (temporary and permanent productivity shocks). It is much more nonlinear in response, highlighting the complexity of implementing and communicating optimal policy under commitment.

The most interesting results relate to the performance of different rules in economies estimated with other rules. This illustrates fiscal dominance mechanics (the significant influence of fiscal policy on monetary policy). Economies with the two hawkish policies imply that fiscal authorities do not care about government debts/assets. Implementing conventional policy in such an economy leads to an explosive trajectory, while implementing hawkish policies in other economies leads to a non-unique solution. This means that the type of monetary policy depends on the type of fiscal policy. Only optimal policy under commitment works without problems in all economies. However, optimal policy under commitment may be non-optimal. The estimated weights of inflation expectation stability, financial stability, and economic stability are used for each case. The loss function for a simple rule may be better than for optimal policy under commitment due to the potential inefficiency of Nash equilibrium. A simple rule could produce expectations (inflation and others) that lead to better results than optimal actions.

Additionally, the experiments show how fragile Ricardian equivalence is. It may be broken even by the form of the monetary policy rule that does not react to government debts/assets.

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# APPENDIX

Table A1. Estimation results. Posterior mode and standard deviation. 2015q1-2019q4

| | Version a | | Version aa | | Version b | | Version c | |
|---|---|---|---|---|---|---|---|---|
| Parameter | Mode | std | Mode | std | mode | std | Mode | Std |
| stderr $\varepsilon_C$ | 3.96E-03 | 1.99E-03 | 3.57E-03 | 1.42E-03 | 3.85E-03 | 1.90E-03 | 3.58E-03 | 1.46E-03 |
| stderr $\varepsilon_{ex}$ | 4.61E-03 | 2.81E-03 | 3.31E-03 | 1.23E-03 | 1.26E-02 | 9.76E-03 | 4.61E-03 | 2.87E-03 |
| stderr $\varepsilon_{pim}$ | 4.57E-03 | 2.79E-03 | 4.56E-03 | 2.59E-03 | 3.94E-03 | 1.85E-03 | 5.17E-03 | 3.38E-03 |
| stderr $\varepsilon_{pw}$ | 5.27E-03 | 1.27E-03 | 5.27E-03 | 1.21E-03 | 5.26E-03 | 1.21E-03 | 5.26E-03 | 1.24E-03 |
| stderr $\varepsilon_R$ | 3.55E-03 | 1.02E-03 | 4.61E-03 | 2.66E-03 | 2.16E-03 | 1.14E-03 | 4.59E-03 | 2.82E-03 |
| stderr $\varepsilon_{rw}$ | 9.55E-02 | 1.71E-02 | 3.53E-02 | 7.24E-03 | 4.53E-03 | 2.53E-03 | 4.57E-03 | 2.79E-03 |
| stderr $\varepsilon_{\theta,F}$ | 4.61E-03 | 2.81E-03 | 4.61E-03 | 2.66E-03 | 4.61E-03 | 2.66E-03 | 4.60E-03 | 2.86E-03 |
| stderr $\varepsilon_{tr}$ | 4.61E-03 | 2.81E-03 | 4.27E-03 | 2.19E-03 | 4.70E-03 | 2.91E-03 | 4.61E-03 | 2.86E-03 |
| stderr $\varepsilon_{tr,W}$ | 4.61E-03 | 2.81E-03 | 4.25E-03 | 2.18E-03 | 4.01E-03 | 1.86E-03 | 3.36E-03 | 1.37E-03 |
| stderr $\varepsilon$ | 2.74E-03 | 8.96E-04 | 3.04E-03 | 7.32E-04 | 2.61E-03 | 8.18E-04 | 2.84E-03 | 8.93E-04 |
| stderr $\varepsilon_{YF}$ | 4.47E-03 | 2.12E-03 | 2.34E-03 | 5.98E-04 | 5.30E-03 | 2.61E-03 | 3.89E-03 | 1.19E-03 |
| $\alpha_K$ | 6.53E-01 | 1.38E-02 | 4.03E-01 | 4.70E-03 | 5.79E-01 | 3.54E-02 | 6.46E-01 | 4.30E-05 |
| $ln(\beta)$ | -1.42E-03 | 1.51E-05 | -9.95E-03 | 4.18E-05 | -9.95E-03 | 5.57E-05 | -2.82E-03 | 1.93E-03 |
| $\varphi_{PF}$ | 3.95E+00 | 3.84E-01 | 1.25E+00 | 7.60E-01 | 2.98E+00 | 5.99E-01 | -3.02E-01 | 1.80E-01 |
| $\gamma_{exp}$ | 3.94E-01 | 1.33E-01 | 5.33E-01 | 3.34E-01 | 9.99E-01 | 3.01E-01 | 1.88E-02 | 5.83E-03 |
| $\gamma_r$ (version a) | 6.50E-01 | 5.06E-02 | | | | | | |
| $\gamma_{rp}$ (version a) | 1.03E+00 | 2.50E-02 | | | | | | |
| $\gamma_{ry}$ (version a) | 3.44E-02 | 7.63E-02 | | | | | | |
| $\gamma_r$ (version aa) | | | 7.74E-01 | 2.12E-01 | | | | |
| $\gamma_{rp}$ (version aa) | | | 1.50E+03 | 7.16E-01 | | | | |
| $\gamma_{ry}$ (version aa) | | | -7.49E-05 | 1.56E-02 | | | | |
| $\gamma_\beta$ (version c) | | | | | | | -9.95E-03 | 9.29E-06 |
| $\gamma_r$ (version c) | | | | | | | 1.37E-02 | 8.72E-03 |
| $\gamma_y$ (version c) | | | | | | | 1.10E-01 | 4.41E-02 |
| $\gamma_{tr}$ | 8.00E-01 | 2.17E-01 | 8.05E-01 | 2.45E-01 | 7.99E-01 | 2.17E-01 | 8.01E-01 | 2.95E-01 |
| $\gamma_{trA}$ (version a,c) | -9.97E-02 | 1.94E-01 | | | | | -1.00E-01 | 1.09E-01 |
| $\gamma_{trA}$ (version aa,b) | | | -6.98E-11 | 9.11E-08 | -9.93E-07 | 2.18E-06 | | |
| $\gamma_{try}$ | 5.42E-05 | 1.53E-04 | -2.60E-02 | 1.33E-01 | -1.34E-02 | 2.53E-01 | -7.99E-04 | 5.07E-04 |
| $\gamma_{ex,fx}$ | 6.31E-01 | 1.82E-01 | 8.61E-10 | 1.21E-06 | 5.24E-11 | 1.30E-07 | 5.25E-05 | 7.25E-05 |
| $\gamma_{pim,fx}$ | 9.57E-01 | 4.04E-02 | 6.95E-01 | 2.84E-01 | 1.85E-01 | 1.28E-01 | 6.75E-02 | 5.20E-02 |
| $\gamma_{rw,bw}$ | -2.81E-01 | 5.22E-02 | -3.15E-02 | 1.74E-04 | -5.61E-02 | 3.34E-01 | -3.48E-01 | 1.50E-01 |
| $H$ | 9.12E-01 | 8.27E-03 | 8.42E-01 | 2.72E-03 | 7.52E-01 | 3.05E-01 | 6.76E-01 | 1.65E-03 |
| $\mu_L$ | 3.25E+00 | 4.00E-02 | -4.54E+00 | 3.85E-03 | 5.99E-01 | 8.62E-02 | -4.36E+00 | 6.54E-04 |
| $\mu_M$ | 8.08E-04 | 2.22E-03 | 4.57E-01 | 1.09E+00 | 2.76E-01 | 1.09E-01 | 1.94E-03 | 1.90E-03 |
| $\eta_{0,C}$ | 4.61E+00 | 2.85E-02 | 2.34E+00 | 2.83E-01 | -8.04E-01 | 1.09E-01 | 1.53E+00 | 6.90E-04 |
| $\eta_{0,ex}$ | -1.34E+00 | 5.11E-02 | 5.46E-01 | 1.31E-01 | 2.04E-01 | 1.86E-02 | -2.42E+00 | 5.74E-03 |
| $\eta_{0,pim}$ | -4.36E-02 | 4.75E-02 | -1.88E-01 | 1.99E-02 | 1.06E+00 | 3.20E-02 | 1.95E+00 | 3.35E-04 |
| $\eta_{0,pw}$ | 4.94E-03 | 1.95E-03 | 5.23E-03 | 1.79E-03 | 5.03E-03 | 1.80E-03 | 5.03E-03 | 3.72E-03 |
| $\eta_{0,R}$ | 1.46E-02 | 4.30E-03 | 2.16E-02 | 1.47E-03 | 2.16E-02 | 1.19E-03 | 1.94E-02 | 8.97E-04 |
| $\eta_{0,\theta,F}$ | 8.90E+00 | 1.21E-01 | 8.58E+00 | 2.42E-02 | 8.86E+00 | 1.03E+00 | 6.04E+00 | 1.26E-02 |
| $\eta_{0,tr}$ | 4.59E-03 | 8.13E-03 | 4.43E-01 | 3.16E-02 | 1.68E+00 | 4.22E-01 | -2.32E-02 | 2.02E-02 |
| $\eta_{0,tr,W}$ | -1.76E-01 | 7.52E-03 | -6.58E-01 | 1.36E-01 | -2.98E+00 | 8.73E-02 | 5.45E-01 | 2.10E-04 |
| $\eta_{0,trY}$ | 4.34E-03 | 1.03E-03 | 4.78E-03 | 1.09E-03 | 4.37E-03 | 9.14E-04 | 5.02E-03 | 1.65E-03 |

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| $\eta_{0,YF}$ | 3.77E+00 | 1.16E-03 | 5.00E-01 | 7.35E-02 | 8.83E-01 | 6.86E-02 | -1.88E+00 | 1.07E-04 |
| $\omega_C$ | 1.69E+00 | 2.51E-02 | 1.52E+00 | 1.03E-01 | 1.55E+00 | 3.57E-01 | 1.49E+00 | 1.28E-03 |
| $\omega_L$ | 1.44E+00 | 2.77E-02 | 1.79E+00 | 6.83E-02 | 1.55E+00 | 3.64E-01 | 1.52E+00 | 6.03E-04 |
| $\omega_M$ | 1.50E+00 | 2.13E-01 | 1.53E+00 | 2.06E-01 | 1.55E+00 | 2.15E-01 | 1.50E+00 | 2.13E-01 |
| *Tax* | 4.05E-01 | 1.20E-02 | 4.63E-01 | 7.10E-02 | 4.11E-01 | 3.42E-02 | 4.00E-01 | 5.13E-04 |
| $\theta_C$ | 9.14E+00 | 4.48E-02 | 4.93E+00 | 8.15E-03 | 6.91E+00 | 9.76E-01 | 9.60E+00 | 1.04E-01 |
| $w_C$ | 5.19E-01 | 4.75E-03 | 4.36E-01 | 2.30E-02 | 8.98E-01 | 1.34E-02 | 6.76E-01 | 1.75E-03 |
| *b_WH_SS* | 3.04E-01 | 4.94E-02 | 2.88E-01 | 6.95E-03 | 1.51E-01 | 7.70E-01 | -1.23E+00 | 6.91E-01 |
| *c_H_SS* | 3.61E+00 | 4.41E-03 | 1.14E+00 | 4.34E-02 | -3.42E-01 | 8.87E-03 | -1.05E+00 | 9.56E-05 |

Table A2. RMSE DSGE/RMSE AR for posterior mode (in-sample). 2015q1-2019q4

| Version | Forecasting horizon | 1 | 2 | 4 | 8 | 12 |
|---|---|---|---|---|---|---|
| A | nominal exchange rate growth | 0.9872 | 1.0991 | 1.1306 | 1.2391 | **0.5793** |
| | Inflation | 1.7975 | 1.8534 | 1.8492 | 1.2633 | 1.1360 |
| | real exchange rate growth | 0.9796 | 1.1792 | 1.2178 | 1.3251 | **0.6134** |
| | GDP growth | 7.5983 | 1.9442 | 0.9157 | 1.1020 | 0.7467 |
| | interest rate | 1.2726 | 1.2480 | **1.2318** | **1.2115** | **1.3333** |
| | Mean | 2.5270 | 1.4648 | 1.2690 | 1.2282 | **0.8817** |
| | Median | 1.2726 | 1.2480 | 1.2178 | 1.2391 | **0.7467** |
| Aa | nominal exchange rate growth | **0.8601** | 0.9686 | **0.9713** | 1.0043 | 0.9932 |
| | Inflation | 1.0870 | 0.9651 | 0.8927 | 1.2131 | 0.9158 |
| | real exchange rate growth | **0.8616** | 0.9858 | **0.9683** | 0.9724 | 1.2936 |
| | GDP growth | **3.8701** | 1.3772 | 0.9125 | 0.9131 | 0.4994 |
| | interest rate | 4.6433 | 2.6524 | 2.2701 | 3.8789 | 4.0990 |
| | Mean | 2.2644 | 1.3898 | 1.2030 | 1.5964 | 1.5602 |
| | Median | 1.0870 | 0.9858 | **0.9683** | **1.0043** | 0.9932 |
| B | nominal exchange rate growth | 0.9316 | **0.8700** | 0.9743 | **0.9953** | 1.0712 |
| | Inflation | **1.0526** | **0.9360** | **0.8726** | 1.1472 | **0.8856** |
| | real exchange rate growth | 0.8808 | **0.8704** | 0.9709 | **0.9667** | 1.3831 |
| | GDP growth | 5.1026 | 1.8760 | 1.2414 | 1.2112 | 0.8487 |
| | interest rate | 5.0040 | 2.6192 | 2.2269 | 3.8018 | 4.0214 |
| | Mean | 2.5944 | 1.4343 | 1.2572 | 1.6244 | 1.6420 |
| | Median | **1.0526** | **0.9360** | 0.9743 | 1.1472 | 1.0712 |
| C | nominal exchange rate growth | 1.1216 | 1.0201 | 0.9997 | 1.0139 | 1.2310 |
| | Inflation | 1.4844 | 1.6668 | 1.2477 | **1.1050** | 1.1691 |
| | real exchange rate growth | 1.0195 | 0.9805 | 0.9767 | 0.9668 | 1.3832 |
| | GDP growth | 4.6493 | **1.1371** | **0.7807** | **0.7888** | **0.4376** |
| | interest rate | **0.9630** | **1.1585** | 1.4332 | 1.2564 | 1.8837 |
| | Mean | **1.8476** | **1.1926** | **1.0876** | **1.0262** | 1.2209 |
| | Median | 1.1216 | 1.1371 | 0.9997 | 1.0139 | 1.2310 |

Table A3. MAE DSGE/MAE AR for posterior mode (in-sample). 2015q1-2019q4

| Version | Forecasting horizon | 1 | 2 | 4 | 8 | 12 |
|---|---|---|---|---|---|---|
| A | nominal exchange rate growth | **0.5549** | 2.3528 | 2.0499 | 1.5027 | **0.4737** |
| | Inflation | 1.6054 | 1.8733 | 2.1032 | 0.9793 | 1.0443 |
| | real exchange rate growth | **0.7774** | 1.5539 | 1.9024 | 1.2326 | **0.7884** |
| | GDP growth | 5.2120 | 1.9588 | 0.7448 | 1.0967 | 1.5063 |
| | interest rate | 1.2298 | **1.0337** | 1.4029 | 1.1965 | **1.9296** |
| | Mean | 1.8759 | 1.7545 | 1.6406 | 1.2016 | **1.1485** |
| | Median | 1.2298 | 1.8733 | 1.9024 | 1.1965 | 1.0443 |
| Aa | nominal exchange rate growth | 0.9034 | **1.2747** | **1.0338** | 1.0035 | 0.9946 |
| | Inflation | **0.5920** | **0.4757** | **0.4407** | **0.7503** | **0.3425** |
| | real exchange rate growth | 1.0552 | 0.9497 | **0.7499** | 0.8602 | 1.5377 |
| | GDP growth | 5.9456 | 1.2154 | 0.5850 | 0.7036 | 0.9055 |
| | interest rate | 6.1024 | 2.9439 | 2.7225 | 4.6343 | 6.6253 |
| | Mean | 2.9197 | 1.3719 | 1.1064 | 1.5904 | 2.0811 |
| | Median | 1.0552 | **1.2154** | **0.7499** | 0.8602 | **0.9946** |
| B | nominal exchange rate growth | 0.9270 | 1.4107 | 1.0511 | **0.9969** | 1.1192 |
| | Inflation | 0.9812 | 0.5865 | 0.5463 | 0.7503 | 0.4491 |
| | real exchange rate growth | 0.8473 | **0.8047** | 0.7668 | **0.8583** | 1.7046 |
| | GDP growth | 7.5190 | 2.4037 | 0.9721 | 1.2525 | 1.7276 |
| | interest rate | 6.6043 | 3.0014 | 2.6528 | 4.5437 | 6.4830 |
| | Mean | 3.3758 | 1.6414 | 1.1978 | 1.6803 | 2.2967 |
| | Median | **0.9812** | 1.4107 | 0.9721 | 0.9969 | 1.7046 |
| C | nominal exchange rate growth | 1.0439 | 1.8004 | 1.3315 | 1.0438 | 1.3845 |
| | Inflation | 1.2980 | 1.3514 | 1.3158 | 0.9820 | 1.1435 |
| | real exchange rate growth | 1.0620 | 0.9193 | 0.7787 | 0.8586 | 1.7049 |
| | GDP growth | **3.7632** | **0.8166** | **0.4227** | **0.5347** | **0.4483** |
| | interest rate | **0.4992** | 1.2290 | **0.7910** | **0.5188** | 2.2617 |
| | Mean | **1.5332** | **1.2233** | **0.9279** | **0.7876** | 1.3886 |
| | Median | 1.0620 | 1.2290 | 0.7910 | **0.8586** | 1.3845 |

Table A4. LPS DSGE for posterior mode (in-sample). 2015q1-2019q4

| Version | Forecasting horizon | 1 | 2 | 4 | 8 | 12 |
|---|---|---|---|---|---|---|
| A | nominal exchange rate growth | 1.4084 | 1.3589 | 1.3541 | 1.3904 | 1.4373 |
| | Inflation | 3.3871 | 3.2968 | 3.2406 | 3.1223 | 2.9562 |
| | real exchange rate growth | 1.3490 | 1.2962 | 1.2988 | 1.3663 | 1.4343 |
| | GDP growth | 3.8175 | 4.0145 | 3.6964 | 3.3521 | 3.2031 |
| | interest rate | -0.7279 | -1.1144 | **-1.3111** | -1.4607 | -1.7832 |
| | all variables | **12.4145** | 11.7148 | 10.8284 | 10.5907 | 9.7234 |
| Aa | nominal exchange rate growth | **1.6986** | **1.8069** | **1.8522** | **1.9277** | **1.9553** |
| | Inflation | **3.9715** | **3.9829** | **3.9951** | **3.9459** | **4.0200** |
| | real exchange rate growth | **1.6675** | **1.7814** | **1.8402** | **1.9311** | **1.9626** |
| | GDP growth | **4.5852** | **4.6932** | **4.7019** | **4.6969** | **4.7315** |
| | interest rate | -2.0813 | -2.0186 | -2.0315 | -2.1223 | -2.1227 |
| | all variables | 12.0862 | **12.3320** | **12.2823** | **12.1719** | **11.9474** |
| B | nominal exchange rate growth | 1.5608 | 1.5409 | 1.5205 | 1.5799 | 1.6005 |
| | Inflation | 3.8051 | 3.7974 | 3.8020 | 3.7841 | 3.8182 |
| | real exchange rate growth | 1.5930 | 1.5576 | 1.5337 | 1.6048 | 1.6270 |
| | GDP growth | 4.3885 | 4.4833 | 4.4589 | 4.4555 | 4.4881 |
| | interest rate | -2.0852 | -2.0694 | -2.0775 | -2.1487 | -2.1494 |
| | all variables | 11.8381 | 11.7432 | 11.5489 | 11.4862 | 11.1388 |
| C | nominal exchange rate growth | 1.3320 | 1.2155 | 1.2256 | 1.2575 | 1.2621 |
| | Inflation | 3.5597 | 3.3351 | 3.3635 | 3.3948 | 3.3744 |
| | real exchange rate growth | 1.3984 | 1.2519 | 1.2560 | 1.2928 | 1.3039 |
| | GDP growth | 4.2109 | 4.2214 | 4.2220 | 4.2205 | 4.2292 |
| | interest rate | **-0.5868** | **-1.0618** | -1.4050 | **-1.1851** | **-1.3500** |
| | all variables | 12.4014 | 11.5102 | 11.1684 | 11.4417 | 10.9196 |

Table A5. Penalty function value, (its std), BK-condition and Ricardian equivalence. 2015q1-2019q4

| | parameters from version a | parameters from version aa | parameters from version b | parameters from version c |
|---|---|---|---|---|
| MP from version a | 3.77<br>(0.45%),r | 0.5<br>(0.23%),e,r | 0.88<br>(0.53%),e,r | -8.06E+40<br>(8.92E+40)~~u~~,r |
| MP from version aa | ~~u~~ | 0.5<br>(0.23%),~~nr~~ | 0.88<br>(0.53%),~~nr~~ | ~~u~~ |
| MP from version b | ~~u~~ | 0.5<br>(0.23%),~~nr~~ | 0.88<br>(0.53%),~~nr~~ | ~~u~~ |
| MP from version c | 8.9<br>(0.45%),r | 8.58<br>(0.48%),e,~~nr~~ | 8.86<br>(0.48%),~~nr~~ | 6.04<br>(0.46%),r |